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A History of Ancient Mathematical Astronomy

Part Three

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Book VI

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Figures and Plates

A. Chronological Concepts

In astronomy, we are concerned, not with defining time, but only with measuring it.

Explan. Suppl. A.E., p. 68

§ 1. Years and Julian Days

The most essential requirement for any measuring unit is its constancy. In historical chronology, however, this condition is satisfied only in three cases: by the Egyptian years of 365 days, by the days themselves, counted consecutively as "julian days" beginning with day $0 = -4712$ Jan. 1, and finally by the seven-day week.

The Egyptian years, although not invented for astronomical purposes, were the favoured time unit for the astronomers of antiquity and the Middle Ages, down to Copernicus and beyond,[1] and long after the replacement in the civil calendar of Egypt by julian ("Alexandrian") years near the beginning of our era. Only in the Persian calendar (Sasanian and Era Yazdegerd) did the Egyptian "wandering" year find public use once more.[2]

The julian days, on the contrary, are a deliberately constructed technical device, introduced by Joseph Justus Scaliger of Leyden in his famous work "De emendatione temporum" (Paris 1583).[3] The sequence of the days in the planetary week, which was never disturbed, is often helpful to distinguish between close chronological alternatives in the dating of documents.

For modern historical purposes the "*julian year*" is used in dates before the 17th century, conventionally counted as years of the Christian era, beginning with A.D. 1 Jan. 1. All julian years are 365 days long unless their number is divisible by 4, in which case they contain 366 days, a 29th day added to February. All these conventions are, of course, of modern origin, made for easy reference but do not necessarily agree with the actual norm used in ancient or mediaeval documents which might, e.g., reckon with julian years not beginning on January 1, or place the intercalary day not at the end of February.

Years before A.D. 1 are conventionally counted as years 1, 2, ... B.C. Consistent arithmetical procedure requires, of course, the use of zero and negative

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[1] Cf., e.g., Huygens at the end of the 17th century (Œuvres Complètes 21, p. 150 or p. 626 et passim).

[2] Cf. Christensen, Hdb., p. 295 or L'Iran, p. 163ff.

[3] In the edition of 1629: p. 359ff. For a biography of Scaliger cf. Bernays, Scal.

numbers.[4] Hence

$$\text{year } (n+1) \text{ B.C.} = -n, \qquad \text{year } n \text{ A.D.} = +n.$$

Under Pope Gregory XIII the "*gregorian years*" were introduced changing the calendar dates such that

$$1582 \text{ Oct. } 5 \text{ jul.} = 1582 \text{ Oct. } 15 \text{ greg.}$$

Otherwise the gregorian years agree with the julian years except for the years which are congruent 100, 200, or 300 modulo 400. These years are ordinary years in the gregorian style instead of leap-years as in the julian calendar. Gregorian years appear in the astronomical literature as early as in Kepler's writings[5] whereas their general acceptance was much delayed for political reasons.[6]

Since Scaliger's choice for the zero point of the julian days is connected with three other important chronological concepts, we shall now describe the procedure which leads to $n_0 = -4712$ as epoch year for the julian days.

Let n be a positive integer, representing the julian year n according to the ordinary historical reckoning of the Christian era. We then have to define three cycles: (a) indictio, (b) solar cycle, (c) golden number.

The "*indictio*" refers to a 15-year cycle (introduced by Constantine for purposes of taxation) beginning with A.D. 313 as indictio 1.[7] Since $313 \equiv -2 \pmod{15}$ we have not only $\text{ind.}(313) = 1$ but also

$$\text{ind.}(-2) = 1.$$

Hence we have the rule for finding the indictio of a year n:

$$\text{If } n \equiv a \pmod{15} \text{ with } -2 \leqq a \leqq 12 \text{ then } \text{ind.}(n) = a + 3. \tag{1}$$

The "*solar cycle*" is a 28-year cycle. Since the number of days

$$\begin{array}{ll} \text{in 1 ordinary julian year is} & 365 \equiv 1 \pmod{7} \\ \text{in 1 leap-year} & 366 \equiv 2 \pmod{7} \end{array}$$

we find for 4 consecutive julian years

$$3 \cdot 365 + 366 \equiv 5 \pmod{7}.$$

Thus the number of days in $7 \cdot 4 = 28$ years constitutes the smallest cycle in which both weekdays and calendar dates repeat. We now define: a leap-year (thus $n \equiv 0 \bmod 4$) for which Jan. 1 = Monday has the solar cycle 1.[8] Such a year was, e.g., the year A.D. 1560. Since $1568 = 56 \cdot 28$ we see that also

$$\text{circ. sol.}(-8) = 1.$$

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[4] The year 0 was introduced by Jacques Cassini in his "Tables astronomiques ..." (Paris 1740), Explication ..., Chap. III, p. 5; cf. also Tables ..., p. 10, p. 22, p. 63, etc. In his "Elemens d'Astronomie" of the same year one still finds (p. 214) an ambiguous terminology: "... l'année 146 avant l'époque de Jesus-Christ dans la forme Julienne, et de l'année 145 avant Jesus-Christ, suivant notre manière de compter. (Voyez les Tables Astronomiques.)"

[5] Cf., e.g., the double-dates in Kepler, Werke 3, p. 146, 29 and 37 for observations made in 1600 and 1602; also Brahe, Opera XIII, p. 246f. where he distinguishes between "stylo veterj" (in 1590) and "stylo novo" (in 1594).

[6] Cf., e.g., Ginzel, Hdb. III, p. 266.

[7] Actually indictio 1 corresponds to the Alexandrian year 312/313 (which begins on Aug. 29 of A.D. 312). For the present purpose, however, we identify julian years and years of indictio.

[8] For details cf. Ginzel, Hdb. III, p. 125, p. 127.

Consequently:

$$\text{If } n \equiv a \text{ (mod. 28) with } -8 \leqq a \leqq 19 \text{ then circ. sol.}(n) = a + 9. \tag{2}$$

The "*golden number*" (=*numerus aureus*) is defined by a 19-year cycle, introduced by Alexandre de Villedieu in his Massa Compoti (1200). As year 1 of this cycle may serve the year A.D. 532 which is the first year of an Easter cycle introduced by Dionysius Exiguus (and is simultaneously the point of departure for his reckoning of the Christian era which is still in use today[9]). Since $532 \equiv 0$ (mod. 19) we have the rule

$$\text{If } n \equiv a \text{ (mod. 19) with } 0 \leqq a \leqq 18 \text{ then num. aur.}(n) = a + 1. \tag{3}$$

Considering these three chronological concepts Scaliger required that the epoch year n_0 for his counting *julian days* should satisfy

$$\text{ind.}(n_0) = \text{circ. sol.}(n_0) = \text{num. aur.}(n_0) = 1.$$

Consequently n_0 must be a solution of the diophantine equations

$$n \equiv -2 \text{ (mod. 15)}, \qquad n_0 \equiv -8 \text{ (mod. 28)}, \qquad n_0 \equiv 0 \text{ (mod. 19)}.$$

From these conditions one finds[10] that

$$n_0 \equiv -4712 \text{ (mod. 7980)}$$

where the modul $7980 = 15 \cdot 28 \cdot 19$ is called the "*julian period*." According to these definitions we know that

$$\text{jul. day } 0 = -4712 \text{ Jan. } 1 = \text{Monday}. \tag{4}$$

In this way one also has established a very simple rule for the determination of the weekday for any given date as soon as its julian day number is known. Indeed it follows from (4) that we only need to know the residue w modulo 7 of the julian day; then:

$w=0$:	Monday	☾	$w=3$:	Thursday	♃	
1:	Tuesday	♂	4:	Friday	♀	(5)
2:	Wednesday	☿	5:	Saturday	♄	
			6:	Sunday	☉	

The concept of "julian day" is particularly useful if one wishes to express a given date in different eras or calendars. Since one knows the number of days which elapsed between different eras (e.g. between the Christian era beginning on Jan. 1 of A.D. 1 = jul. d. 1721424 and the era Hijra beginning A.D. 622 June 15 = jul. d. 1948439) one can tabulate the julian days for the beginning of each year and each month in both calendars. The transformation of one calendar date into its equivalent in another system is then reduced to finding the same number in the corresponding table. Tables of this type are most conveniently arranged in R. Schram's "Kalendariographische und chronologische Tafeln" (Leipzig 1908) where all technical details for the manifold use of such tables are explained.[11]

[9] Ginzel, Hdb. III. p. 137.

[10] For the methods of solving linear diophantine equations cf. below p. 1117ff.

[11] For additional literature and for tables concerning special eras cf. below p. 1074f.

In the present work "*julian days*" are always reckoned as "*civil days,*" i.e. from midnight to midnight. Since it had been customary in modern astronomy up to 1925 to reckon days from noon to noon the "*astronomical Julian Day*" is still reckoned from noon to noon. Hence the astronomical *J.D.* begins 12 hours later than the "*j.d.*" used here for historical purposes.[12] The historical norm of reckoning is necessary if one wishes to transform civil days of one calendar into civil days of another or for the determination of weekdays by the above given rules.

§ 2. Special Calendars and Eras

1. The Egyptian Calendar

The "years" of ancient Egyptian history consisted of 12 months of 30 days each and 5 additional ("epagomenal") days at the end.[1] This "Egyptian year" of always 365 days length was modified by Augustus to a year with a "julian" intercalation pattern by adding a sixth epagomenal day every fourth year. This "alexandrian year" begins three times with August 29, then once, after a leap year, with August 30. The first deviation of the alexandrian from the Egyptian year occurs in the year -21:

$$-21 \text{ Aug. } 29 = \text{Eg. Thoth } 1 = \text{alex. epag. } 6 \tag{1}$$

hence

$$-21 \text{ Aug. } 30 = \text{alex. Thoth } 1 = \text{Eg. Thoth } 2.$$

This implies that alexandrian years ending in August of a year n (julian) are intercalary if n satisfies the condition

$$n \equiv 3 \mod 4. \tag{2}$$

Then Aug. 29 year n = epag. 6 and alex. Thoth 1 = Aug. 30.

It is a fortunate accident that the Diocletian era, using the alexandrian calendar, follows the same pattern, i.e., the Diocletian year n is a leap year if n satisfies (2). Hence, e.g.,

A.D. 287 Aug. 30 = Diocl. 4 Thoth 1 (alex).

The Greeks and the Romans used the same names both for the months of the Egyptian and of the alexandrian years. It seems plausible to assume that eventually the latter type of years prevailed though it is often impossible to decide in economic and private documents which form of the calendar was used. In astronomical and astrological texts, however, this assumption cannot be made a priori and only agreement or disagreement with lunar, solar, and planetary positions can lead to a decision.

2. The Seleucid Calendar

The Seleucid calendar in the form which is well known to us from the cuneiform documents of the last centuries B.C. is built on a 19-year cycle of 12 ordinary

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[12] Cf. below p. 1068.
[1] Cf. above p. 560.

years (which contain 12 lunar months) and 7 intercalary years of 13 lunar months. The first month of every year is Nisan, always kept near to the vernal equinox. Six of the intercalary years add a second 12th month (Adar); one intercalation, however, is made near the autumn equinox by adding a second 6th month (Tishri).

The Seleucid calendar is not only the first to use the 19-year cycle, the prototype of many similar later arrangements, e.g., the Easter computations, but it is also the earliest case of a civil era, called by us the *Seleucid Era*, because it continues to count the regnal years of Seleucus I after his death. With the help of the Seleucid era (abbreviated: S.E.) it is easy to define the position of the intercalary years: a second twelfth month (XII_2) is given to the years

$$\text{S.E.}\quad 1\quad 4\quad 7\quad 9\quad 12\quad 15$$

and a second sixth month (VI_2) to S.E. 18. All subsequent intercalations are obtainable from this initial set by adding multiples of 19.

The relation of the Seleucid era to the Christian era is given by the equation

$$\text{S.E. } 0 = -311/-310.$$

Consequently, if $k=n-311$, then

$$\text{S.E. }(n)-311=\text{year } k/k+1,$$

e.g., S.E. 200 $= -111/-110$ and S.E. 400 = A.D. 89/90.

When the Parthians took over the rule of Mesopotamia they introduced an era of their own, called *Arsacid Era* (abbreviated: A.E.). Its calendar, however, remained identical with the Seleucid calendar, except for the constant difference of 64 years between the initial dates:

$$\text{A.E. } 0 = \text{S.E. } 64.$$

Thus, if $k=n-64$, then

$$\text{S.E. }(n)-64=\text{A.E. }(k).$$

Furthermore: if $k=n-247$

$$\text{A.E. }(n)-247=\text{year } k/k+1,$$

e.g., S.E. 200 = A.E. 136 $= -111/-110$ and S.E. 400 = A.E. 336 = A.D. 89/90.

While the Seleucid era in Mesopotamia began the year with Nisan, following Babylonian tradition, the western part of the Seleucid empire, which remained under Greek domination after the loss of the eastern parts to the Parthians, adopted a beginning of the year with the month Tishri, the seventh month of the Mesopotamian calendar. The years of this western Seleucid era precede the years of the eastern norm by half a year.

3. Synopsis of Eras

The following table gives a list of some of the most commonly used eras and their epoch dates. Such a list cannot be exhaustive nor take local variation into consideration. For all questions of details monographs must be consulted (cf. for literature below, p. 1074).

year 1 of	day 1	julian date	julian day	form of year[5]
Byzantine World Era	Sept. 1	−5507 Sept. 1	9 709 870	julian
Kaliyuga	Chaitra 1	−3101 Febr. 18	588 466	(solar years)
Nabonassar	eg. Thoth 1	−746 Febr. 26	1 448 638	egyptian
Philip	eg. Thoth 1	−323 Nov. 12	1 603 398	egyptian
Seleucid West[1]	Tishri 1	−311 Oct. 1	1 607 739	(luni-solar)
Seleucid East	Nisan 1	−310 Apr. 3	1 607 923	(luni-solar)
Spanish Era[2]	Jan. 1	−37 Jan. 1	1 707 544	julian
Augustus	alex. Thoth 1[3]	−29 Aug. 30	1 710 707	alexandrian
Incarnation	Dec. 25	0 Dec. 25	1 721 417	julian
Christian Era	Jan. 1	1 Jan. 1	1 721 424	julian
Diocletian	alex. Thoth 1	284 Aug. 29	1 825 030	alexandrian
Hijra	Muharram 1	622 July 15[4]	1 948 439	(lunar)
Yazdegerd	Farwardin 1	632 June 16	1 952 063	(egyptian)
Alfonso X	June 1	1252 June 1	2 178 503	julian

In Monumenta 13, 3, p. 368/9 Usener gave a synchronistic table for the eras Nabonassar, Philip, Augustus. and Diocletian until A.D. 644.

4. The "Era Dionysius"

The Almagest is our only source for an era κατὰ Διονύσιον which is based on a calendar that denotes the months by the zodiacal signs of the corresponding solar travel.

This peculiar era and calendar is mentioned in the Almagest in connection with observations of planetary positions, eight in all. Since Ptolemy gives its equivalent in the era Nabonassar and the Egyptian calendar it is easy to determine the julian dates as follows:

No.	Dionysius year	Dionysius month and day	julian date	Almagest	
1	13	♑ 25	−271 Jan. 18	X, 9 Heib. II	352,5
2	21	♏ 22	−264 Nov. 15	IX, 10	288,9
3	21	♏ 26	−264 Nov. 19	IX, 10	289,2
4	23	♒ 29	−261 Febr. 12	IX, 7	264,18
5	23	♉ 4	−261 Apr. 25	IX, 7	265,9
6	24	♌ 28	−261 Aug. 23	IX, 7	267,3
7	28	♊ 7	−256 May 28	IX, 7	266,2
8	45	♍ 10	−240 Sept. 4	XI, 3	386,17

[1] Often called in Islamic works "Era of Alexander" or "Era of the Two-Horned."
[2] Also called "Era of Caesar."
[3] = eg. Thoth 0 = epagomenal day 5.
[4] This is the norm used by astronomers; historians use July 16. Cf. Neugebauer, The Astronomical Tables of Al-Khwārizmī, Danske Vidensk. Selskab, Hist.-filos. Skrifter 4,2 (1962), p. 10, Fig. 1.
[5] ( ) indicate that a special discussion is required for an accurate definition.

The change of the Dionysian year number between No. 5 and No. 6 suggests the summer solstice as the starting point of the year. Indeed, if we assume that the month "Cancer" had 30 days we obtain, reckoning back from No. 6, for "year 24 Cancer 1" the date −261 June 27 which agrees very well with the entry of the sun into Cancer (i.e. $\lambda_{\odot} = 90$).

Having established that much one finds that the year Dionysius 1 began at the summer solstice of the year −284. The first of Thoth which belongs to this year has as julian equivalent −284 Nov. 2. This Thoth 1 is also the beginning of the first regnal year of Ptolemy II Philadelphus.[1] Consequently the years of the Dionysian era agree for about 8 months each year with the regnal years of Philadelphus.[2]

All day numbers from the Dionysian calendar, quoted by Ptolemy, are single numbers, that is to say they are never of the type "from n to $n+1$," used commonly by Ptolemy. Consequently it is impossible to determine from what point of the day (e.g. sunrise or sunset) these dates are reckoned.

Much speculation has been spent in attempts to reconstruct the complete Dionysian calendar, a rather valueless enterprise as long as we have no hope of checking these highly hypothetical constructions on additional material.[3]

§ 3. The Reckoning of Days

1. Epoch

It is not surprising to find an agricultural population, e.g. in Ancient Egypt, count their days from sunrise. If then a "*morning epoch*" becomes the calendaric norm, even lunar months can be adapted to such a scheme by being reckoned from the *last* visibility of the moon. And fortunately the heliacal rising of a star (e.g. Sirius at the time of the beginning of the inundation) is also a morning phenomenon.

If, however, a calendar becomes strictly lunar, the reckoning of the months from the *first* visibility of the moon seems most natural. Such a practice also induces an "*evening epoch*" for the reckoning of the days. Such is the case in the Mesopotamian civilizations.

Both morning and evening epoch are inconvenient for mathematical astronomy. Not only is the length of daylight or night subject to seasonal variations but the moments of sunrise or sunset can be greatly influenced by the deviations of natural conditions from an ideal horizon. Consequently the development of a mathematical astronomy leads to a better defined and more convenient reckoning of days by introducing either a *midnight* or a *noon epoch*.

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[1] Cf., e.g., Skeat, Ptolemies, p. 10.

[2] This has been interpreted as an act of flattery and a whole string of totally unfounded "conclusions" were derived from it (e.g. an embassy of this Dionysius to India!). In fact the Dionysian years are neither called after the king nor are they identical with his regnal years — not to mention the rapidly increasing discrepancy between the years of the Egyptian civil calendar and astronomically defined solar years.

[3] It may suffice to quote Ideler, Astron. Beob. (1806), p. 260–274 and Böckh, Sonnenkr. (1863), p. 286–340. Borchardt, Zeitm. (1938), p. 8–11 ignored these predecessors but increased the list of unprovable hypotheses by adding lunar dates to the chaos.

Both norms require the determination of the local meridian, which is easily found, e.g., by the observation of shadow lengths. The midnight epoch is used in the Babylonian lunar theory of "System B,"[1] the noon epoch is the consistent norm in the Almagest and in modern astronomy until 1925.

Modern astronomy adhered so long to the Ptolemaic tradition because all observations made in the same night carried the same date. For historical studies, however, the deviation from the generally accepted modern midnight epoch of the civil calendar is of great inconvenience; it is therefore rather unfortunate that Ginzel based his tables on the astronomical noon epoch (meridian of Greenwich). Fig. 1 illustrates[2] the relation between this astronomical noon epoch, valid until the end of 1924, and the norm of the civil calendar and of astronomical tables after 1924.

Ptolemy did not use the argument of non-changing dates for nightly observations. On the contrary: since the Egyptian-hellenistic calendar used morning epoch the whole night belongs to the preceding daytime. Ptolemy, however, operates with noon epoch, extending his day to noon of the next calendar date. Thus almost all his observations carry double dates in the form "day n to $n+1$."

In reducing Ptolemy's dates to julian dates (using midnight epoch) one has to be careful with respect to hours before or after midnight. This can be best illustrated by an example. Fig. 2a concerns an observation of Mercury as evening star, i.e. a moment shortly after sunset.[3] Ptolemy gives the date as "from the 5th to the 6th of Pharmouthi" because his day begins in the 5th and ends in the 6th of the civil calendar. For us an event in the first half of the night belongs to the civil day which overlaps 3/4 of the Egyptian civil date; in our example -256 May 28 = julian day 1627702. Fig. 2b illustrates the case of an observation of Mercury as morning star,[4] thus an event in the second half of the night, called by Ptolemy "Thoth 27 to 28" but belonging to the civil date Thoth 27. The modern equivalent, however, is the date that corresponds in modern tables to Thoth 28 because the second half of the night is always counted with the following 3/4 of a civil day. Disregard for the differences in epoch have caused many errors in the modern literature. Equivalents of modern and ancient dates should therefore never be accepted without careful checking.

This is not the place to discuss the enormous variety of calendaric rules for different localities and in different periods. Only as a warning may be added the remark that non-astronomical documents from hellenistic Egypt also contain double-dates of the form "night of n to $n+1$," unrelated to Ptolemy's terminology.[5]

Crude errors in chronological matters are not a prerogative of modern literature. Also ancient popularizing works simplify the actual complexity of calendaric institutions and resort to free inventions (e.g. of a Babylonian morning epoch) when they do not know better. Much confusion has been inherited in this way by modern scholarship. The investigation of astronomical documents, e.g. of ephemerides or of eclipse computations, is the by far safest method of establishing the

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[1] Above p. 492 and p. 496.

[2] Here, as always in this work, the heavily drawn sections of the time axis represent night time.

[3] Almagest IX, 7 (Heiberg II, p. 266, 2).

[4] Almagest IX, 7 (Heiberg II, p. 268, 1).

[5] Cf. Neugebauer-Van Hoesen, Gr. Hor., p. 167 ff.

underlying norm for the reckoning of days. For example the use of a morning epoch in Byzantium, at least from the 11th to the 14th century, can be securely deduced from astronomical texts.[6]

2. Hours and Other Divisions

It is customary to call "*seasonal hours*" the subdivisions of daylight or night-time in 12 equal parts, respectively. Only at the equinoxes are these hours of daytime and night of the same length; hence the hours used today, which are 1/24th of the solar day are also called "*equinoctial hours.*"

The seasonal hours must have been originally of very uneven length within each day as is evident from the primitive schemes for their determination by shadow lengths, or by water clocks, or by observations of stars. Only by a proper understanding of the solar motion with respect to a given horizon and to the equator could one construct theoretically correct sun dials and thus measure time intervals of equal length. The existence of a spherical astronomy is a prerequisite for a reasonably accurate division of time.

Quite independent of the problem of actual time measurement is the definition of convenient units of time in mathematical astronomy. If, e.g., the "day" is defined by the solar motion from noon to noon this interval can be subdivided sexagesimally and related to the corresponding motions of the celestial bodies without any need for a direct measurement of these smaller units. The use of mean values for the motion of celestial bodies lends itself to the definition of units of constant length which is all that is required for the independent variable in mathematical astronomy. This does not imply, however, the existence of instruments or observations designed to reproduce these units. Such is clearly the situation which prevailed in the astronomy of Mesopotamia in the Persian-hellenistic period.

The trend toward a purely mathematical form of definition of units of time is particularly recognizable in another creation of late Babylonian astronomy, the "*lunar days*" or "*tithis.*" Since the Babylonian calendar was based on real lunar months which form a very irregular sequence of full (30 days) and hollow (29 days) months the concept of calendar month was very inconvenient for the computation of ephemerides. Thus one introduced "lunar months" of a fixed length, satisfying a relation of the type k months $= n$ days, empirically found from a large number of actual months. These "mean synodic months" were subdivided for arithmetical convenience in 30 equal parts, each of which is a little shorter than one solar day. We call these units "lunar days" or "tithis," the latter term being taken from the Sanscrit name of these lunar days which play an important role in Indian astronomy. In this way the Babylonian astronomers had at their disposal an exactly defined and convenient computational unit, fractions of which could be identified with the same fraction of a solar day without committing a serious error. But no instrument or clock would show tithis or indicate their constantly shifting beginnings within a civil day.

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[6] Suggested by Mentz [1908], p. 475; confirmed, e.g., by Marc. gr. 325 fol. 15^v, 21 or Par. gr. 2425 fol. 269^v, 22. Also Nicephoros Gregoras, Hist. Byz. IX, 12 (Corpus Script. Hist. Byz., Vol. I, p. 455, 1).

Neither from Babylonian nor from Greek astronomy do we have a definite value for the length of the tithi in relation to the day. Obviously this length depends on the norm one accepts for the mean synodic month; nevertheless, the range of these variants is very limited and for practical purposes it will suffice to assume for the mean synodic month the commonly used value $29;31,50,8,20^d$. Consequently

$$1^\tau = 0;59.3,40,16,40^d$$

can be used unless some other information is available. The equivalent inverse relation is

$$1^d = 1;0,57,13,\ldots^\tau.$$

3. Astronomical Time Units

"*Universal Time*" is derived from what is called "solar time" which underlies the familiar civil time reckoning. For an accurate definition it has to be related to "sidereal time" which is amenable to direct observation by star transits. Transits depend on the actual rotation of the earth; thus fluctuations in the latter affect the former. Because these fluctuations are irregular, and hence neither predictable nor accurately known, a strictly uniform parameter had to be introduced, known as "*Ephemeris Time.*" This is the independent variable in the dynamical equations of the planetary motions and therefore by definition uniform. The practical units of ephemeris time (E.T.) have been chosen conveniently to agree as nearly as possible with universal time (U.T.) during the 19th century.[1] For antiquity, however, $\Delta T = \text{E.T.} - \text{U.T.}$ reaches an appreciable amount, e.g. for -500 about 4^h. Hence a lunar position computed for E.T. can be about 2° ahead of the position obtained for the moment denoted by the same time units reckoned in U.T. For details cf. Clemence [1965], p. 96ff. or the Explan. Suppl. A.E., p. 66–82. In 1967 ephemeris time was replaced by a new definition of uniform time in terms of atomic constants (cf. Trans. of the Intern. Astron. Union, Vol. 13B, p. 40/41 and p. 182; also Clemence [1971]).

It has already been mentioned[2] that before 1925 Jan. 1 days were reckoned in astronomy from noon to noon, such that the civil day n begins at midnight 12 hours earlier than the astronomical day n (cf. Fig. 1, p. 1433).[3] This change of definition must be kept in mind when reading older literature; on the other hand Oppolzer's Canon der Finsternisse (1887) reckons time in "Weltzeit," i.e. in Greenwich civil time which is practically equivalent to Universal Time.

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[1] The number of "ephemeris seconds" in the tropical year 1900 is the same as the number of seconds in universal time. In this way the variable length of the tropical year is measured in units of the year 1900.

[2] Above p. 1064; p. 1068.

[3] Consequently the "astronomical julian day" (cf. above p. 1064) is more accurately to be defined as "Julian Ephemeris Day" (J.E.D.), since it is used in connection with Ephemeris Time; cf. Explan. Suppl. A.E., p. 71.

§ 4. The Foundations of Historical Chronology

In the preface of his "Chronology" E. J. Bickerman writes: "we say that Caesar was assassinated on 15 March 44 BC. How do we know it? To answer this kind of question, we have to understand the calendar systems used by the ancients and their time reckoning." But a little probing will make it evident that it cannot suffice for establishing the fact implicit in the above made statement, that the date in question precedes by 709595 days Jan. 1 (gregorian) 1900 to possess a list of the Roman consuls and to know that the Ides of March correspond to (julian) March 15. Indeed, historical chronology rests on an interplay of theoretical astronomy and historical conditions, far more intricate than professional historians usually realize – to the great detriment of their insight into the very foundations of their field.

The first and foremost requirement for the establishment of an "absolute chronology" (i.e., a chronology which is based on astronomically fixed dates in contrast to a "relative chronology" which tells us only the length of certain intervals, e.g., the total of regnal years in a dynasty) is a reasonably accurate theory of eclipses. To develop such a theory one must have a sufficiently accurate understanding of the motion of the moon, that in turn can only be obtained from the analysis of a long sequence of dates of syzygies and observed eclipses. For this an undisturbed and precisely known local calendar is a necessary prerequisite. Such conditions were satisfied in Mesopotamia through the archives of the Late-Assyrian and Neo-Babylonian kings, archives maintained through the Persian and Greek period. Conversely the increasing understanding of the motion of the moon made the lunar calendar amenable to calculation, eventually leading to systematically regulated calendaric intercalations as well as to a cyclic theory of eclipses.

Hence, by the beginning of the Greek rule over Mesopotamia existed a lunar theory, based on excellent parameters which were only slightly refined when Ptolemy put the whole theory on a new cinematic foundation. For chronology this means that an accurately known astronomical system had established a sequence of fixed points, distributed over some 900 years and dated in a uniform (the Egyptian) calendar, directly controllable by modern computations.

The data from the Almagest provide the backbone for all modern chronology of antiquity. Copernicus, Scaliger, Kepler, and Newton had at their disposal Ptolemaic dates, accurate enough to establish within limits of less than one day their own distance from any of Ptolemy's eclipses or related data. The uninterrupted use of the Egyptian calendar made it possible to express these intervals in a precisely known form as "years," "months," and days. At the same time the increase in distance permitted new improvements in the basic elements and the recognition of slow systematic deviations.

Long before the 16th century a similar process had taken place at least twice. The ancient tradition was continued unbroken into the Byzantine period which kept the conventional astronomical list of rulers up to date, though now usually based on Alexandrian (i.e. "julian") years. The use of some arbitrary (i.e. theologically motivated) "era of the world" stabilized also the relative chronology. Thus one now had an astronomically secure "royal canon" based on the Babylonian-

Ptolemaic canon of the Era Nabonassar, eventually extended until the 15th century.

Once again the Ptolemaic chronology had to serve as the foundation for a new chronological structure in the 9th century in Islamic astronomy. It is again the comparison with the data of the Almagest (or the equivalent "Handy Tables") which established the dates within the era of the Hijra. Fortunately again a precisely regulated calendar allows an exact coordination with "julian" dates, e.g. in Byzantine documents. Hence Byzantium and the Muslim world between Spain and India operated with chronological systems that are directly connected with astronomically secure data.

This is the foundation upon which rests the specific work of the modern historian. The chronology of the european Middle Ages would be chaos were it not for the contact with the Byzantine and Islamic dates. Of course, there are always additional astronomical events recorded which allow independent checks of certain documents or local annals. Ginzel's investigations of eclipses ([1882], [1883], [1884]) demonstrate how much can be gained in this way, both historically and astronomically. It is, however, only at this level that the problem of local calendars and time reckonings enters the picture. As we said before, the astronomical calendars and eras are rarely used in specific documents. Hence one must find the transition from the astronomical fixed points to the variety of local usages in contemporary documents, annals, and historical narrative. The complexity of mediaeval chronological habits, liturgically influenced and based on antiquated cyclical computations, easily obscures their dependence on the exact data from ancient astronomy. In principle one still has to operate as Ptolemy did in the Almagest when he identified a local date by its equivalent in the Egyptian calendar and the Era Nabonassar.

What we have discussed so far would concern the absolute chronology from, say, -800 to $+1500$. That modern historical research was able to extend this interval back to nearly -3000 within reasonable limits of safety (i.e. no longer with an accuracy of a single day but at least within a few decades) is due only to the lucky accident of the undisturbed reliability of the Egyptian calendar whose uniformly slow rotation, like the hand of a clock, fixes (within narrow limits) the julian date by its passing by the fixed point of the heliacal rising of Sirius. The existence of Egyptian king lists in combination with a great wealth of archaeological evidence thus made it possible to establish a reasonably secure chronology back to about 3000 B.C. Documentary evidence and archaeological evidence relating Egypt to its nearer and more distant neighbours in Syria and Mesopotamia then provided the substructure for the Near-Eastern chronologies. The total lack of observational data from ancient Egypt – not a single eclipse record is extant – make a refinement of the Sirius-based chronology impossible and the Near-Eastern situation before about -800 is not much better; the famous Venus tablets of Ammizaduqa of the "first Babylonian dynasty" are not much help in themselves. Hence it is in fact only the consistency of the Egyptian calendar that made an extension over two millenia feasible.

The modern reader will perhaps say that radio-carbon dating should have made these purely chronological methods obsolete. In fact the opposite is the case: it is essentially on the basis of the well established Egyptian chronology that the

radio-carbon method obtains its standards. It is only in areas where none, or only extremely loose, astronomical data are available that radio-carbon dates can supplement the classical methods of chronology. As soon as real astronomical data become available, e.g. from a simple horoscope or from an eclipse or occultation, the date can be established within a day or even hours, a precision never obtainable from radiation measurements.

The chronology of ancient and mediaeval India is connected with the Near-East, ever since the Persian-Hellenistic period. The pre-Arian "Indus-civilization" is archaeologically related to Mesopotamia in the period around 2000 B.C., establishing at least some chronological estimate for early Indian antiquity. For the period between the Graeco-Roman contact with India and the Islamic conquest of the Punjab in the 11th century A.D. one has the advantage of the consistent use of the Śaka-Era in the Indian astronomical literature. Its first year begins in A.D. 78, as we know through the epoch constants in astronomical treatises as early as the 6th century, e.g. in the Pañca-Siddhāntikā for Ś.E. 428, agreeing with A.D. 505. In these cases data from theoretical astronomy, ultimately derived from Greek as well as from Babylonian sources, take the place of observational records which are totally lacking in India for the pre-Islamic period.

All chronological data discussed so far are seen to rest in the final analysis on the Babylonian-Greek observations which can be directly controlled by modern astronomy. Entirely independent of these western sources is Chinese chronology which is based on annals within a strictly maintained 60-year cycle. In this way a secure chronology was established back to the 9th century B.C. while references to eclipses on the famous "oracle-bones" permitted the extension of dates into the middle of the second millenium.[1] The chronological parallelism in the cultural development of the Far-East and the West is a remarkable fact that should not be obscured by attempts to construct contacts where there are clearly none.[2]

We mentioned before that the systematic analysis of observational records led the Babylonian astronomers to devise computational methods accurate enough to make it possible for the Greeks to connect their observational data with these older data and thus to improve on the accuracy of the basic parameters. The same process continued through the Islamic period until Ulugh Beg in the 15th century, to be taken up again most successfully in the European Renaissance. It is still going on today. The creation of a celestial mechanics has made it possible to distinguish clearly between effects within the planetary system and variations of conditions of terrestrial origin. Now securely established chronological data of ancient observations can provide the basis for the checking of long-range extrapolations for effects only recently recognized as existing. The great wealth of observational records assembled in Babylonia during the last three or four centuries B.C. will be a test case for modern parameters similar to the Ptolemaic data for an earlier phase of modern astronomy.

In conclusion one may say that chronology is not only the backbone for the writing of history but that chronological facts belong to the very few elements of history which can be established objectively.

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[1] For a summary and references cf., e.g., Needham SCC I, Sec. 5 (p. 73–99).

[2] A favored topic is the lunar mansions which are again and again "discovered" in Babylonian sources. Needham SCC III, Sec. 20 is vitiated by these imaginary Babylonian associations.

§ 5. Literature

1. General

For the conversion of a great variety of calendaric dates to julian (or gregorian) dates, or of one type of dates directly into another (without an intermediary julian date) the most convenient work is

R. Schram "Kalendariographische und chronologische Tafeln" (Leipzig 1908).

The basis of all its procedures is the tabulation of julian days[1] which are the equivalents of characteristic dates in each individual calendar (e.g. the zero-day of each month). Schram's tables contain complete instructions for their use and thus represent a self-sufficient tool for the solution of calendaric transformations.

Nevertheless for the understanding of the historical background of the calendaric concepts of different nations and different periods a work like

F. K. Ginzel "Handbuch der mathematischen und technischen Chronologie" (3 vols., Leipzig 1906–1914[2])

is indispensable. This is a work of outstanding scholarship[3] and should be studied by every historian. Ginzel's "Handbuch" is actually a vastly revised version of an earlier brilliant chronological study, namely Ideler's "Handbuch" in two volumes (1825/6).[4] It is now much too antiquated to be used without great caution but it contains many references to ancient and mediaeval (also Islamic) sources which are discussed nowhere else.

An excellent guide to the practice of chronological computations, including planetary positions, eclipses, stellar phases, etc. with a very useful survey of modern chronological tables is found in

P. V. Neugebauer[5] "Astronomische Chronologie" (Vol. I, Berlin 1929).

To it belong, for actual computations, the "Tafeln zur astronomischen Chronologie" (in 3 vols., Berlin 1912–1922, with supplements in Astr. Chron., vol. II, Berlin 1929),[6] and the same authors "Hilfstafeln zur astronomischen Chronologie" A.N. 261 (1936/7).

Finally al-Bīrūnī's monumental work "Vestiges of the Past" should be mentioned, written in A.D. 1000 and available in an English translation by C. E. Sachau under the title "Chronology of Ancient Nations" (London 1879). The colossal amount of historical and chronological material assembled and discussed by al-Bīrūnī is an inexhaustible source for the study of mediaeval oriental chronology.

There exist, of course, many specialized chronological tables, e.g. for the direct conversion of Islamic to julian dates the "Vergleichungs-Tabellen" of Wüstenfeld-Mahler (1854–1887), now in an enlarged version by Spuler and Mayr

[1] Cf. above p. 1063.

[2] Reprinted 1960.

[3] Of course certain sections are antiquated, e.g. on Egyptian and Babylonian data.

[4] An abridged version appeared in 1831 as "Lehrbuch der Chronologie."

[5] No relative of the present author.

[6] For simplified planetary tables of the same author cf. P. V. Neugebauer [1932].

(1961) which also includes the Persian calendar and conversion tables for several oriental eras (Byzantine, Seleucid, Alexandrian-Ethiopic).

W. Kubitschek "Grundriss der antiken Zeitrechnung" (in the Handbuch der Altertumswissenschaft, 1928) is a rather mediocre compilation. V. Grumel "La chronologie" (in the Bibliothèque Byzantine, Paris 1958) contains an enormous mass of information on mediaeval chronology, including Islamic and other oriental material. Unfortunately the work is badly organized, overburdened with theoretical speculations and difficult to use because of the lack of indices. Bickerman, Chronology of the Ancient World (1968) contains a great wealth of references to the more recent chronological literature but is again not easy to use by a not already well informed-reader. Samuel, Greek and Roman Chronology, is the latest handbook (1972) on the subject.

2. Chronological Tables

All works mentioned in the preceding section contain tables needed for the most common chronological transformations (e.g. Roman calendar, era Hijra, etc.) and bibliographical references to more specialized tools and discussions. The sole purpose of the following list is therefore to provide the reader with a few titles of easily accessible works which he may find useful in connection with topics discussed in the present work.

Babylon. Julian dates for the days of first visibility of the moon are given in Parker-Dubberstein, B.C. In principle these are the dates of day 1 of consecutive lunar months for Babylon. Since, however, these dates are obtained by computation with uniform visibility conditions the actual days may occasionally differ from the computed dates by ± 1 day but such discrepancies cannot lead to accumulative errors. For the years from -625 onward the tables are arranged by regnal years until the Seleucid (and Arsacid) era takes over (to A.D. 76). For this latter half a list of Seleucid and Parthian rulers is given (p. 24).

Egypt. The julian equivalents of Thoth 1 for the Egyptian wandering years from -746 to $+300$ are given in Bickerman, Chron. Table IV. Otherwise use P. V. Neugebauer, Hilfstafeln (No. 22, from Nabonassar 1 to A.D. 687). Alexandrian dates are most easily determined with Schram, Tafeln (p. 107–157).

Greek. In the study of Greek calendars one meets the paradoxical situation that the apparent abundance of sources – epigraphical, literary, historical – has proven to be far from sufficient to gain a clear picture of the working of Greek calendars before the Byzantine period. In fact it is certain that a reasonably secure control of the numerous Greek calendars will never be reached since it would require a month by month record of calendaric data which are neither governed by actual astronomical facts nor by arithmetical patterns. Modern hypotheses about the existence of a Greek "astronomical" calendar beside the chaos of local civil calendars are only wishful thinking. If Hipparchus or Ptolemy were able to utilize Greek data it must have been in the form of double dates of local civil dates

and Egyptian calendar dates, the latter being the natural reference system for astronomical records.[1]

Given this state of affairs it would be pointless to give here a list of publications concerning Greek calendars. A reader who wishes to obtain a first impression of the complexity of the problems which confront a student of the best known Greek calendar may consult Pritchett, The Choiseul Marble (1970) and Meritt, The Athenian Year (1961). The latter work also contains a list of archons (from −345 to −80) on which Athenian chronology hinges.[2] A Study by Jon D. Mikalson on the Athenian calendar is to appear in the Princeton University Press. 1975.

As early as the Byzantine period the coordination of months and dates among local calendars in Greece and of hellenistic cities in Asia caused great difficulties and schematic concordances were constructed, often without any reliable basis; cf., e.g. the discussion by Hanell [1931] or Kubitschek, Kalenderbücher.[3]

For the Macedonian calendar in Egypt cf. Samuel, Ptol. Chron. Special attention to Byzantine problems is given in Grumel, Chronol. Cf. also Gardthausen. Griech. Pal., e.g. for a list of indictions from A.D. 800 to 1599.[4]

Roman Calendar. Tables for the transformation of Roman calendar dates of the imperial period to julian dates are given in many works, e.g. in Ginzel, Hdb. II, p. 179–181. or Bickerman, Chron. Table V, or Grumel, Chronol., p. 298f.

Bickerman, Chron. Table IV gives the julian equivalents for the Varronic era of the foundation of Rome (753 B.C.) and for the Olympiads (from 776 B.C.) until A.D. 300. Table VIII is a list of Consuls from 509 B.C. to 337 A.D.

For the problems connected with the earlier history of the Roman calendar cf. Michels. Cal. Rom. Rep.

Middle Ages. For the European Middle Ages (e.g. Easter-computus) cf. Ginzel, Hdb. III Chap. XIV or Van Wijk, Nombre d'or. For Syriac-Islamic concordances see the contemporary chronological tables of Bar Šinaya (written A.D. 1018).[5] Smaller calendaric tables belong to the equipment of practically all Islamic astronomical tables.[6] For Indian astronomy and chronology cf. Pillai, Ephemeris (6 vols.).

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[1] Cf. for details and deviating opinions above p. 617.
[2] For a detailed discussion of the archon lists cf. Samuel, Chron., p. 195–237.
[3] Cf. also above p. 973.
[4] Vol. II. p. 488–497; indictions are also given in Grumel from A.D. (312/)313 to 1525 (p. 240–264).
[5] Delaporte. p. 147–376.
[6] Cf. Kennedy. Survey. p. 139.

B. Astronomical Concepts

§ 1. Spherical Coordinates

The unit sphere with the observer in its center is called the "*celestial sphere.*" If the contrary is not stated it is assumed that the observer and the center of the earth coincide. Only for questions of parallax is it necessary to distinguish between positions with reference to the observer and with reference to the center of the earth (or even to the sun). For all pre-telescopic astronomy, however, such a distinction is only of interest in the case of the moon and (indirectly) of the sun.

Following ancient custom we call a celestial object simply a "*star,*" even if it is a planet or the sun or the moon — in the two latter cases the center of the apparent disk defines the position. The line of sight from the observer to the star meets the celestial sphere in a point whose location with respect to a system of orthogonal spherical coordinates requires two arcs which are conventionally named depending on the special coordinate system.

In general a system of spherical coordinates can be described by means of the following three elements (cf. Fig. 3):

a) a certain great circle is selected as "fundamental great circle" and a "positive" sense of rotation on it is chosen

b) one of its two poles is selected as "visible pole" (P)

c) a point of the fundamental great circle is chosen as "zero point" (Z).

With reference to these elements the position of a star on the celestial sphere is determined by two arcs σ and τ, σ counted in degrees from 0° to 360° beginning at the zero point (Z) of the fundamental circle and τ from 0° to +90° at the visible pole, and to −90° at the opposite pole. Since antiquity the following three coordinate systems are in use:

1. The Horizon System (cf. Fig. 4)

Fundamental great circle: the *horizon*,

visible pole: the *zenith* Z, exactly overhead,

zero point: the *north point* N of the horizon (or the diametrically opposite point S); the great circle SZN is called the *meridian*.

The corresponding coordinates are:

azimuth A (counted positive from the north toward east),

altitude a or h.

The coordinate A plays practically no role in ancient or mediaeval astronomy whereas the altitude of an object, especially of the sun, is obtainable by direct observation. Its complement $z = 90° - a$ is the *zenith distance*.

The points 90° distant from S or N on the horizon are called east (E) and west (W). Azimuths with respect to E are called *rising amplitudes* or *ortive amplitudes*.[1]

2. The Equator System (cf. Fig. 5)

Fundamental great circle: the celestial *equator*,
visible pole: the *north pole* N (i.e. the pole of the daily rotation of the celestial sphere),
zero point: the *vernal point* ♈.
The corresponding coordinates are:
right ascension α, counted in counter clockwise direction seen from N (also called from West to East)
declination δ.

This coordinate system is the most important modern system because instruments are mounted to follow the rotation of the celestial sphere about the axis ON. Pre-telescopic astronomy knows only observations of positions at given moments. Nevertheless the equatorial system was of great importance because it relates time to the uniform change of right ascension. For this reason α is not only reckoned in degrees but also in hours such that 24 hours correspond to 360°. Consequently

$$1^h = 15°, \qquad 0;4^h = 1°.$$

3. The Ecliptic System (cf. Fig. 6)

Fundamental great circle: the *ecliptic* (i.e. the plane of the yearly solar orbit with respect to the fixed stars)
visible pole: the *pole of the ecliptic* P (belonging to the northern hemisphere)
zero point: the *vernal point* ♈.
The corresponding coordinates are:
longitude λ counted counterclockwise seen from P (also called from West to East),
latitude β.

Longitudes are not only counted in degrees from 0° to 360° but also in 30° sections called *zodiacal signs* or *signs* for short. We write

$$1^s = 30°.$$

[1] In German: Morgenweite.

The conventional names and symbols for the zodiacal signs are:

λ	Name	Symbol	λ	Name	Symbol
0 to 30	Aries	♈	180 to 210	Libra	♎
30 to 60	Taurus	♉	210 to 240	Scorpius	♏
60 to 90	Gemini	♊	240 to 270	Sagittarius	♐
90 to 120	Cancer	♋	270 to 300	Capricorn	♑
120 to 150	Leo	♌	300 to 330	Aquarius	♒
150 to 180	Virgo	♍	330 to 360	Pisces	♓

The zodiacal signs have nothing to do with the constellations (i.e. areas which contain certain configurations of stars) which carry the same name although, of course, the names of the signs originated from their relation to the constellations. In the usage of ancient and mediaeval astronomy, however, signs are nothing but names for sections of longitudes, counted from a properly defined vernal point.[2] It is a mistake, often made by modern historians, to interpret a sentence like "the planet entered Leo on this and this date" as an expression for the planet's position with respect to the constellation "Leo" — although definite boundaries of constellations were unknown in antiquity. In fact the above quoted phrase can only mean that the planet had the longitude $\lambda = 120°$. If one wished to express a relation with respect to fixed stars one would say, e.g., that the planet was 1 1/2° to the east of Regulus, crossing at the given moment the line from star A to star B, etc.

In Islamic and mediaeval astronomy there existed a certain picture of concentric shells, called "spheres" or "heavens," among which the planetary spheres were located in their proper order. In such a model the zodiac with its stars belongs to a sphere outside the planetary spheres, i.e. also outside the solar and lunar sphere. Since the coordinates λ and β refer to the plane of the solar orbit a special terminology was invented to denote inner circles which are concentric and coplanar with the ecliptic, e.g. the term "*parecliptic*."[3] In fact it only means a circle of reference for the coordinates λ and β.

4. Relations Between the Systems (cf. Fig. 7)

The significance of the different coordinates used in the three systems described becomes evident when one places them in their relative position with respect to the same celestial object.

The equator intersects the horizon in the points *East* (E) and *West* (W) and reaches its greatest altitude SC in the *culminating* point C.

Ecliptic and equator intersect in the vernal point ♈ (and in the point $\lambda = \alpha = 180°$) such that the points of the ecliptic with longitudes from 0° to 180° have positive declinations, i.e. lie to the north of the equator. The angle between equator and ecliptic is called the *obliquity of the ecliptic* and customarily denoted by ε. Both α and λ are counted in the direction opposite to the daily rotation.

[2] In Indian and Islamic astronomy one finds also any 30-degree arc (e.g. on an epicycle) denoted as "one sign." Cf. also above p. 299.

[3] As far as I know this term was introduced by Nallino in his edition of al-Battānī (I, p. 45, n. 3).

Tables for the slowly decreasing values of ε (beginning at -3000) and for the direct conversion of ecliptic coordinates λ, β to equatorial coordinates α, δ are given in Ahnert, Tafeln XVa and XXXVIII–XLIII, respectively.

Ecliptic and horizon intersect in the *ascendant* H and in the *descendant* Δ.

Using the culminating point C of the equator as zero point one calls the arc from C, counted in the direction of the daily rotation the *hour angle* H. The hour angle of ♈ ($\lambda=0°$) is called *sidereal time* θ. Consequently (cf. Fig. 8)

$$\theta=|\alpha|+H.$$

With respect to an observer O at the geographical latitude φ the celestial sphere with its coordinates is situated as shown in Fig. 9. The altitude of the culminating point C of the equator is $\bar{\varphi}=90-\varphi$. This is also the noon altitude of the sun when it is in the equator, i.e. at the equinoxes ($\lambda=0°$ and $180°$). The altitude of the north pole N and the zenith distance of C are φ.

A special situation prevails if $\varphi=0°$, i.e. for an observer on the equator. This case is known as the case of *sphaera recta* in contrast to the general case of *sphaera obliqua*. At sphaera recta the north pole N lies in the horizon and the equator is perpendicular to the horizon. Since the meridian is also at sphaera obliqua perpendicular to the equator the meridian at sphaera obliqua plays the role of the horizon at sphaera recta.

Note on velocities. For the computation of lunar and planetary longitudes ancient astronomy makes the simplifying assumption that the orbital plane coincides with the ecliptic. It is therefore of interest to determine the relation between the velocity component $d\lambda/dt$ of a body which actually moves with the velocity $d\omega/dt$ in an orbital plane of inclination i, ω being the distance from the node, β the latitude (cf. Fig. 10).

Since

$$\tan\lambda=\tan\omega\cdot\cos i, \qquad \cos\omega=\cos\lambda\cdot\cos\beta \tag{1}$$

we have

$$\frac{d\lambda}{dt}=\frac{\cos^2\lambda}{\cos^2\omega}\cos i\,\frac{d\omega}{dt}=\frac{\cos i}{\cos^2\beta}\cdot\frac{d\omega}{dt}. \tag{2}$$

In the special case of mean motion

$$\frac{d\omega}{dt}=c \tag{3}$$

we see from (2) that the motion in longitude increases from $c\cos i$ at the node to $c/\cos i$ at maximum latitude $\beta=i$. Somewhere near the middle of each quadrant the longitudinal velocity must be the same as the true velocity hence from (2)

$$\cos^2\omega=\cos^2\lambda\cos i. \tag{4}$$

With (1) one finds

$$\frac{1}{\cos^2\lambda}=\tan^2\lambda+1=\tan^2\omega\cdot\cos^2 i+1$$

hence from (4)

$$\frac{\cos i}{\cos^2\omega}=\frac{1}{\cos^2\lambda}=\frac{\sin^2\omega\cos^2 i+\cos^2\omega}{\cos^2\omega}$$

and finally

$$\sin^2 \omega = \frac{1}{1+\cos i}. \tag{5}$$

For the lunar orbit, $i \approx 5°$ one finds from (5) that at $\omega \approx 45;1°$ the two velocities are equal ($\cos i = 0;59,46,18$).

5. Equation of Time

Since the sun does not move with constant velocity one has to distinguish between the true sun and a "*mean sun.*" In ancient and mediaeval astronomy the "mean sun" is a point of the ecliptic which has from the solar apogee the distance $\bar{\kappa} = \bar{v} \cdot \Delta t'$ where $\bar{v}$ represents the mean velocity of the sun and $\Delta t'$ the time elapsed since the true sun was in the apogee.[4]

In modern astronomy the "mean sun" coincides with the true sun at the vernal point and is a point of the equator with the right ascension $\bar{\alpha} = \bar{v} \cdot \Delta t$, Δt being reckoned with respect to the vernal point.

"*True* (or *apparent*) *solar time*" is defined by the hour angle H of the true sun, "mean solar time" by the hour angle $\bar{H}$ of the mean sun $\bar{\mathrm{S}}$ (cf. Fig. 11). The difference

$$E = H - \bar{H}$$

is called the "*equation of time.*" If α is the right ascension of the true sun we can write for the sidereal time, i.e. for the hour angle of the vernal point, either $H + \alpha$ or $\bar{H} + \bar{\alpha}$ (cf. Fig. 11). Consequently $H + \alpha = \bar{H} + \bar{\alpha}$ and therefore

$$E = H - \bar{H} = \bar{\alpha} - \alpha.$$

If λ is the longitude of the true sun and if we set $c = \lambda - \bar{\alpha}$ then we have finally

$$E = \lambda - \alpha - c.$$

The term $\lambda - \alpha$ is called the "*reduction to the equator*"; it depends on the obliquity of the ecliptic and thus produces for modern times values which differ slightly from values computed for antiquity. The second term c is the "*equation of center*" which depends on the parameters for the solar orbit, thus in particular on the position of the solar apogee. Consequently the relative position of the two components $\lambda - \alpha$ and c of the equation of time changes and this modifies the resultant function much more than the small variations in the single terms.[5]

6. "Polar" Coordinates

For historical reasons a system of spherical coordinates must be mentioned which combines equatorial and ecliptical elements (cf. Fig. 12). The hour circle NSU through the star S intersects the ecliptic at a point T. The arc $\mathrm{TS} = b$ (in

[4] Cf. above p. 60.

[5] Cf. Fig. 57 (p. 1222) where the curve for $-c$ has to be moved toward greater longitudes if one wishes to obtain later conditions.

mediaeval terminology[6] the "*basis latitudinis*") and VT $= m$ ("*mediatio coeli*"), V being the vernal point, are taken as coordinates of S. In connection with Indian astronomy these coordinates are also called "polar latitude" and "polar longitude", respectively.[7]

§ 2. Years, Months

1. The Year

The term "year" has either a calendaric or an astronomical meaning. Calendaric years contain an integer number of days. e.g. 354 days in certain lunar calendars, or 365 days as in the Egyptian or in the Persian year. The discussion of the great variety of calendaric years belongs to the field of technical chronology and does not concern us here.[1] Astronomical years. however, are defined in relation to the periodicity of the solar motion itself and are therefore intimately related to the development of mathematical astronomy. In modern times the direct correlation between the actual solar motion and the units of time measurement had to be loosened and had to be replaced by much more complex definitions. For historical discussions, however, much simpler concepts will suffice; we may, e.g., assume that the relative configurations of the "fixed stars" remain unchanged and are therefore fitted to provide an accurately defined reference system for all celestial motions. This makes it possible to define "sidereal periods" as returns to the same fixed star without further specification and to act as if all celestial coordinates were directly represented by properly located fixed stars.

There are two phenomena which obviously reflect a periodicity of the solar motion: the recurrence of the same length of daylight and the return of the same constellation to the same position at the same part of the night (e.g. midnight). The first phenomenon is not easy to associate with a definite moment since the seasonal variations are slow and ill defined. Hence the sidereal periodicity seems to be the ideal phenomenon to allow us to establish accurate limits for the length of the periodic solar motion. e.g. by the appearance or disappearance of bright stars in their relation to the sun. The resulting period is now called the "*sidereal year*," schematically defined by the return of the sun to the same fixed star (cf. Fig. 13, p. 1436).

The definition of "year" as recurrence of the same season is easily expressible (though not directly observable) in terms of the celestial spherical coordinates. The return of the sun in its travel in the ecliptic to the intersection with the equator at the vernal point produces equality of length of daylight and night, at least in principle, i.e. assuming an ideal horizon and disregarding all atmospheric influences. This type of year is called the "*tropical year*."

The recognition of the fact that the sidereal year is longer than the tropical year is equivalent to the discovery of the "*precession of the equinoxes*." The basic

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[6] Cf., e.g., Kepler, Epitome Astronomiae Copernicanae III, 5 (Werke 7, p. 217f.).

[7] This terminology is modern (introduced by E. Burgess, S.S., p. 320; Calcutta edition of 1935, p. 203).

[1] Cf. for the literature above p. 1074f.

observations, due to Hipparchus.[2] make it easy to remember the relative length of the two types of years. Hipparchus found that the ecliptic coordinates of fixed stars (i.e. the longitudes counted from the vernal point and the latitudes) behave in the same way as the ecliptic coordinates of the sun: the longitudes are always increasing, the latitudes remain constant. But while the longitude of the sun increases 360° in one (tropical) year the longitude of a fixed star increases only about 1° per century (according to Hipparchus' estimate, actually 1° in about 72 years) thus requiring 36000 years (actually about 26000 years) for one rotation.

Fig. 13 illustrates this situation. While the sun moves in the ecliptic from the vernal point F_1 eastwards the vernal point is displaced with respect to the fixed stars to F_2 (about 0;0,36° per year according to Hipparchus, actually about 0;0,50° per year). Thus the sun returns sooner to the vernal point than to the same fixed star and the tropical year is shorter than the sidereal year by the amount it takes the sun to travel from F_2 to F_1, i.e. about $0;0,40^d = 0;16^h$ (actually $0;0,50^d = 0;20^h$). The length of the tropical year was estimated by Hipparchus to $365;14,48^d$ [3]; thus the sidereal year would be about $365;15,30^d$. The modern values are

$$\begin{aligned} &\text{tropical year: } 365;14,31,55,12^d \\ &\text{sidereal year: } 365;15,23,\ 2,24^d. \end{aligned} \tag{1 a}$$

The schematic year which underlies the julian calendar is $365;15^d$ long, accidentally falling between the sidereal and the tropical year.

Since the motion of the sun in the ecliptic is not uniform, one has to distinguish a third period, the *anomalistic year*, defined by the return of the sun to the same velocity. Ancient astronomers always considered the place of minimum velocity, the apogee A, as point of reference. The question then arises whether A, located by Hipparchus at ♊ 5;30, has a fixed distance from the vernal point, or from a fixed star, or whether it moves independently. Ptolemy came to the conclusion that the longitude of A remains constant, thus identifying anomalistic and tropical year. Thābit ibn Qurra (9th century), however, convinced himself that anomalistic and sidereal year are equal. Finally Azarqiel (11th century) realized the independence of the movement of the solar apogee. The modern value for the length of the anomalistic period is

$$\text{anomalistic year: } 365;15,34,33,36^d. \tag{1 b}$$

This shows that the longitude of A increases faster than that of fixed stars.

2. Months

Similar periods can be distinguished for the moon, obtained as mean values over long intervals of time. The sidereal month is the time elapsed between consecutive returns of the moon to the same fixed star, about $27\ 1/3^d$ long. The anomalistic month is slightly more than $27\ 1/2^d$, corresponding to a motion of the apogee in the direction of increasing longitudes. The nodal or draconitic month, however, is slightly shorter than the sidereal month, measuring the return to a

[2] Cf. above p. 293.
[3] Cf. above p. 54.

node of the lunar orbit, i.e. to an intersection of the orbital plane with the ecliptic (cf. Fig. 14).

Finally one has the synodic month of about $29\,1/2^d$ which represents the interval between consecutive conjunctions, i.e. moments of equal longitude, of moon and sun. This interval is the basis for the calendaric months of 29 ("hollow" month) or 30 ("full" month) days.

Sufficiently accurate mean values are:

$$\begin{array}{lll} \text{draconitic:} & 27^d & 5;\ 5,35,48^h = 27;12,43,59,30^d \\ \text{sidereal:} & 27^d & 7;43,11,30^h = 27;19,17,58,45^d \\ \text{anomalistic:} & 27^d & 13;18,33,\ 6^h = 27;33.18,22,45^d \\ \text{synodic:} & 29^d & 12;44,\ 2,48^h = 29;31,50,\ 7,\ 0^d. \end{array} \tag{2}$$

In historical context one can expect at least three different meanings of the word "month": (a) a schematic month of 30 days, e.g. in business transactions or in general context (e.g. 5 months = 150 days); (b) the synodic month of 29 1/2 days, or, calendarically of either 29 or 30 days; (c) a sidereal month of 27 1/2 days.[4]

The tables by H. H. Goldstine for Full and New Moons (1973) give all true syzygies from −1000 to +1651 (in Babylon civil time).

§ 3. Fixed Stars

1. Proper Motion

The term "fixed star" is derived from the common experience that the relative position of the stars remains unchanged, in marked contrast to the "wandering stars," the planets. Greek astronomers were by no means convinced, however, that the apparent invariability of the positions of the fixed stars was a fact in the strict mathematical sense. When Hipparchus found that the longitudes of stars near the ecliptic had increased in the course of time he considered the possibility that stars near the ecliptic were in fact very slow moving planets[1] until he convinced himself that this motion was common to all fixed stars — now called precession. And we know from Macrobius (around A.D. 400) that there existed a school of astronomers who thought that only the vastness of the universe and the length of time prevents us from observing the motion of individual stars.[2] But only modern astronomy could furnish the proof for the correctness of this ancient hypothesis. Halley, in 1718,[3] investigating the change of latitudes of fixed stars caused by the decrease, since Ptolemy, of the obliquity of the ecliptic by about 20 minutes of arc, found that three stars (Sirius, Arcturus, Aldebaran) showed variations opposite to the expected trend, an observation confirmed beyond doubt twenty years later by J. Cassini.[4]

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[4] Cf., e.g., Neugebauer [1963].

[1] Cf. above p. 296.

[2] Commentary to Cicero's Dream of Scipio, I, Chap. 17, 16 (ed. Willis, p. 69, 23–30; trsl. Stahl, p. 158).

[3] Halley [1718], p. 736.

[4] Cassini [1738], p. 331–346.

These observations concern, of course, only the displacement of stars in a direction perpendicular to our line of sight (now called "*proper motion*"). The orthogonal component, the "*radial velocity*" of the star, has no influence on the apparent configurations and could only be detected by the Doppler effect in stellar spectra.[5]

There are only some 50 stars known whose proper motion exceeds 1 second of arc per year (three stars move more than 4 seconds), among which Sirius has a proper motion of about 1 1/3″ per year; thus the displacement of Sirius during two millenia amounts to about 40 minutes of arc.[6]

For historical purposes proper motion will rarely be of significance. There are only very few bright stars which show displacements of more than 1 minute of arc per century; Fig. 15 illustrates the change of position of Procyon and of Sirius from the time of Hipparchus (black dots) to modern times (white dots) in relation to Orion (for which proper motions cannot be shown in the scale of our diagram). Data for proper motions of all Bright Stars can be found in the Yale catalogue.

2. Yearly Parallax

We now ignore proper motion and assume that the sun is in a fixed relation to all other stars. We may also assume that the orbit of the earth with respect to the fixed stars is a circle of fixed position with the sun at its center. An observer on the earth O sees the sun S traverse a circle of radius r in one year. The observer in O, exactly as in the geocentric description of planetary motion, would also assign to any fixed star F a circular motion of radius r in a plane parallel to the plane of the solar motion (i.e. the ecliptic) about a center C which corresponds to the center of a planetary epicycle with the only difference that C now has the mean motion zero (cf. Fig. 16). In fact, however, no such yearly displacement of the fixed stars could ever be detected by visual observation. Only two explanations are possible: either the earth has a fixed position with respect to the stars (and consequently the sun has not), or the distance R of F from the sun is so great that the diameter $2r$ of the earth's orbit, seen from F, i.e. the "epicycle" of the star seen from O, subtends an angle which is smaller than the smallest angle distinguishable on a sighting instrument. The latter assumption leads to such colossal distances for the stars that the first alternative seemed to be more plausible. The fantastic emptiness of the universe, expressed in terrestrial dimensions, is indeed a conclusion which no sober scientist could have accepted (e.g. Brahe and Kepler) without overwhelming empirical evidence to the contrary.

The above described periodic displacement is known to modern astronomy as "*yearly parallax*" of the fixed stars. It was observed for the first time by

[5] First observed in 1868 by W. Huggins for the hydrogen spectrum of Sirius (Huggins [1868], p. 549).

[6] The term "proper motion" does not differentiate between the various causes of the apparent change in the position of fixed stars. Hence it includes not only the motion of a star itself but also the effect of the motion of the sun within our galaxy, as well as of the displacement of our galaxy with respect to extra-galactic objects.

F. Struve in 1836[1] and by F. W. Bessel in 1838[2] who found some of the comparatively very few stars which are near enough to show an optically measurable parallax.

Let F be a star of the ecliptic, λ its observed longitude, and $\odot$ the longitude of the sun at a given moment. Then the parallactic displacement p' of F with respect to its mean position C is obviously given (cf. Fig. 17) by $\mathrm{CD} = R \sin p' \approx R p'$. But since CF is always parallel to OS (as the radius of the epicycle for an outer planet) we can also express CD in the form $\mathrm{CD} = r \sin(\lambda - \odot)$. Consequently we have for the parallax

$$p' = \frac{r}{R} \sin(\lambda - \odot) = p \sin(\lambda - \odot)$$

where p denotes the maximum parallax

$$p = \frac{r}{R}. \tag{1}$$

If we now assume that the star F lies outside the ecliptic at a latitude β the plane of the circle of parallactic displacement is no longer seen edge on but from above or from below. For the same distance $\mathrm{OC} = R$ the longitudinal component CD of the parallax remains the same, but a latitudinal displacement p'' (cf. Fig. 18) will now appear, such that $\mathrm{DE} \approx R p''$. Since DF is parallel to the ecliptic we have also $\mathrm{DE} = \mathrm{DF} \sin \beta$. But from Fig. 17 we find $\mathrm{DF} = r \cos(\lambda - \odot)$, hence

$$p'' = \frac{r}{R} \sin \beta \cos(\lambda - \odot) = p \sin \beta \cos(\lambda - \odot).$$

This formula also holds near $\beta = 90$, i.e. if F is the pole of the ecliptic. Then the star is seen to describe a circle. Thus for all stars

$$\begin{aligned} p' &= p \sin(\lambda - \odot) \\ p'' &= p \sin \beta \cos(\lambda - \odot) \end{aligned} \tag{2}$$

which is the equation of an ellipse with half major axis p and half minor axis $p \sin \beta$. It is called the "*parallactic ellipse.*"

The purpose of measuring parallaxes is, of course, to find the distance R of the stars from our solar system, i.e.

$$R = \frac{r}{p}. \tag{3}$$

The value of R which corresponds to a parallax of one second of arc is called one "*parsec.*" It is the commonly used astronomical unit of distance. To travel at the velocity of light the distance of 1 parsec takes 3.26 years (one "*light year*" corresponds to a little less than $6 \cdot 10^{12}$ miles). All fixed stars are farther away from us than one parsec. The greatest parallax known is 0.8″ (proxima Centauri). The parallax of Sirius is 0.36″. A list of fixed stars nearer than 5 parsecs is given by Van de Kamp [1969].

[1] F. G. W. Struve, Stellarum duplicum et multiplicium mensurae micrometricae ... annis a 1824 ad 1837 .... Petropoli 1837 (p. CLXXII).

[2] First announcement (Oct. 1838) in M.N. 4; cf. Bessel [1838]. Then in A.N. 16 (Nos. 365, 366) col. 65 to 69 (cf. Bessel [1839]: reprinted, with some additions, Bessel, Abh. II, p. 217–236).

3. Names and Constellations

The first catalogue of stars in the modern sense of this term is found in the Almagest (VII, 5/VIII, 1), recording the positions of 1022 stars in longitude and latitude (for A.D. 137), distributed among 48 constellations which provide the background for the names of the individual stars, e.g., Sirius is the "very bright one in the mouth" of the "Constellation of the Dog." As I have discussed at length[1] there is no basis whatever for the assumption that a catalogue of this type existed before Ptolemy. On the other hand all later catalogues eventually descend from the Almagest.

The description of stars did not deviate in essential points from the inherited classical fashion until Joh. Bayer's "Uranometria" (1603). It was Bayer who introduced Greek letters (and, if needed, Latin letters) as designation of the single stars, alphabetically arranged according to brightness, such that, e.g., Sirius becomes "α Canis majoris."[2] Finally the never sharply defined boundaries of the pictorial constellations were replaced (by international agreement in 1928) by a system of arcs of constant right ascension and declination (for the equinox of 1875). At present the names also have been normed (for the sake of computers) to three letter words, e.g., CMa = Canis major. A list of these abbreviations and references to the literature which defines the now adopted boundaries is found, e.g., in Explan. Suppl. A.E., p. 495, or in $BS^{(2)}$ Appendix 2.

A list for 300 stars of positions in right ascension and declination from century to century from -4000 to $+1900$ is given in P. V. Neugebauer, Tafel I, in revised and modified form in Baehr, Tafeln. Schoch, Planetentafeln (p. 13 M) gives ecliptic coordinates for 49 stars of the zodiacal area in steps of 500 years between -2500 and 0. For highly accurate postitions the Yale catalogue of 9091 bright stars ($BS^{(3)}$) will be useful. Information concerning star catalogues for the professional astronomer can be found in Explan. Suppl. A.E., p. 147f.

Reliable data for the history of star catalogues are not easy to obtain. Knobel's paper [1877] is trustworthy only for the time after 1600 (p. 24ff.); for the earlier periods it can be used only to track down sources of continuously repeated mistakes. For Islamic catalogues cf. Kennedy, Survey; for the Byzantine period Kunitzsch, Sternverz.

The history of Arabic star names and their influence on the western terminology has obtained a solid basis through the work of Paul Kunitzsch (Sternnamen, Sternnomenklatur, and Alm.). For the Indoeuropean terminology cf. Scherer, Gestirnnamen. Allen, Star-names (1899), is still a useful work when consulted in conjunction with specialized modern publications.

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[1] Cf. above I E 2, 1 B and 1 C.

[2] Bayer gave 49 plates with fanciful pictures of constellations, framed by scales for degrees of longitude and latitude (presumably for 1603). Each plate is associated with one or two pages of text in which are given the names of the single stars in the Ptolemaic tradition but arranged according to magnitude. An initial column gives the order number in Ptolemy's list, a second column the new Greek letters. Kepler knew Bayer's work and quoted his notation (e.g. Werke 16, p. 41, 2). In the heading of his text Bayer collected what he considered to be names of a constellation; cf. for the resulting mistakes Boll, Sphaera, p. 450f., p. 456, p. 277. Bayer's text contains also astrological data; cf. above p. 954, n. 28.

§ 4. Geocentric Planetary Motion

Since ancient mathematical astronomy is concerned with the description of the planetary phenomena as seen by a terrestrial observer it is only natural to use a local coordinate system, e.g. with reference to a given horizon. With the discovery of the sphericity of the earth the advantage of real geocentric coordinates became evident. Heliocentric coordinates revealed their usefulness only after it had been realized that the earth and the planets were satellites of the sun. Finally gravitational theory made heliocentric procedures the only reasonable ones (lunar theory always excepted). One generally has this fact in mind when one speaks about the "simplification" introduced by the Copernican system in comparison to the Ptolemaic.

In the actual sequence of historical events such a simplification never existed, considering equivalent problems. Neither Ptolemy nor Copernicus had the faintest concept of dynamical conditions prevailing in the solar system. For their purely cinematic purposes, however, it is irrelevant at what step one introduces the transformation to geocentric coordinates which are eventually needed as long as one wishes to describe what an observer is expected to see. And in fact it is by far simpler to analyze planetary phenomena like stations and retrogradations by means of a geocentric model than for a moving earth; and the same holds for the basic relations between sidereal and synodic periods, phenomena which are significant only for a stationary observer. Hence here, as always, the choice of a coordinate system depends on the problem one wishes to solve and since ancient astronomy is mainly concerned with the cinematics of planetary motion and with the related phases the use of geocentric coordinates is usually preferable. And geocentric coordinates are unavoidable for the theory of sun dials and related problems (cf., e.g. Fig. 17, p. 1376).

In the course of our discussion of Greek astronomy we repeatedly had to make use of the transformation from heliocentric to geocentric coordinates and vice versa. Hence we do not need to repeat these considerations but can restrict ourselves to references to preceding chapters.

Ancient astronomy separated the theory of latitude from the determination of longitudes, a simplification justifiable by the smallness of the orbital inclinations (cf. Fig. 214, p. 1276 and I C 7, 2 B Figs. 220 and 221, p. 1280). The additional assumption of circular orbits is in principle of much greater significance; its relation to the actual elliptic orbits is shown below in Fig. 34, p. 1443. We simply have to accept this postulate as valid for our subsequent discussions.

The cinematic equivalence of heliocentric and geocentric motion is illustrated in Fig. 128, p. 1246. Since eccenters and epicycles are cinematically equivalent (cf., e.g., Fig. 51, p. 1220) a heliocentric model can also be transformed directly into a geocentric eccenter model.

For the description of the apparent motions as projected onto the celestial sphere distances play no role; consequently it was possible to remove the (mean) sun from the center of the epicycle of an inner planet and the famous accident of misjudging the actual distance of the sun (cf. above p. 112) made it possible to create a world picture of nested planetary orbits, i.e. a physical theory of geocentricity. Had it not been for this vital numerical error nothing in ancient

astronomy would have prevented the construction of a basically correct planetary system, e.g. of Tycho Brahe's type.

In I C 1, 2, p. 146 we listed the epicycle radii in their relation to the actual heliocentric planetary distances. As the latter increase the epicycles become smaller; for fixed stars the epicycles which correspond to the earth's heliocentric motion become the "yearly parallax" (cf. Fig. 16, p. 1437). For numerical data cf. below VI B 7, 2.

The Babylonian astronomers had discovered simple numerical relations between the number of the sidereal rotations of an outer planet and the number of synodic periods contained in the corresponding number of (sidereal) years; e.g. for Saturn

$$59 \text{ years} = 57 \text{ synodic periods} + 2 \text{ sid. rot.}$$

(cf. above p. 151; p. 390; p. 420, etc.). For the Greek cinematic models such a relation is a direct consequence of the rule that the radius of the epicycle CP must be parallel to the direction from O to the mean sun (cf. Fig. 158, p. 1257). On the basis of these considerations it is also easy to derive a theory of stations and retrogradations. Fig. 19, e.g., shows that ideally (ignoring, e.g., solar anomaly) the first and second stations of an outer planet must be symmetrically located to the opposition (Θ). For an inner planet it is clear that the stations are nearer to inferior conjunction than the maximum elongations. All this can also be represented in simple velocity diagrams (cf., e.g., Figs. 21 and 23, p. 1438 f.).

In the theory of latitudes the equivalence of geocentric epicycles and of a heliocentric model would remain valid as long as no eccentricities are involved (cf. Figs. 206 and 207, p. 1272). But because the planetary orbital planes go through the true sun, models based on mean conditions require complicated corrections, a fact particularly annoying in Copernicus' theory of latitudes. The insight into this situation constitutes a decisive progress in Kepler's theory of planetary motion.

Babylonian planetary theory is mainly directed toward the prediction of the planetary phases (the so-called Greek-letter phenomena; cf. p. 386 and Fig. 16, p. 1319). It is remarkable that one came very near to this goal by a purely numerical analysis of empirical data. The Greek geometrical models, now including latitudes, permitted Ptolemy to develop a general theory of first and last planetary visibility for arbitrary geographical latitudes (cf. I C 8, 5 and V C 4, 5 C; for fixed star phases V B 8, 1). Modern astronomy cannot improve on these results since they depend on climatic and other local conditions removed from our empirical or theoretical control.

With the introduction of elliptic orbits the simplicity of the heliocentric-geocentric transformations vanishes. It remains, of course, irrelevant whether one lets the sun or the earth move in an elliptic orbit about the other body as focus. It is also easy to derive the relations between the longitudes obtained for a Kepler motion as compared with an eccenter (cf. VI B 7, 4 and 5). But as soon as additional planets are involved no convenient direct transformation to geocentric coordinates (including latitudes) exists. The increase in observational accuracy demanded by the Renaissance astronomers, extended to all points of an orbit, not only to some characteristic phases, and made the traditional techniques obsolete beyond repair.

§ 5. Planetary and Fixed Star Phases

As "phases" of the planets or of fixed stars we denote phenomena which are related to the boundaries between visibility and invisibility due to the position of the star relative to the sun. The inner planets become invisible twice in each synodic revolution (cf. Fig. 20), the outer planets only once (Fig. 23). Fixed stars near the ecliptic will behave like extremely distant outer planets but for stars at a greater distance from the ecliptic such a generalization loses its value.

1. Planetary Phases

For the inner planets it is obvious from Fig. 20 that the period of invisibility at inferior conjunction (C) must be much shorter than at superior conjunction (S); this is equally evident from the velocity diagram Fig. 21. Since the planet is invisible whenever it is inside a cone with the line observer-sun as its axis it is also clear that the planet's latitude must greatly influence the duration of invisibility near inferior conjunction, in particular for Venus where OC is about 1/3 of the distance from O to the sun and where latitudes near $\pm 8°$ bring the planet close to the upper or lower rim of the cone of invisibility (cf. Fig. 22). Mercury, on the other hand, is so near to the sun that its maximum elongation may not be sufficient to remove the planet from the brightness of the sun. We have described in I C 8, 3 the ancient theory of these "paradoxical phases" of the two inner planets.

An outer planet (cf. Fig. 23) will be invisible for a comparatively short time when its motion in longitude is small. Therefore Saturn and Jupiter will not be invisible much longer than a nearby fixed star, i.e. about one month, while Mars follows the earth so closely that it remains hidden from sight about four times longer.

These qualitative considerations must be greatly refined before any numerical data can be obtained. For the planetary phases the variable inclination of the ecliptic to a specific horizon must come into play, combined with the planet's latitude. This was done in great detail by Ptolemy in Alm. XIII, 10 and his tables for the planetary phases at each of the seven climata from the Handy Tables remained standard during the Middle Ages (cf. above I C 8, 5, p. 259f. and V C 4, 5 C, respectively).

2. Fixed Star Phases

To begin again with a qualitative discussion let us assume a fixed star of longitude λ^* and near to the ecliptic. Then we will find in every year an interval of invisibility (cf. Fig. 24) between heliacal setting (Ω) and heliacal rising (Γ), in antiquity traditionally estimated to correspond to 30° of solar motion.[1] These two phases are of great significance for all primitive calendars, felt with Hesiod,[2]

[1] Cf. above IV D 3, 4 and V A 3.
[2] Cf. p. 573.

or in the role of Sirius in Egypt[3] and Mesopotamia.[4] Two more phases are important, known as acronychal rising (Θ_1) and setting[5] (Θ_2), respectively (cf. Figs. 24 and 25), similar to the lunar phases near full moon[6] or to planetary opposition[7] in Babylonian astronomy. In the interval between Θ_1 and Θ_2 neither the daily rising of the star nor its setting can be seen because the sun is not sufficiently deep below the horizon to make stars visible.

If we now consider stars which are no longer on (or near to) the ecliptic the sequence $\Omega \rightarrow \Gamma \rightarrow \Theta_1 \rightarrow \Theta_2$ of the phases can be changed. These changes in the order of the phases were already discussed in the early treatises on spherical astronomy, e.g., in the "Risings and Settings" by Autolycus (4th cent. B.C.). We have described the details in IV D 3, 4 and schematically represented in Fig. 56, p. 1368. Other formulations were given, e.g., by Tannery or by O. Schmidt in their publications on Autolycus.[8]

We have remarked before[9] that Ptolemy realized the possibility of such permutations in the order of the phases, similar to the case of Venus near inferior conjunction in combination with a high value of the planet's latitude.[10]

3. Tables

Modern tables for visibility problems, i.e. for the phases of the moon, of the planets, and of the fixed stars are all based more or less on the work of C. Schoch (1927, 1928) and P. V. Neugebauer (1922, 1929). Important criticism and modifications of the methods developed by these authors are given by van der Waerden [1942] and [1954, 2].

Schoch's tables (Planetentafeln and Ammiz.) were designed for problems of Babylonian chronology, in particular for the phases of Venus in combination with a strict lunar calendar, i.e. with the phases of the moon.[1] Clear directions for the use of these tables are found in P. V. Neugebauer, Astron. Chron. I, p. 167–172; cf. also van der Waerden [1942].

The adaptation of these tables (which include all five planets[2]) to the latitude of Babylon is not a serious limitation of their usefulness since it is only Babylonian astronomy that the planetary phases play an important role[3] which requires extensive numerical computations. Outside Babylonia planetary phases may occasionally play a role for chronological problems. Then P. V. Neugebauer,

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[3] Cf. III. 1.
[4] Cf. II In. 3, 3.
[5] Called "cosmic setting."
[6] Cf. p. 538.
[7] Cf. p. 386.
[8] Tannery, Mém. Sci. II. p. 228–232 (1886); Schmidt [1949].
[9] Cf. above p. 930.
[10] Above p. 241; cf. also above p. 1090.
[1] It is of no concern in the present context that Schoch's method was much too artificial to solve the chronological question for which it had been developed (cf. Neugebauer [1929]).
[2] The validity of dates for Mercury may be doubted; cf. P. V. Neugebauer [1938] col. 313 and Neugebauer [1951], p. 115f.
[3] Cf. above p. 386f.

Tafeln III (§ 26) and the revisions in Astron. Chron. I (§ 19) will provide the necessary information, extended to a wider range of geographical latitudes in P. V. Neugebauer [1938, 1]. Further revised and simplified tables are given in Baehr, Tafeln (p. 12–14).

Essentially the same works have to be consulted for fixed star phases: P. V. Neugebauer, Astron. Chron. I (§ 17), Baehr, Tafeln (p. 14f.). For Ptolemy's fixed star phases[4] Vogt, Griech. Kal. V is of primary importance; cf. in particular his table of the phases for 30 stars of first and second magnitude for the climata I to V and the time of Antoninus 1 (A.D. 137/138).[5]

Much in the very extensive modern literature on planetary and fixed star phases is without practical value since both the ancient sources and the modern attempts at mathematical formulations must introduce strong schematisations which may be wrong in individual cases. In fact the results of all computations can hardly be more than estimates of plausible mean values which can never reach the reliability and usefulness for chronological problems inherent in planetary positions or eclipses.[6]

§ 6. Lunar and Solar Eclipses

We consider in Figs. 26 and 27 the line earth-sun, i.e. the axis of the earth's shadow cone, as line of reference for the motion of the moon and for the axial rotation of the earth, N being its north pole. It is of no interest for our qualitative description that the line earth-sun rotates by a small angle with respect to the fixed stars during the time of the eclipse. Also with respect to scale and relative inclinations our figures are strongly schematized. Finally we disregard all refinements, e.g. the distinction between umbra and penumbra, i.e. between exterior and interior tangent cones.

The case of a lunar eclipse is described in Fig. 26. The moon moves around the earth in the direction from A to B, A being the position of first contact with the earth's shadow, B representing the end of the eclipse. An observer, located on the night side of the earth, sees the shadow entering the surface of the moon on its eastern side and leaving it at the western rim.

A solar eclipse (Fig. 27), as seen by an observer on the day side of the earth, presents the opposite situation. The dark disk of the moon meets the sun at its western side and leaves the sun at its eastern rim. If the solar eclipse is total, i.e. if the axis of the moon's shadow cone hits the earth, then the first impact of the shadow occurs at the boundary between illuminated and dark side of the earth that is to say at the point of sunrise. Similarly, the shadow leaves the earth at a point of sunset. Thus the curve which connects the points on earth met by the axis of the moon's shadow cone, the so-called curve of centrality (where the eclipse appears total), begins in the west and ends in the east. The details of its location

4 Cf. above V B 8, 1 B.

5 Griech. Kal. V, p. 54–61.

6 The numerous discussions about the arcus visionis of Sirius in Egypt are without interest for historical questions; cf. Neugebauer [1939].

depend, of course, on the relative position of the moon to the equator during the time of the eclipse.

Size and distance of the moon in relation to the sun are such that the vertex of the shadow cone of the moon can fall short of the earth — this will be the case when the moon is near the apogee of its orbit. The apparent diameter of the moon is then slightly smaller than the apparent diameter of the sun and the eclipse appears "annular" for an observer on the curve of centrality. It is also possible that a solar eclipse is total only for the middle section of its path, but annular at the beginning and at the end.

The maps in Oppolzer's "Canon der Finsternisse" (1885)[1] give approximate paths for the total (and annular) solar eclipses between −1207 and A.D. 2161, if visible to the north of the parallel 30° south. For the area of Egypt, Mesopotamia, and Asia Minor the paths of total eclipses are plotted for the time from −4204 to −900 in P. V. Neugebauer's "Spezieller Kanon der Sonnenfinsternisse" (1931). The period from −900 to A.D. 600 for the Mediterranean area is covered by F. K. Ginzel "Spezieller Kanon der Sonnen- und Mondfinsternisse" (1899) and the European area between A.D. 601 and 1793 is represented in J. Fr. Schroeter's "Spezieller Kanon" (1923). The tables of "Solar and Lunar Eclipses of the Ancient Near East from 3000 B.C. to 0" by Kudlek-Mickler (1971) are not only inconvenient to use (entry in maps: julian day numbers, not civil dates) but contain systematic errors in dates of lunar eclipses (cf. Sachs [1975]).

A list of solar and lunar eclipses mentioned in ancient sources between −771 and +592 is given by Boll in his article "Finsternisse" in R.E. 6, 2 (1909) col. 2329–2364. Cf. also the discussion of eclipse reports by Ginzel [1882/1884].

The idea of investigating the total path of a solar eclipse (instead of determining the magnitude and other circumstances for a given locality) is of modern origin — probably developed in the time of J. Cassini under the influence of the great theoretical interest of the Venus transits of 1761 and 1769.[2] The modern method of computation goes back to Bessel's "Astronomische Untersuchungen." II (Königsberg 1842).

Lunar eclipses, even of small magnitude, are easily discovered by a casual observer because the indentation of the illuminated disk of the full moon is very marked. The blinding brightness of the sun, however, allows partial eclipses to go unnoticed until more than half of the disk is obscured. Ginzel, Kanon, p. 14, gives details, reckoning 9 digits as limit for naked eye discovery of a partial solar eclipse.

Remark. It is an often repeated statement — from Aristotle[3] to modern textbooks — that the sphericity of the earth is demonstrated by the fact that the earth's shadow on the moon is always bounded by a convex arc.[4] This, of course, is mathematically inconclusive, quite aside from the fact that nobody ever explains

[1] For accurate references cf. the bibliography VI D 2.

[2] According to Lalande (Astron. II, p. 358, No. 1799; Bibl., p. 256, 1644) Dom. Cassini constructed in 1664 for the first time the path of a solar eclipse (visible in Ferrara) on a terrestrial map. But there was no total solar eclipse in 1664 and no publication of Cassini with the title quoted by Lalande seems to be known.

[3] De caelo II, XIV (Loeb, p. 252/253, Budé, p. 100).

[4] ἀεὶ κυρτὴν ἔχει τὴν ὁρίζουσαν γραμμήν.

how to establish the accurate nature of the observed curve. But even if we take it for granted that the shadow of some object on another unknown surface appears as a circle one should remember that there exists an unlimited number of shadow casting and shadow receiving bodies which produce identical shadow limits. Furthermore, assuming the sphericity of earth, moon, and sun the shadow curve on the moon is the intersection of a circular cone with a sphere, thus an algebraic space curve of the fourth order[5] and part of its projection on the celestial sphere is what we see as the boundary of the shadow.

The Saros. Two periodic functions must always have a common period (or at least as nearly common as one wishes) but ordinarily its length will be by far larger than the single components, even if one operates with reasonably close approximations. And all practical limits are rapidly transgressed if one considers more than two functions.[1] It is therefore an extraordinarily lucky accident that four elements which are decisive for the occurrence and special circumstances of eclipses have a nearly common period of only about 18 years. That is to say: within a period of 223 lunations the following elements return to their original values except for the following small corrections[2]:

$$\begin{aligned} \text{argument of latitude:} &\ -\ 0;28,36^\circ \\ \text{lunar mean anomaly:} &\ -\ 2;49,52 \\ \text{solar mean anomaly:} &\ +10;29,38 \end{aligned} \tag{1a}$$

to which corresponds a change in the distance from the node of the

$$\begin{aligned} \text{lunar perigee:} &\ +\ 2;21,11^\circ \\ \text{solar perigee:} &\ -10;58,16. \end{aligned} \tag{1b}$$

This implies the nearly simultaneous completion of

$$223 \text{ lunations} \approx 242 \text{ draconitic months} \approx 239 \text{ anomalistic months} \tag{2}$$

a fact which is the cornerstone of the Babylonian theory of eclipses,[3] in modern astronomy known under the name "*Saros*."[4] The data in (1a) show that two eclipses one Saros apart (i.e. about 6585 days apart) will be of very similar appearance since not only the lunar latitude is almost the same (changing only by about 2 1/2 minutes) but, most important, because also the lunar anomaly is nearly restored, a fact of which the Babylonian astronomers were fully aware, rediscovered by Newcomb.[5]

Since, however, the Saros does not exactly restore all elements, its repetition will slowly change the character of consecutive eclipses and finally lead to a situation which no longer corresponds to an eclipse. Hence one must construct higher

[5] A special case of such a curve is the "Hippopede" of Eudoxus; cf. above p. 678.

[1] This fact is very advantageous for the historical dating of Babylonian material (cf. Neugebauer [1937, 2] and ACT, p. 35–37).

[2] These data are taken from Newcomb [1879], p. 8.

[3] Cf. for additional details above II B 4, 2.

[4] Cf. for this terminology above p. 497, note 2.

[5] Newcomb [1879], p. 7: "There are, however, two remarkable chance relations connected with the Saros, which, so far as I know, have never been remarked."

cycles if one wishes to account for the slowly changing aspects of eclipses from Saros to Saros. For lunar eclipses one such cycle exceeds 800 years, for solar eclipses even 1200 years. This shows that for most historical purposes the "Saros" is an excellent guide in the search for related eclipses in records which will rarely cover more than a few centuries.

In practice short range relations between eclipses will be of greater interest than the repetition of Saros cycles. Such data for short intervals can be easily derived from approximations to the Saros, e.g. in the form of continuous fractions (as shown, e.g., below p. 1124).

§ 7. Kepler Motion

1. Definitions

We call "*Kepler Motion*" a cinematic model in which a celestial body P moves in an elliptic orbit (cf. Fig. 28), S being one of the two foci of the ellipse. If it were not for the mutual perturbations of the members of our planetary system the sun as well as the moon would move in this fashion with respect to the earth, and each planet with respect to the sun. For our purposes it suffices to assume this simplified situation, i.e. we consider each case individually as corresponding to a "two-body-problem" only. Consequently S represents the earth if P is the sun or the moon; but S is the sun if P is a planet. In the first case Π is the "*perigee*," A the "*apogee*." In the second case these words stand for "*perihelium*" and "*aphelium*", respectively.

Let C be the center of the elliptic orbit of P and let the time t being counted from a moment when P is at Π. If T is the "*orbital period*" of P, i.e. the time between two consecutive passings of Π by P then

$$n = 2\pi/T \tag{1}$$

is the "*mean motion*" of P and

$$M = nt = 2\pi t/T \tag{2}$$

the "*mean anomaly*." The "*true anomaly*," however, is the angle

$$v = \mathrm{PS}\Pi \tag{3}$$

and the difference

$$\theta = v - M \tag{4}$$

is the "*equation of center*."[1]

If P is the sun the orbital plane is by definition the ecliptic. This is also true in the other cases where latitudes are ignored as is common practice in ancient astronomy, at least in first approximation. Then the direction from S to the vernal point ♈0° belongs to the orbital plane and we can reckon "true longitudes" λ of P from this direction. Finally we define a "mean longitude" $\bar{\lambda}$ by means of

$$\bar{\lambda} = \lambda - \theta \tag{5}$$

[1] This term is of Arabic origin (*ta'dīl*). Cf. Nallino, Batt. I, p. 213 and II, p. 330.

which we can represent geometrically as longitude of a "mean" body $\overline{\mathrm{P}}$ which is in Π simultaneously with P and moves with the mean velocity M in a circular orbit with center S (cf. Fig. 28). Obviously

$$\mathrm{PS}\overline{\mathrm{P}} = \theta. \tag{6}$$

2. Parameters

All parameters which determine the planetary motion are subject to small secular variations, determined in modern astronomy by a complex interplay of dynamical theory and empirical data. Since for historical purposes high accuracy is usually not required[1] we consider the data listed in Table 1 sufficient. The values for A.D. 100 are computed on the basis of formulae given in the Explan. Suppl. A.E. p. 98 (for the sun) and p. 112f. (for ☿, ♀, ♂) and by Gaillot[2] (for ♃ and ♄). Roundings to the nearest minute of arc suffice for our purpose. The eccentricities given are, of course, the eccentricities of the elliptic orbits, not of the corresponding approximations by eccentric circles.

Table 1

	☉		
	A.D. 100	A.D. 1900	Δ per cent.
obliquity of ecl.	23;41°	23;27°	−0; 0.47°
long. of perigee	250;16°	281;13°	+1;43, 9°
eccentricity	0.0175	0.01675	−0.000042
precession			+1;23°

	☿			♀		
	A.D. 100	A.D. 1900	Δ per cent.	A.D. 100	A.D. 1900	Δ per cent.
incl. of orbit i	6;58°	7; 0°		3;23°	3;24°	
ascend. node Ω	25;52°	47; 9°	+1;11°	59;43°	75;47°	+0;54°
perihelium π	48; 0°	75;54°	+1;33°	104;49°	130;10°	+1;24°
eccentricity e	0.205	0.206		0.008	0.007	

	♂			♃			♄		
	A.D. 100	A.D. 1900	Δ per cent.	A.D. 100	A.D. 1850	Δ per cent.	A.D. 100	A.D. 1850	Δ per cent.
i	1;52°	1;51°		1;25°	1;19°		2;33°	2;30°	
Ω	34;54°	48;47°	+0;46°	81;25°	98;56°	+1; 0°	97; 4°	112;21°	+0;52°
π	301; 8°	334;13°	+1;50°	344; 6°	11;55°	+1;36°	55;52°	90; 7°	+1;57°
e	0.091	0.093		0.045	0.048		0.062	0.056	

[1] The situation is quite different when historical data are utilized for the correction of secular coefficients.

[2] Taken from P. V. Neugebauer, Astron. Chron. II, p. VIIf.

Table 2

	half major axis	eccentricity	sidereal period	syn. p.
☿	0.39 ≈ 0;23.20	0.206 = 0;12.22 ≈ 1/5	87.9ᵈ	115.9ᵈ
♀	0.72 ≈ 0;43.24	0.007 = 0; 0.25 ≈ 1/146	224.7ᵈ	583.9ᵈ
♂	1.52 ≈ 1;31.10	0.093 = 0; 5.35 ≈ 1/11	687.0ᵈ = 1.88ʸ ≈ 2ʸ	779.9ᵈ
♃	5.20 ≈ 5;12. 0	0.048 = 0; 2.53 ≈ 1/21	4332.6ᵈ = 11.86ʸ ≈ 12ʸ	398.9ᵈ
♄	9.54 ≈ 9;32.20	0.056 = 0; 3.18 ≈ 1/18	10759.2ᵈ = 29.46ʸ ≈ 30ʸ	378.1ᵈ
☉	1.00	0.017 = 0; 1. 1 ≈ 1/60	365.2564ᵈ	
☾	0.0026 ≈ 0;0.9.15	0.055 = 0; 3.18 ≈ 1/18	27.32166ᵈ	29.53059 = 29;31.50,7ᵈ

Comparison with the constant of precession shows that all apsidal lines show a small positive sidereal motion whereas the nodes recede.

Table 2 gives approximate data for the dimensions of the planetary system in astronomical units, i.e. in the scale of the earth's orbit.[3] The half major axis of an elliptic orbit can serve as the mean radius for a circular orbit traversed during the same time T in which the actual planet completes one sidereal rotation. The periods given are again only mean values. The sidereal period is the time between two consecutive returns of the planet to a direction from the sun (or from the earth) to a certain fixed star (the size of the earth's orbit being negligible for fixed star distances); the synodic period brings the planet back to the same phase with respect to the sun, e.g. conjunction. Note that the synodic periods tend toward the length of the year with increasing heliocentric distance.[4]

3. Kepler's Laws

The planet P moves in an ellipse (cf. Fig. 29), the sun S being in one focus ("*Kepler's first law*"), Π being the perihelium. We denote

half major axis ... a
half minor axis ... b
eccentricity ... e, thus $CS = ea$.

Construct QPR perpendicular to $C\Pi$ with $CQ = a$. Then the angle

$$QC\Pi = E$$

is the "*eccentric anomaly*."[1]

For corresponding segments in ellipse and circle we have

$$\text{area } C\Pi P = \frac{b}{a} \text{ area } C\Pi Q = \frac{b}{a} \cdot a^2 \pi \cdot \frac{E}{2\pi} = 1/2\, abE. \tag{1}$$

On the other hand

$$\text{area } C\Pi P = \text{triangle } CSP + \text{area } S\Pi P \tag{2}$$

[3] Introduced by Gauss, Theoria motus, Werke 7, p. 14. For the modern definition cf. Clemence [1965], p. 107/108.

[4] The limes represents, of course, the period of the yearly parallax of a fixed star; cf. above p. 1085.

[1] The angle $PS\Pi = v$ is called "*true anomaly*" (cf. p. 1095).

where

$$\text{triangle CSP} = 1/2\, e a \cdot \text{PR} = 1/2\, e a \cdot \frac{b}{a} \cdot a \sin E = 1/2\, a b e \sin E \tag{2a}$$

and according to the area theorem ("*Kepler's second law*")

$$\text{area } S\Pi P = ct \tag{2b}$$

where c is a constant and t the time such that for $t=0$ the planet was in Π. Hence from (1) and (2)

$$1/2\, a b E = 1/2\, a b e \sin E + ct$$

or

$$E - e \sin E = \frac{2c}{ab} \cdot t \tag{3}$$

which is "*Kepler's equation.*"

The quantity M in

$$\frac{2c}{ab} \cdot t = M = n \cdot t \tag{4}$$

is the "*mean anomaly*" as defined on p. 1095. Obviously

$$nT = 2\pi \tag{5}$$

gives the time of revolution T of the planet.

It can be shown that it follows from Newton's law of gravitation that

$$n = k \sqrt{1+m} \cdot a^{-3/2} \tag{6}$$

where k is a universal constant, m the mass of the planet in units of the solar mass. From (5) and (6) one obtains

$$n^2 a^3 = k^2 (1+m) = 4\pi^2 a^3 / T^2. \tag{7}$$

For the earth $m \approx 1/354\,700$. If we ignore for two planets P_1 and P_2 their masses we obtain from (7)

$$T_1^2 / T_2^2 = a_2^3 / a_1^3. \tag{8}$$

This is "*Kepler's third law*" in its original form,[2] the accurate relation (7) being only the result of Newton's dynamics.

4. Approximations

Since the mean anomaly $M = nt$ increases linearly with time it is natural to seek an expression for true anomaly v and for the eccentric anomaly E in terms of M. This will also lead to a comparison of an eccenter model with the corresponding Kepler motion.[1]

For our historical problems approximative solutions suffice in which we ignore terms containing the factor e^3 or higher powers of the eccentricity e. All subsequent computations make use of this simplification.

[2] Kepler, Epitome. Werke VII, p. 291, 9–21.

[1] Below p. 1100f.

We first establish the following lemma: if for angles α, β, γ, and given e

$$\beta=\alpha+e\sin\gamma \tag{1a}$$

then

$$e\sin(\alpha+e\sin\beta)\approx e\sin\alpha+1/2\,e^2\sin 2\alpha. \tag{1b}$$

Indeed:

$$\begin{aligned} e\sin(\alpha+e\sin\beta)&=e\sin\alpha\cos(e\sin\beta)+e\cos\alpha\sin(e\sin\beta)\\ &=e\sin\alpha(1-1/2e^2\sin^2\beta+\cdots)+e\cos\alpha(e\sin\beta-\cdots)\\ &\approx e\sin\alpha+e^2\cos\alpha\sin\beta. \end{aligned}$$

But it follows from (1a) that

$$\begin{aligned} \sin\beta&=\sin(\alpha+e\sin\gamma)=\sin\alpha\cos(e\sin\gamma)+\cos\alpha\sin(e\sin\gamma)\\ &=\sin\alpha(1-1/2e^2\sin^2\gamma+\cdots)+\cos\alpha(e\sin\gamma-\cdots). \end{aligned}$$

The only term free from a factor e or e^2 is $\sin\alpha$. Thus

$$\begin{aligned} e\sin(\alpha+e\sin\beta)&\approx e\sin\alpha+e^2\cos\alpha\sin\alpha\\ &=e\sin\alpha+1/2e^2\sin 2\alpha \end{aligned}$$

q.e.d.

We now apply this lemma to the Kepler equation[2] (above p. 1098 (3) and (4))

$$E=M+e\sin E \tag{2a}$$

which we iterate

$$E=M+e\sin(M+e\sin E). \tag{2b}$$

Hence from (1):

$$E=M+e\sin M+1/2e^2\sin 2M \tag{3}$$

which gives the eccentric anomaly in terms of the mean anomaly.

Next we express v in terms of E and then, by means of (3), in terms of M. Let Fig. 30 represent an ellipse of major half axis 1. Then

$$\begin{aligned} &r'+r=2 &&\text{hence } r'^2=4-4r+r^2\\ &r'\cos v'=2e+r\cos v\\ &r'\sin v'=r\sin v &&\text{hence } r'^2=4e^2+4er\cos v+r^2 \end{aligned}$$

and thus

$$1-r=e^2+er\cos v.$$

But (Fig. 30)

$$e+r\cos v=\cos E$$

thus

$$1-r=e\cos E$$

or

$$r=1-e\cos E$$

and

$$r\cos v=\cos E-e.$$

Hence

$$\begin{aligned} r(1+\cos v)&=(1-e)(1+\cos E)\\ r(1-\cos v)&=(1+e)(1-\cos E) \end{aligned}$$

[2] The "Kepler equation" appeared already in Islamic astronomy (in the theory of parallaxes) and was solved by an iteration method. Cf. Kennedy-Transue, A Medieval Iterative Algorism. Amer. Math. Monthly 63 (1956), p. 80–83.

and by division

$$\tan\frac{v}{2}=\sqrt{\frac{1-e}{1+e}}\;\tan\frac{E}{2}. \tag{4}$$

This relation can again be expanded in a series of powers of e, resulting in[3]

$$v=E+e\sin E+1/4e^2\sin 2E. \tag{5}$$

For E we now substitute (3):

$$\begin{aligned} v=M&+e\sin M+1/2e^2\sin 2M\\ &+e\sin(M+e\sin M+\cdots)\\ &+1/4e^2\sin(2M+2e\sin M+\cdots). \end{aligned}$$

Using lemma (1) for the second line we have

$$\begin{aligned} v=M&+e\sin M+1/2e^2\sin 2M\\ &+e\sin M+1/2e^2\sin 2M\\ &+1/4e^2\sin(2M+2e\sin M+\cdots). \end{aligned}$$

In the third line

$$\sin(2M+2e\sin M)=\sin 2M\cos(2e\sin M)+\cos 2M\sin(2e\sin M)$$

contributes only the term $\sin 2M$ which is free from e, hence

$$v=M+2e\sin M+5/4e^2\sin 2M \tag{6}$$

is the expression of the true by the mean anomaly, correct to e^2.

5. Eccenter Motion

Fig. 31 shows that

$$\sin\theta=e\sin\kappa=e\sin(\bar\kappa+\theta) \tag{7}$$

hence

$$\theta=\arcsin\theta=e\sin\kappa+1/6e^3\sin^3\kappa+\cdots.$$

Again disregarding terms with e^3 and higher powers of the eccentricity we have

$$\theta\approx e\sin\kappa=\sin\theta \tag{8}$$

and thus

$$\theta=e\sin(\bar\kappa+\theta)=e\sin(\bar\kappa+e\sin\kappa).$$

Making use of lemma (1), p. 1099 we have

$$\theta=e\sin\bar\kappa+1/2e^2\sin 2\bar\kappa$$

or

$$\kappa=\bar\kappa+e\sin\bar\kappa+1/2e^2\sin 2\bar\kappa \tag{9}$$

as expression of the true anomaly by the mean anomaly.

We are now in a position to compare an eccenter model with the Kepler motion. We assume that the periodic time T is the same for both models and that

[3] Cf., e.g., Smart, Spher. Astr., p. 118.

the planet is in Π for $t=0$; hence

$$\bar{\kappa}=M. \tag{10}$$

We denote the eccentricity of the elliptic orbit by e_K, of the eccenter model by e_P. Thus we obtain from (6) and (9):

$$\begin{aligned}\kappa-v&=\bar{\kappa}+e_P\sin\bar{\kappa}+1/2\,e_P^2\sin 2\bar{\kappa}\\&\quad-M-2e_K\sin M-5/4\,e_K^2\sin 2M\\&=(e_P-2e_K)\sin M+(1/2\,e_P^2-5/4\,e_K^2)\sin 2M.\end{aligned}\tag{11}$$

If we design the eccenter model in such a fashion that its eccentricity equals the distance between the two foci in the corresponding Kepler orbit (cf. Fig. 32, p. 1442), i.e., if

$$e_P=2e_K \tag{12a}$$

then we find,[1] that, accurate to e^2

$$\kappa-v=3/4\,e_K^2\sin 2M. \tag{12b}$$

Hence the maximum deviation occurs in the octants ($M=45+k\cdot 90°$) and amounts to

$$\max|\kappa-v|=3/4\,e_K^2. \tag{12c}$$

Application to the solar theory. Ptolemy assumed[2] for the sun an eccentricity

$$e_P=1/24=0;2,30=0.0417$$

whereas actually, at his time[3]

$$2e_K=0.0350.$$

Hence the error (11) of his solar model can reach near the quadratures

$$\kappa-v\approx e_P-2e_K=0.0067\cdot\frac{180°}{\pi}=0.384°\approx 0;23°$$

while exact adjustment according to (12a) would reduce the maximum error (now at the octants) to about

$$\frac{180°}{\pi}\cdot\frac{3}{4}\cdot 0.0175^2\approx 0.013°\approx 0;0.45°.$$

Al-Battānī found for the solar eccentricity[4]

$$e_P=0;2.4.45=0.0346$$

hence reducing the error near quadratures to about

$$\frac{180°}{\pi}(0.0346-2\cdot 0.01717)=\frac{180°}{\pi}\cdot 0.0003=0.017°\approx 0;1°.$$

The optimum would be again about 0;0.45° (near the octants, for $e_P\approx 0;2,3,30$).

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[1] Aaboe [1958], p. 212ff. has shown that (12a) can also be obtained by the requirement that the planet in both models is not only at the same time in the apsidal line but also in the quadrature ($M=90°$).

[2] Above p. 58.

[3] Cf. above p. 1096, Table 1.

[4] Nallino, Batt. I, p. 47.

Errors of this order of magnitude fall, of course, below the limits of accuracy of direct observations in antiquity. The only way to detect such discrepancies would be their influence on the time of eclipses but then the errors in the theory of the lunar motion would again obscure the situation.

The "Equant". Let us assume that an epicyclic model is properly adjusted to its corresponding elliptic model, i.e. that

$$e_P = 2e_K$$

or, to say the same (cf. Fig. 32), that the center D of the eccenter is the second focus of the ellipse of center C, the observer O=S being located at the other focus. We then know that within the square of the eccentricity the true anomalies are the same in both models:

$$v_P \approx v_K.$$

In other words, seen from O the longitude of the planet varies in the same way, regardless of whether the planet moves according to Kepler's laws on the ellipse (P_K) or according to the ancient model on the circle (P_P) of center D=T. Hence it is clear that within the same limits of accuracy $TP_P \approx TP_K$. But the radius TP_P rotates with mean velocity; hence the second focus in a Kepler ellipse functions as "*equant*" of the motion. i.e. an observer in T would see the planet move with constant angular velocity.

It should be remarked. however, that an eccenter model (even for small eccentricities) represents distances far less accurately than longitudes. Fortunately for ancient theory distances play practically no role in it.

The term "equant" does not occur with Ptolemy who uses only expressions like "center for the eccenter which produces the uniform motion" or similar circumlocutions. A more concise terminology apparently originated in Arabic astronomy, as early as with al-Farghânî (about A.D. 850).[5] To Copernicus the term "aequans" seems comparatively new since he says in Revol. V, 25[6] "(circulum) quem recentiores appellant aequantem." Kepler uses "aequans" freely; he say, e.g., "erit C punctum aequantis" or he speaks about "eccentricitas aequantis."[7]

6. "Elliptic" Orbits

It is only natural that in all discussions of Kepler motions the orbits are drawn as elongated ellipses. i.e. for eccentricities considerably greater than in the actual orbits. In order to give the reader a feeling for the difficulty of determining the true character of planetary orbits it will be useful to give some scale drawings of the actual conditions.

Fig. 33 shows one quadrant of an ellipse of semi-axes a and b, respectively. OF represents the eccentricity $e=\sqrt{a^2-b^2}$ and the perpendicular from D to the diagonal AB intersects the axes in the centers of curvature C_a and C_b for the

[5] Cf. Nallino. Batt. I. p. 237. note 1 and II. p. 238.

[6] Gesamtausg., p. 339, 9f.

[7] Werke III. p. 73. 12 and p. 172. respectively.

vertices A and B, respectively. The ellipse itself lies inside the larger and outside the smaller of these two circles of curvature.

In Fig. 34 we represent one quadrant of the orbit of Mercury in the same fashion. Table 2 (p. 1097) tells us that OF $\approx 12\ 1/2$ when OB = FA = 60. The resulting circles of curvature are AA′ with center C_a and BB′ with center C_b. The actual ellipse bridges the narrow gap between these two circles. Below the diagram for Mercury there is drawn, in the same scale, the triangle OC_aC_b for the "highly eccentric" orbit of Mars. It is clear that the thickness of the lines in our drawings would suffice to connect the two circles of curvature into one "ellipse," consisting of two almost identical circular arcs. For the remaining planets the scale of our drawing does not allow us to distinguish O from the centers of curvature and the orbital ellipse would have to be drawn as one single circle.

I think it is obvious from these diagrams that the elliptic shape of the planetary orbits could never have been detected on purely geometrical grounds, were it not for the variation of the velocity which depends on the eccentricity e and not on the shape of the curve with respect to O. The dimensions shown in Fig. 34 go far in justifying the use of an eccenter model.

§ 8. The Inequalities of the Lunar Motion

Since "ancient" – that is pre-Newtonian – astronomy is only concerned with the development of cinematic models a discussion of dynamical principles could be completely avoided here. Nevertheless, it seems desirable to describe at least in general outlines the connection between the phenomena established one by one during about two millennia of astronomical theory and practice and the explanations furnished by the theory of gravitation based on concepts of dynamics. Such an attempt is the more justified as it can give at least some idea of the type of argument which Newton used in his monumental discoveries, arguments which are usually no longer transparent in the modern form of analytical presentation which incorporates all of the enormous progress made by the great mathematicians of the 18th and 19th century.

The following is not more than a sketch, in simplest possible terms, of some typical applications of principles of mechanics to the solar perturbations of the earth-moon system. But this should suffice to make such empirically established facts as the rotation of the apsidal line or the recession of the nodes intelligible as elements of a much larger picture that eventually was to include the finest details in the motion of the planets and their satellites.

It is obvious that the forces of mutual attraction lie in the plane determined by the three bodies earth (E, mass m_1), moon (M, mass m_2), and sun (S, mass m_3), acting in the directions of the sides of the triangle EMS (cf. Fig. 35). For the understanding of the perturbations to which the moon is subject one is particularly interested in forces which do not coincide with EM and ES because such forces alone cause only fixed elliptic orbits for M and S. In order, furthermore, to use the earth as the reference system one must add forces acting equally on all three bodies but in a direction opposite to the attractions on E by M and by S. Conse-

quently not only a force $(m_1+m_2)/r^2$ is acting on M, beside one m_3/s^2 toward S, but also one m_3/r'^2 in a direction parallel to SE (cf. Fig. 35). Hence the perturbations of S on M consist in a force (depending only on m_3 and the distances r' and s) directed toward ES, i.e. toward the ecliptic. Since the latter will be used as plane of reference it is reasonable to split this force of perturbation into orthogonal components, one being vertical to the ecliptic (hence influencing the moon's latitude and nodes), the others in the plane of the ecliptic acting on the Kepler ellipse which one may assume for the undisturbed motion. We ignore the perturbations acting on the sun, assuming consequently that the sun moves in a fixed ellipse (or circle) about E.

Since we are aiming only at qualitative explanations we shall introduce simplifications whenever convenient, without worrying about the possible errors in a quantitative treatment. The eccentricity, e.g., of the lunar orbit is only about 0.055 hence we shall occasionally consider the orbit as simply circular.[1] The inclination of the orbit can also be ignored unless we are dealing specifically with latitudes and nodes. The angle θ at S in Fig. 35 is at most about 9 minutes of arc and thus may be assumed to be zero in certain cases. On the other hand our figures must greatly exaggerate small forces or eccentricities, badly distorting relative distances. For $r\approx 2$, e.g., one should have $r'\approx s\approx 900$. The masses are $m_1=1$. $m_2\approx 10^{-2}$, $m_3\approx 3\cdot 10^5$ thus the vectors at E should be $4\cdot 10^{-2}$ and 3/8, at M 1/4 and 3/8, at S $8\cdot 10^{-5}$ and $8\cdot 10^{-7}$.

In a discussion of the discoveries of lunar inequalities actual numerical data are required. For this aspect of the problem the reader should turn to p. 1106ff.

It must be our first goal to obtain an overall impression of the forces acting on the moon. We assume a circular lunar orbit and call a radial force R positive when it is directed away from E, while a tangential force T is reckoned positive when agreeing with the orbital motion of the moon. Let us assume that all constants are normed in such a fashion that $R=-1$, $T=0$ for the undisturbed circular motion. Let furthermore η be the elongation of the moon from the sun (cf. Fig. 36) and w the square of the ratio n'/n where n and n' denote the rotational velocities of sun and moon respectively, thus

$$w=\left(\frac{n'}{n}\right)^2\approx\left(\frac{27.32}{365.26}\right)^2\approx\frac{1}{178.7}\approx\frac{1}{180}. \tag{1}$$

It is then possible to show by elementary means[2] that

$$\begin{aligned} R&=-1+\frac{3w}{2}\cos 2\eta+\frac{w}{2}\\ T&=-\frac{3w}{2}\sin 2\eta. \end{aligned} \tag{2}$$

This tells us that in the syzygies ($\eta=0$) the moon is pulled away from E with the force $2w$, in the quadratures ($\eta=90°$) attracted to E with the force w. Formula (2)

[1] Cf. p. 1443, Fig. 34.

[2] Cf., e.g., Möbius, Werke 4, p. 155–165 or Herschel, Outlines art. 675f. (giving erroneously $\eta=64;14$ instead of (3)).

also allows a simple geometrical construction of the perturbing forces $R' = R + 1$ and T at intermediate elongations (cf. Fig. 36). Obviously $R' = 0$ for $1 + 3\cos 2\eta = 0$ which is the case for

$$\eta = 54;44°. \tag{3}$$

Hence we find for the perturbations a distribution as shown in Fig. 37, reminiscent of the tidal forces which produce high tides always at two diametrically opposite points of the earth.

The forces of perturbation are constructed in Fig. 37 to scale among themselves. A scale relation to the constant attraction $R = -1$, however, is out of the question since (1) shows that the constant vector ME should be about 180 times as long as w. It is a most remarkable fact that the relative minuteness of the disturbing forces in combination with almost invisible deviation from circular orbits produces such drastic effects as the motion of the apsidal line or of the line of nodes.

In order to describe the effect of the tangential and of the normal component of a perturbing force one makes use of a relation which connects the length a of the major semiaxis of a Kepler ellipse with the velocity v in the orbit at a point which is at a distance r from the focus E:

$$\frac{1}{a} = \frac{2}{r} - \frac{v^2}{c} \tag{4}$$

c being a constant. This formula shows that for a given r the semiaxis a increases when v increases and that the rate of change of a is given by

$$\frac{da}{dt} = \frac{2a^2 v}{c} \cdot \frac{dv}{dt}. \tag{5}$$

Let Fig. 38 represent the instantaneous elliptic orbit of M moving in counterclockwise direction. Let an additional force operate at M, acting in tangential direction. Such a force will increase v and therefore a but it will not change the direction of the tangent. Therefore the angles of EM and of MF with the tangent remain the same and the new semiaxis $a' > a$ will move the second focus from F to F′ such that $MF' = 2a' - r$. Consequently the apsidal line will recede from the position EF to EF′ and the eccentricity will increase from $e = 1/2\,EF$ to $e' = 1/2\,EF'$.

If M were to receive the same tangential acceleration at any other position of its orbit the locus of F′ would be a little circle with center F and diameter $a' - a$. The apsidal line will then experience a maximal displacement when M is located practically perpendicular above or below F but then e remains unchanged. The effect on e, however, has a maximum for M in apogee or perigee when the apsidal line coincides with the original one.

If M were subject to the same tangential perturbation during a complete revolution the apsidal line and e would oscillate about a mean position. Fig. 37, however, shows that the tangential forces of the solar perturbations vary greatly in amount and direction during each synodic revolution. The mathematical theory of perturbation shows that the accumulated effect of these varying forces is a progressive displacement of the apsidal line, well-known since antiquity.

The effect of a normal force will tend to change the curvature of the orbit, i.e. it will change the direction of the tangent. The velocity, however, will not be affected by a force normal to the direction of motion; thus (5) shows that a remains unchanged. Consequently (cf. Fig. 39) the direction MF will be changed by twice the amount of the change in tangential direction but the new focus F′ will be at the same distance from M, i.e. MF = MF′. Hence the apsidal line recedes to a position EF′ and the eccentricity increases to $e' = 1/2\,EF'$.

To obtain by this kind of arguing even qualitatively correct results it is necessary to make e small enough (still much larger than in fact in order to remain recognizable) such that the orbit becomes nearly circular (cf. Fig. 39). That means that MF = MF′ is nearly constant and not very different from $EM = r(\approx a)$. For the same change of direction at different positions of M the arc FF′ will remain nearly of constant length and almost perpendicular to MF. Consequently F′ is seen to move in a small circle about F. The maximum displacement of the apsidal line will now occur when M is at the apogee or perigee, with no change of e; a position of M nearly vertically above or below F causes a maximal effect on e but none on the apsidal line. The variability of the perturbing forces again greatly modifies this simple picture.

Since the orbit of the moon is inclined toward the ecliptic the perturbation caused by the sun also produces a component perpendicular to the moon's orbit directed toward the ecliptic (cf. Fig. 35, p. 1443). Such a force will tend to change the inclination of the instantaneous orbit and with it the position of the nodal line. In a situation as depicted in Fig. 40, e.g., the new nodal line will come nearer to M in comparison to the undisturbed orbit; thus, with respect to the motion of M, the nodal line recedes. If one applies the same argument in each of the four quadrants with respect to the line of syzygies one sees that a force directed toward the ecliptic always causes a recession of the nodal line. Thus is explained a phenomenon well known since antiquity, comparatively easy to detect through the shifting positions of lunar eclipses. The changes of inclination of the orbit, however, is only periodic since opposite effects take place on opposite nodes. Hence the period of this perturbation is only about 14 days. Another and much larger periodic change of the inclination depends on the relation of the direction of the nodal line to the direction to the sun; consequently this period amounts to about half a year.

1. Longitude

The modern theory of perturbations gives the following major terms for the correction of the mean longitude $\bar{\lambda}$ of the moon, leading to the true longitude λ:

$$
\begin{array}{ll}
\text{(I)} & \lambda = \bar{\lambda} - 6;17,19^\circ \sin\bar{\alpha} + 0;12,48^\circ \sin 2\bar{\alpha} \\
\text{(II)} & \qquad - 1;16,26^\circ \sin(2\bar{\eta} - \bar{\alpha}) \\
\text{(III)} & \qquad + 0;39,30^\circ \sin 2\bar{\eta} \\
\text{(IV)} & \qquad + 0;11,\ 9^\circ \sin\bar{\alpha}_\odot \\
\text{(V)} & \qquad - 0;\ 6,54^\circ \sin 2\bar{\omega} \\
\text{(VI)} & \qquad - 0;\ 2,\ 5^\circ \sin\bar{\eta} + \cdots
\end{array}
$$

where we are using the same notation as in I B for the Ptolemaic model, i.e. $\bar{\alpha}$ for the mean anomaly,[3] $\bar{\eta}=\bar{\lambda}-\bar{\lambda}_{\odot}$ for the mean elongation; $\bar{\omega}=\bar{\lambda}-\bar{\Omega}$ where $\bar{\Omega}$ is the (mean) longitude of the ascending node.

In the syzygies, i.e. for $\bar{\eta}=0$ or $180°$, the terms (I) and (II) combine to $\approx -5° \sin\bar{\alpha}$ which is the "first inequality" of the ancient lunar theory. The term (II) by itself is called the "evection"; it maximizes the effect of the anomaly in the quadratures, i.e. at $\bar{\eta}=\pm 90°$. For its relation to Ptolemy's "second inequality" cf. below p. 1108. The term (III) is the "variation" discovered by Tycho Brahe (cf. below p. 1109f.). For the "annual equation" (IV) which has the anomalistic year for its period cf. below p. 1110.

The next term, (V), is the "reduction to the ecliptic," expressing the fact that the plane in which the moon moves is inclined to the ecliptic in which longitudes are measured. Ptolemy knew this term but considered its effect small enough to be ignored.[4] The last term, (VI), is called the "parallactic equation" because its coefficient is proportional to the ratio of the mean distance of the moon to the mean distance of the sun, i.e. to the ratio of the sine of the solar to the sine of the lunar parallax.[5] Its effect falls below the accuracy of naked eye observations.

2. Latitude

The description of the lunar latitude β requires only two terms:

$$\begin{array}{ll} \text{(I)} & \beta \approx 5;9° \sin\omega \\ \text{(II)} & \quad + 0;8,48° \sin(2\eta-\omega)+\cdots \end{array}$$

since the coefficients of all subsequent terms remain between $\pm 0;0.30°$. The term (I) corresponds to the ancient model of an orbital plane of fixed inclination. Term (II) is the analogue to the evection of the longitudes. For Tycho Brahe's account of this effect cf. below p. 1111.

As a consequence of the two above given terms one can derive equivalent relations for an instantaneous position of the orbit.[6] Thus one finds that the orbital inclination i undergoes periodic variations around the mean value $\bar{i}=5;9$:

$$i=\bar{i}+0;8,48° \cos 2(\lambda_{\odot}-\bar{\Omega}). \tag{1}$$

Hence, when the sun is in the nodal line, the inclination takes its extremal value of about 5;18°, but i is at its minimum, $\approx 5;0°$, when the direction to the sun is perpendicular to the nodal line.

Similarly the recession of the nodal line is not uniform (as was assumed until Tycho Brahe) but the true longitude Ω of the ascending node oscillates about the

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[3] Modern astronomy counts the anomaly always from the perigee whereas our $\bar{\alpha}=0$ at the apogee; consequently signs in (I), (II), and (IV) differ from the modern norm.

[4] Almagest IV, 6 (Man. I. p. 219, 10–15).

[5] Cf., e.g., Moulton, Cel. Mech. No. 196 (p. 352).

[6] Cf., e.g., Moebius, Mech. d. H., § 142.

mean value $\bar{\Omega}$ according to

$$\Omega \approx \bar{\Omega} + 1;38° \sin 2(\lambda_{\odot} - \bar{\Omega}). \tag{2}$$

For eclipses, when the sun lies in the nodal line, this effect vanishes.

3. Bibliographical and Historical Remarks

The discovery and clear distinction of all lunar perturbations which lie within the limits of accuracy inherent in naked eye observations must be counted among the most remarkable achievements of early science. Thus was prepared the basis upon which Newton's dynamics could build and uncover a unifying principle of explanation for a great variety of apparently disconnected effects.

The following is intended to relate the discovery of the major short periodic perturbations to some of the data in the modern theory.

A. Evection

The two greatest contributions of Ptolemy to celestial mechanics are undoubtedly his analysis of the lunar inequality now known as "evection" and the introduction of the "equant" into planetary theory.[1] In I B 4 we discussed in detail Ptolemy's dealing with the "second inequality" of the lunar motion. The customary identification with the modern "evection" ((II) on p. 1106) is not strictly correct in a mathematical sense since Ptolemy assigned to the second inequality a maximum of 2;39° beyond the 5;1° of the "first inequality" (our (I)). It is therefore only the total 7;40° and the phase which agree closely with the modern sum (I)+(II) ≈ 7;34° (cf. p. 1106).

The situation is still more complicated by the effect of the so-called prosneusis ("inclination"[2]). Stumpff has shown[3] that Ptolemy's lunar theory is the equivalent of the following expansion (ignoring the reduction to the ecliptic, i.e. (V) on p. 1106):

$$\begin{array}{ll} \text{(I)} & \lambda = \bar{\lambda} - 6;14° \sin \bar{\alpha} + 0;19° \sin 2\bar{\alpha} \\ \text{(II)} & \qquad - 1;16° \sin (2\bar{\eta} - \bar{\alpha}) + \cdots \end{array}$$

which not only compares favorably with the expansion given above (p. 1106) but which would produce a term with $2\bar{\eta} + \bar{\alpha}$ as argument had the mean anomaly not been reckoned according to the norm of the prosneusis.

The only essential improvement of Ptolemy's lunar theory during the Middle Ages consists in the replacement of his crank mechanism[4] by a nearly equivalent double epicycle arrangement which had the great advantage of avoiding the exaggerated changes of the geocentric distance of the moon, vitiating Ptolemy's

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[1] For the origin of the term "evection" cf. below p. 1109; for the "equant" above p. 155.
[2] Cf. I B 4, 2 B.
[3] Stumpff, Himmelsmech. I, p. 38–41. Bullialdus, Astr. Phil. Book III, Chap. XII, p. 173f. suggested to see in Ptolemy's prosneusis an effect of the third inequality, the "variation."
[4] Cf. above p. 85 and Fig. 78 there.

model. This improvement is due to Ibn ash-Shāṭir of Damascus (about 1350)[5] and again, some 150 years later, to Copernicus.[6]

Appendix. The term "evection". Ptolemy's "second inequality" of the lunar motion received its now generally accepted name "evection" by Ismael Boulliau (1605–1694) because of a peculiar model of planetary and lunar motion, designed by him to reconcile Kepler's elliptic orbits with the doctrine of uniform circular motion which alone was supposed to maintain itself eternally. The basic idea, as described in the "Astronomia Philolaica" of 1645,[7] consists in considering an elliptic orbit as the result of the intersection of a certain skew circular cone by the orbital plane. The inclination of the cone to the plane of its circular base is chosen such that the axis of the cone meets the orbital plane in its second focus while the sun (for the planetary orbits) or the earth (for the moon) occupy the other focus. The motion of the planet in its orbit is regulated by the uniform rotation[8] of the generating line of the cone passing through the planet. In this way the planet participates in the uniform and circular rotation of the conic surface, resulting in an orbital motion slow at the top and fast at its lower end, i.e. at the perihelium. This "explains" the first inequality in a Kepler orbit, ignoring, of course, the equal area law.[9]

Real difficulties are caused, however, by the additional inequalities of the lunar motion. Here Boulliau takes refuge in a desperate remedy:[10] he makes the second focus movable on a little circle such that this focus coincides with the earth only once in each rotation (which progresses with the velocity of the double elongation). Fortunately the details of this construction do not need to be described here; we only need to mention the fact that Boulliau accounts for the second lunar inequality by a periodic removal of the second focus from the earth, a process which suggested the name "*evectio*." By similar arguments he gave the third inequality the name "*reflectio*"[11] but here Brahe's and Kepler's "*variatio*" prevailed.

B. Variation

It was only with Tycho Brahe that the lunar theory transgressed the traditional framework. In a letter of August 12, 1595, to Hagecius[12] he announced the

[5] Roberts [1957].

[6] Cf. Neugebauer [1968, 2].

[7] This huge tome of 725 pages of text and tables also contains the first publication of observations made around A.D. 500 and ascribed by Boulliau to "Thius" (Astron. Philol., Book III, p. 172), discussed later on by Delambre in HAA I, p. 318f. and shown by Tannery (Mém. Sci. II, p. 125f.) to belong to Heliodorus and his contemporaries. Cf. above p. 1039.

Of importance is also the publication of the "Persian Tables" (Astron. Philol., second part p. 211–232), brought to Constantinople and translated around 1300 by Gregory Chioniades and commented on in the middle of the 14th century by Georgios Chrysokokkes. Cf. Pingree [1964] and Kunitzsch [1964].

[8] In fact the motion is uniform only in so far as the generating line progresses with constant angular velocity along the circular base. But since the axis of the cone is inclined to the plane of the base the rotation is neither circular nor uniform with respect to the axis.

[9] Delambre, HAM II, p. 151f. computed the errors (which reach only about 0;8° for Mars). For good measure he added the errors for Pallas and Uranus to the sins of poor Boulliau.

[10] Bullialdus, Astron. Philol. Book III, Chap. I, p. 104f. and Chap. X, p. 155ff. Also Tables, p. 127–134.

[11] Astron. Philol. Book III, Chap. XI, p. 160 and 161, respectively.

[12] Brahe, Opera VII, p. 370, 17–29; English translation: Thoren [1968], p. 165.

discovery of a new periodic inequality which he called "variatio."[13] This perturbation depends, as the evection, on the elongation, but not on the anomaly (cf. above p. 1106 (III)); its effects are described by Brahe in his "Progymnasmata."[14] The circumstances of the discovery are fairly well known thanks to the investigations of V. E. Thoren [1968].

It still does not seem superfluous to mention a controversy in the French Academy, started in 1836 by L.-Am. Sédillot who insisted that Abū'l Wafā (about A.D. 970) should be credited with the discovery of the "variation." This assertion, based on some fragmentary passages out of context, was finally disproved by Carra de Vaux [1892] who published the whole available text. From this it is clear that it was only a superficial description of Ptolemy's "prosneusis" which was mistaken by Sédillot to refer to a new inequality.

C. *Annual Equation*

The existence of an irregularity in the motion of the moon with the anomalistic year as period (cf. above p. 1106 (IV)) was discovered independently by Kepler and by Brahe. Kepler was led to the problem by his attempt to explain by a little fraud[15] the discrepancy between his predictions and the actual events at the solar eclipse of March 7, 1598.[16] Speculating about a delay of the lunar motion in the winter, an acceleration in the summer he came close to the explanation of the phenomenon as caused by increased solar attraction near the perigee of the solar orbit.[17]

About the same time Brahe had already come to a more accurate description of this new inequality, of course without looking for any physical cause. He realized the necessity of either further complicating the cinematic model, or — an even more desperate remedy[18] — of modifying the equation of time when applied to the motion of the moon by dropping the component which is caused by the solar anomaly.[19] Consequently Brahe tabulated for the moon an "equation of time" depending only on right ascensions.[20] The resulting correction has proper phase and sign but its amplitude is only about 0;4,30° instead of 0;11°.[21] Kepler knew since 1598/1599 of Brahe's discovery of an annual equation and the

[13] Kepler, Werke 7, p. 461, 8.

[14] Brahe, Opera II, p. 101, 4–19; German translation: Anschütz [1886/1887], p. 168.

[15] For these rather comical events cf. Anschütz [1886/1887], p. 202–207.

[16] Kepler (in Graz) was not only wrong with respect to time and magnitude of the eclipse but he also gave a path from ... Spain, Sardinia, Greece, Egypt, Jerusalem, Babylon, to Persia (Kepler, Opera I, p. 396) which he could not have found by any computation (cf. Schröter, Kanon, Chart 121a and our Fig. 41). A year later he changed the path to the "gefrornen Meer hinder Schotland, Nordwegen, Moschau" (Kepler, Opera I, p. 409; Anschütz l.c. p. 204), thus again simply assuming a west-eastern direction.

[17] Letter to Herwart of Jan. 29, 1599 (Kepler, Werke 13, p. 284, 137–155; translated: Anschütz [1886/1887], p. 209–210. Cf., however, for Kepler's final opinion Anschütz l.c. p. 4–12.

[18] Apparently suggested by Longomontanus; cf. Kepler, Werke 15, p. 343, 37–44; also Anschütz [1886/1887], p. 165f.

[19] Cf. for the equation of time above p. 1081.

[20] Progymnasmata I; Brahe, Opera II, p. 101f.

[21] Cf. above p. 1106 (IV). In a letter to Herwart (August 1600) Brahe estimated, however, this inequality correctly as a little in excess of 0;10° (Brahe, Opera VIII, p. 345, 25f.).

attempts of adjusting the lunar model and the tables accordingly.[22] It seems clear that it was only through Brahe's work that the annual equation became a recognized part of the lunar theory.[23]

D. Latitude and Nodes

Around 1588 Brahe had come to the conviction that the inclination i of the lunar orbit is variable.[24] In 1599, in a letter to Herwart,[25] he specified as limits 4;58° at syzygies and 5;20° at quadratures. At the same time he stated that also the nodes are subject to vibrations around their mean positions with 1;35° as amplitude.

In the Progymnasmata[26] the limits for i are given as 4;58,30° and 5;17,30°, respectively and a definite cinematic model is constructed based on these two empirical parameters (cf. Fig. 42). Let A be the pole of the ecliptic, B the pole of the mean lunar orbit, thus $AB=\bar{i}$. The pole P of the instantaneous orbit is assumed to rotate about B on a small circle of radius r with the angular velocity 2η of the double elongation. At syzygies P is at C and the inclination of the orbit is at its minimum $\bar{i}-r$; at quadratures P is at D and the inclination is greatest $\bar{i}+r$. In both cases the ascending node N is at its mean position, i.e. N is the pole of the circle ACBD. In Brahe's model $\bar{i}=5;8°$ and $r=0;9,30°$. In the octants, however, i.e. when $2\eta=90$ or 270, the inclination is at its mean value but the pole, e.g. at E, moves the orbital plane up such that $NF=BE=r$ and the node is displaced from N to G. Hence $NG=r/\sin\bar{i}=0;9,30/0;5,22=1;46°$ which is Brahe's value for the amplitude of the displacement of the true node with respect to the mean node, the "prosthaphaeresis nodorum."

It is also easy to show[27] that Brahe's model is an essentially correct representation of the relations (I) and (II), p. 1107 and therefore also of the variation of the inclination (1) and the position of the nodes (2). Let in Fig. 42 M be the moon on its instantaneous orbit, thus $PM=90°$. Let $\bar{M}$ be a position on the mean orbit of nearly the same distance $\omega\approx\bar{\omega}$ from the mean node N. Thus $\bar{M}B=90°$ and $M\bar{M}\approx PP'$ where $\bar{M}PP'=90°$. Consequently we have a right triangle BP'P in which $PP'=r\sin\gamma$, γ being the angle P'BP. Since the arc CP is, by construction, 2η we have

$$\gamma=90-(2\eta-(90+\omega))=180-(2\eta-\omega)$$

thus $\sin\gamma=\sin(2\eta-\omega)$. All angles are so small that $M\bar{M}$ can be taken as the change of lunar latitude between instantaneous and mean orbit. Hence we have

$$\beta\approx 5;8\sin\omega+0;9,30\sin(2\eta-\omega)$$

which is indeed a close approximation of (I)+(II), p. 1107.

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[22] Cf., e.g., letter of Herwart to Kepler of July 25, 1600 (Kepler, Werke 14, p. 138, 61–72; also Anschütz [1886/1887], p. 164).

[23] Cf. Anschütz [1886/1887], p. 5ff.

[24] Cf. his letter to Rothmann of 1589 Febr. 21 (Opera VI, p. 170, 3–18) in which he reports about a correspondence with Brucaeus (Opera VII, p. 151, 28ff.). Cf. also Opera XI, p. 163 (observations in 1587).

[25] Opera VIII, p. 161, 13–19.

[26] Opera II, p. 121, 40–122, 1; p. 122, 38–40; p. 123, 1.

[27] Cf. Dreyer in Brahe, Opera II, p. 447 (following Lalande, Astronomie II, 2nd ed. (1771), p. 244, 3rd ed. (1792), p. 191, No. 1495). Cf. also Dreyer, Brahe, p. 344, n. 2 and Herz, Bahnb. II, p. 116.

E. Bibliographical Notes

A reader with an independent mathematical training will not look for references on celestial mechanics or lunar theory in the present work. It might be useful, however, to point to the existence of some "antiquated" literature in which an attempt had been made to explain in an elementary fashion the physical basis for the theory of perturbations. Such works were written in order to help in the study of Newton's "Principia" in which the lunar perturbations were explained much in the same way. Thus the following books may be mentioned:

George Bidell Airy, Gravitation, an elementary explanation of the principal perturbations in the solar system. London 1834.[28]

August Ferdinand Möbius, Die Elemente der Mechanik des Himmels auf neuem Wege ohne Hülfe höherer Rechnungsarten dargestellt. Leipzig 1843. Reprinted in Möbius, Gesammelte Werke IV, p. 1–318.[29]

John F. W. Herschel, Outlines of Astronomy. London 1849. Chaps. XII to XIV concern the theory of perturbations.

Hugh Godfray, An elementary treatise on the Lunar Theory, with a brief sketch of the history of the problem before Newton. 3rd ed., London 1871.

Norbert Herz, Geschichte der Bahnbestimmung von Planeten und Kometen. I, Die Theorien des Altertums. II, Die empirischen Methoden. Leipzig, Teubner. 1887, 1894. (The title is misleading since also the theory of the moon is discussed in detail; both parts are largely historically oriented.)

The thesis of P. Kempf [1878] is particularly concerned with Ptolemy's lunar theory. Of modern works on celestial mechanics one should mention K. Stumpff, Himmelsmechanik I (Berlin 1959) which contains two introductory chapters on the development from Ptolemy to Kepler. F. R. Moulton, An Introduction to Celestial Mechanics (2nd ed., New York 1914) gives many references to the historical development, in particular for the theory since Newton.

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[28] Airy (1801–1892), Astronomer Royal (1834–1881). Cf. Autobiography of Sir George Biddell Airy, ed. by Wilfrid Airy. Cambridge 1896.

[29] Möbius (1790–1868), a pupil of Gauss, professor of astronomy and mathematics in Leipzig; cf. Klein, Entw. d. Math. I, p. 116–119.

C. Mathematical Concepts

§ 1. Sexagesimal Computations

Numbers expressed in a system of basis 60 are called *sexagesimally* written numbers. We write the single digits which range between 0 and 59 in our ordinary decimal notation.[1] We separate consecutive digits from each other by commas; we put a semicolon between integers and fractions. Thus 1,25 means 85 and 1;30 = 1 1/2 = 1.5.

Following ancient custom we often deviate from a strictly sexagesimal notation by writing integers decimally, fractions sexagesimally. Hence 125;17,20 instead of 2,5;17,20.

Metrological units are indicated only once, always with reference to the integers. Hence we write 5;24,20° and not 5°24′20″ or 7;30^h instead of 7^{h}30^{min}.

Cuneiform texts have a special symbol for zero, rendered in our transcriptions as a period because it originated from a separation mark. Hence 2..5 means the same as 2.0.5 and .,5 is the same as 0,5.

In working with sexagesimally written texts it is essential not to convert the numbers to decimals, to carry out operations decimally and only to change results back to sexagesimals. For example a division by 3 results in an infinite decimal fraction whereas the sexagesimal division gives a finite number of digits. Thus roundings in one system do not mean the same in the other and accurate parameters given in sexagesimals may be altered by the transition through decimal computations.

Numbers n which contain no other prime factors than 2, 3, and 5 are called *regular* numbers. To be regular is the necessary and sufficient condition for $1/n$ to be expressible by a finite number of sexagesimal digits.

The sexagesimal place value notation, including a symbol for zero, is of course of Babylonian origin. By its adoption in Greek astronomy it also became the standard method in Indian, Islamic, and western European treatises and tables. The method of writing the single digits is insignificant. The alphabetic notation is used in Greek and Arabic texts, Roman numerals in Latin, Hindu numerals in Sanscrit. The essential point, common to all, is the place value notation and the use of a zero symbol. The modification of this notation to decimally written numbers as well, which took place in India, produced the "Hindu numerals" which we use now and which appear in slowly increasing frequency in the later Middle Ages in Arabic as well as in Byzantine and Latin texts. For the computational methods this is of very little importance since it does not matter in what form the individual digits are written.

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[1] Cf. Neugebauer [1933] and [1936, 2], p. 521 f.

§ 2. Square Root Approximations

There exist at least two simple methods in antiquity for the approximation of square roots. Assume that

$$c = a^2 + b$$

where a represents some obvious approximation of $\sqrt{c}$. Then

$$\sqrt{c} = \sqrt{a^2 + b} \approx a + \frac{b}{2a} \tag{1}$$

because $\left(a + \frac{b}{2a}\right)^2 = a^2 + b + \frac{b^2}{4a^2} = c + \frac{b^2}{4a^2}$ where the last term measures the error committed in (1); it will be small if b is small in comparison to a^2.

The second procedure is based on the idea that, if a represents an approximation of $\sqrt{c}$ then also c/a will be an approximation of $\sqrt{c}$; it will be larger than the accurate value of $\sqrt{c}$ if $a < \sqrt{c}$ and vice versa. In both cases $\sqrt{c}$ lies between $\alpha_1 = a$ and $\beta_1 = c/a$. Thus

$$\alpha_2 = 1/2\,(\alpha_1 + \beta_1)$$

is an approximation nearer to $\sqrt{c}$ than α_1 and β_1. Again: with α_2 also $\beta_2 = c/\alpha_2$ is an approximation of $\sqrt{c}$ and these two approximations lie on opposite sides of the accurate value. Hence we can form $\alpha_3 = 1/2\,(\alpha_2 + \beta_2)$, etc. This procedure is known as alternating between "arithmetical and harmonic means."

Examples. The first procedure suggests itself, e.g., when one deals with Pythagorean triangles. In order to find c from[1]

$$c^2 = 53;13^2 + 2;41^2 = 53;13^2 + 7;12,1$$

we use $a = 53;13$ $b = 7;12$ and obtain from (1)

$$\sqrt{c} \approx 53;13 + \frac{7;12}{1,46;26} \approx 53;17,3.$$

Ptolemy gives 53;17.

As an example for the second method we compute $\sqrt{2}$ and $\sqrt{3}$, using in both cases $\alpha_1 = 1;30$ as a first crude approximation.

$$\begin{array}{lll} \sqrt{2}: & \alpha_1 = 1;30 & \beta_1 = \frac{2}{1;30} = 1;20 \\ & \alpha_2 = 1;25 & \beta_2 = \frac{2}{1;25} \approx 1;24,42,21,\ldots \\ & \alpha_3 = 1;24,51,10 & \text{etc.} \\ \sqrt{3}: & \alpha_1 = 1;30 & \beta_1 = \frac{3}{1;30} = 2 \\ & \alpha_2 = 1;45 & \beta_2 = \frac{3}{1;45} \approx 1;42,51,26,\ldots \\ & \alpha_3 = 1;43,55,43 & \beta_3 \approx 1;43,55,3,\ldots \\ & \alpha_4 = 1;43,55,23 & \text{etc.} \end{array}$$

[1] From Alm. XIII, 4 (Heib. II, p. 556).

Both approximations are used in the table of chords in the Almagest (II, 11) where we find[2]

$$\text{crd } 90° = \sqrt{2} \approx 1;24,51,10, \qquad \text{crd } 120° = \sqrt{3} \approx 1;43,55,23.$$

Of course, agreement with the numerical result is not a proof for the identity of methods. In our specific example the value for $\sqrt{2}$ is not only found in the table of chords of the Almagest but also in an Old-Babylonian text[3]; in neither case do we know how the value was obtained. For the method expressed by (1) one can only say that it leads in the majority of cases to the same value as given by Ptolemy.

§ 3. Trigonometry

Ancient trigonometry is originally based on the function Crd α which is defined by (cf. Fig. 43)

$$\text{Crd } \alpha = 2R \sin \frac{\alpha}{2} \tag{1}$$

where R in Greek astronomy is usually chosen as $60 = 1,0$.

The history of the tables of chords and their use has been described in I A 1, 2 and need not be repeated here. Fig. 44 shows that the following relations hold for the solution of right triangles

$$\begin{aligned} a &= \frac{c}{2,0} \text{Crd } 2\alpha = \frac{c}{2} \text{crd } \alpha \\ b &= \frac{c}{2,0} \text{Crd } 2\beta = \frac{c}{2} \text{crd } (180 - 2\alpha) = \frac{c}{2} \text{crd } 2\bar{\alpha} \end{aligned} \tag{2}$$

where $\bar{\alpha}$ denotes the angle $90 - \alpha$. Hence the equivalent of our function $\tan \alpha$ is given by

$$\frac{a}{b} = \frac{\text{crd } 2\alpha}{\text{crd } 2\bar{\alpha}}. \tag{3}$$

Unfortunately this function was never tabulated in antiquity.

For the general triangle (cf. Fig. 45) the analogue of the sine theorem is frequently used

$$\frac{a}{b} = \frac{\text{crd } 2\alpha}{\text{crd } 2\beta}. \tag{4}$$

Ancient trigonometry is made clumsy not only by the lack of a tabulated tangent function but also by the difference in units for the measurements of distances (R) and arcs (degrees). This is, e.g., evident if one considers the differences in a table of sines. Since the derivative of $\sin \alpha$ is $\cos \alpha$ one would have in the differences an easy check for the computation of a table of sines. If, however, α is measured in degrees and if we denote $R \sin \alpha$ by Sin α, $R \cos \alpha$ by Cos α, we see

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[2] We ignore here the factor $R = 1,0$. Cf. above p. 22.

[3] Neugebauer-Sachs, MCT, p. 42 f. (YBC 7289).

that

$$\frac{\Delta \operatorname{Sin} \alpha}{\Delta \alpha} = \frac{R \Delta \sin \theta}{\frac{180}{\pi} \Delta \theta} \approx \frac{R\pi}{180} \cos \theta \tag{5}$$

(θ being measured in radians).

In Hipparchus' table of chords and in certain Indian trigonometric tables[1] the radius R is chosen to be

$$R = \frac{180}{\pi} \approx 57;18 \tag{6}$$

such that the coefficient in (5) becomes the value 1.[2] Nevertheless the table for $\operatorname{Sin} \alpha$ has only $\cos \alpha$ as its difference sequence instead of $R \cos \alpha = \operatorname{Cos} \alpha$.

Similarly detrimental for the development of trigonometry was the selection of special units for g in the "shadow" function $g \cot \alpha$ which prepared the way to the function $\tan \alpha$ and thus to the systematization of trigonometry.

For spherical trigonometry cf. p. 26ff.

§ 4. Diophantine Equations; Continued Fractions

1. Euclidean Algorithm

In the following all letters denote non-negative integers. We also assume always that

$$a > b > 0 \tag{1}$$

and that a and b are relatively prime, i.e. that their greatest common divisor is 1.

The following sequence of divisions which begins with the division of a by b, producing a quotient q_0 and a residue r_1, is known as the "*Euclidean algorithm*"[1]:

$$\begin{aligned} a &= q_0 b + r_1 \\ b &= q_1 r_1 + r_2 \\ r_1 &= q_2 r_2 + r_3 \\ &\vdots \\ r_{n-1} &= q_n r_n + r_{n+1}. \end{aligned} \tag{2}$$

Obviously the remainders form a decreasing sequence

$$r_1 > r_2 > r_3 > \cdots \geqq 0.$$

If r_{n+1} is the last positive remainder in this sequence and if it had a value

$$r_{n+1} = c > 1$$

then we would have

$$r_n = q_{n+1} r_{n+1} = q_{n+1} c$$

[1] Cf. below p. 1132 and Table 8 there.

[2] The value of π which would lead exactly to (6) is 3;8,28,54,... (instead of 3;8,29,44,...).

[1] Cf., e.g., Heath, Euclid II, p. 299.

and

$$r_{n-1} = q_n r_n + r_{n+1} = (q_n q_{n+1} + 1)\, c$$

would have in common with r_n the factor $c > 1$. Therefore

$$r_{n-2} = q_{n-1} r_{n-1} + r_n$$

would also contain c and so forth to r_1, b, and finally a; but this is a contradiction to our assumption that a and b were relatively prime. Thus the Euclidean algorithm for relatively prime numbers a and b must end in

$$r_{n-1} = q_n r_n + 1.$$

If we wish to add one more step in the sequence of divisions we may write

$$r_n = q_{n+1} r_{n+1}, \qquad r_{n+1} = 1.$$

Example: $a = 242$, $b = 223$.

$$\begin{aligned} 242 &= 1 \cdot 223 + 19 \\ 223 &= 11 \cdot 19 + 14 \\ 19 &= 1 \cdot 14 + 5 \\ 14 &= 2 \cdot 5 + 4 \\ 5 &= 1 \cdot 4 + 1 \\ 4 &= 4 \cdot 1 \end{aligned}$$

or

i	0	1	2	3	$4 = n$	5
q_i	1	11	1	2	1	4
r_i		19	14	5	4	1

Remark. The Euclidean algorithm can be used for the determination of the greatest common divisor c of any pair of integers a' and b'. If $c > 1$ one can therefore always replace the pair a' b' by a pair of integers $a = a'/c$, $b = b'/c$ which are relatively prime.

2. Linear Diophantine Equations

To solve a linear diophantine equation means to find integers x and y which satisfy the relation

$$a x - b y = c. \tag{3}$$

Obviously we have solved (3) if we are able to solve

$$a x - b y = 1 \tag{4}$$

because if x and y are solutions of (4) then cx and cy will be solutions of (3). It is equally obvious that (4) has no solution if a and b are not relatively prime since a common factor $c > 1$ on the left-hand side of (4) cannot produce 1 on the right-hand.

The method for solving (4) described in the following goes back at least to Bhaskara (about 1150 A.D.)[2] but might well be many centuries older.[3] Essentially the same procedure was rediscovered by Bachet in 1624.[4]

In describing the general idea for solving (4) we use the above notation for the quotients and residues of the Euclidean algorithm. Accordingly

$$\frac{a}{b}=q_0+\frac{r_1}{b}$$

and hence

$$y=\frac{ax-1}{b}=q_0 x+\frac{r_1 x-1}{b}. \tag{4a}$$

This shows that y will be an integer if x is an integer such that

$$y_1=\frac{r_1 x-1}{b} \tag{4b}$$

is an integer. In other words, we have solved (4) if we can solve another diophantine equation, namely

$$b y_1 - r_1 x = -1. \tag{5}$$

This equation has smaller coefficients than (4) because $a>b>r_1$.

Repeating this argument we see that the solution of (5) depends on solving

$$r_1 y_2 - r_2 y_1 = 1 \tag{6}$$

and so forth until

$$r_{n-1} y_n - r_n y_{n-1} = (-1)^n \tag{7}$$

where

$$r_{n-1} = q_n r_n + r_{n+1} = q_n r_n + 1 \tag{8}$$

because we had to assume that a and b are relatively prime. Solving (7) for y_{n-1} we find with (8) that

$$y_{n-1}=\frac{r_{n-1}}{r_n} y_n - \frac{(-1)^n}{r_n} = q_n y_n + \frac{y_n-(-1)^n}{r_n} \tag{8a}$$

will be an integer if

$$y_{n+1}=\frac{y_n-(-1)^n}{r_n}$$

is an integer, i.e. if we can find an integer solution of

$$r_n y_{n+1} - y_n = -(-1)^n. \tag{9}$$

Let us assume that n is even, i.e. that

$$(-1)^n = 1. \tag{10}$$

Then a solution of (9) is

$$y_{n+1}=0, \qquad y_n = 1. \tag{11}$$

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[2] In the Lilavati. Cf. Colebrooke AAM, p. 112ff.; Datta Singh HM II, p. 110ff.

[3] A process for solving linear diophantine equations was known to Āryabhata (Ar. II, 31–33, Clark, p. 41ff.), about 500 A.D.

[4] Dickson, Hist. II, p. 44f.

But knowing y_n we can use (8a) to find y_{n-1} and going in the same fashion backwards we can finally reach (4).

Before establishing the pattern of this recursive process we show that the assumption (10) does not constitute a restriction of generality. Indeed, if n is odd we do not conclude the Euclidean algorithm with

$$r_{n-1}=r_n q_n+1, \qquad r_n=q_{n+1}$$

but with

$$r_n=(q_{n+1}-1)\cdot 1+1, \qquad r_{n+1}=q_{n+1}.$$

Thus we may always assume that the Euclidean algorithm requires an even number of steps.

We now return to the sequence of diophantine equations which led from (4) to (9) and to the solution (11) of (9). All these diophantine equations are, for $i<n$, of the type

$$r_i y_{i+1}-r_{i+1} y_i=(-1)^{i+1}.$$

Consequently

$$y_{i+1}=\frac{r_{i+1} y_i+(-1)^{i+1}}{r_i}$$

or, because of (2),

$$r_{i+1}=r_{i-1}-q_i r_i$$

and

$$y_{i+1}=-q_i y_i+\frac{r_{i-1} y_i+(-1)^{i+1}}{r_i}.$$

But again

$$r_{i-1} y_i-r_i y_{i-1}=(-1)^i=-(-1)^{i+1}$$

and therefore

$$r_{i-1} y_i+(-1)^{i+1}=r_i y_{i-1}.$$

Hence

$$y_{i+1}=-q_i y_i+y_{i-1}$$

or

$$y_{i-1}=q_i y_i+y_{i+1}. \tag{12}$$

This recursive formula not only holds for $i<n$ but also for $i=n$ because substituting $y_n=1$ from (11) in (8a) gives $y_{n-1}=q_n$ which we can write, because of (11) as $y_{n-1}=q_n y_n+y_{n+1}$.

Thus, starting with the values (11) we can find by means of (12) integers y_{n-1}, $y_{n-2}, \ldots, y_2, y_1$ where y_1 satisfies the relation

$$y=q_0 x+y_1 \tag{13}$$

as is clear if one substitutes (4b) in (4a). If we call

$$x=y_0, \qquad y=y_{-1} \tag{14}$$

we have instead of (13)

$$y_{-1}=q_0 y_0+y_1 \tag{15}$$

which is (12) for $i=1$.

Hence we have shown that the solutions of (4) are given by

$$x=q_1 y_1+y_2, \qquad y=q_0 x+y_1 \tag{16}$$

where y_2 and y_1 are known by the recursive formula (12) which begins with the values (11):

$$y_{n+1}=0, \qquad y_n=1.$$

Having found one solution x, y of

$$ax-by=1 \tag{4}$$

it is clear that we have found infinitely many solutions because also

$$x'=x+kb, \qquad y'=y+ka \qquad (k \text{ a positive or negative integer}) \tag{17}$$

satisfies (4); but it is also obvious that these are all possible solutions. In other words: all solutions x are congruent to one another modulo b, all solutions y modulo a.

Example: $a=242$, $b=223$.

The quotients q_i $(i=0, 1, \ldots, 5)$ are known to us from the Euclidean algorithm (cf. p. 1117); and since $r_5=1$ we have $n=4$. Consequently the recursion (12) begins with $y_5=0$, $y_4=1$. Since $q_4=1$ we have

$$y_3=q_4 y_4+y_5=1.$$

Then, with $q_3=2$

$$y_2=q_3 y_3+y_4=2\cdot 1+1=3$$

and so forth as shown in the following tabulation:

i	-1	0	1	2	3	$4=n$	5	i
q_i		1	11	1	2	1	4	q_i
y_i	$51=y$	$47=x$	4	3	1	$y_4=1$	$y_5=0$	y_i

All solutions of

$$242x-223y=1$$

are therefore

$$x=47+k\cdot 223, \qquad y=51+k\cdot 242.$$

The smallest positive solutions are $x=47$, $y=51$, the smallest negative solutions $x=-176$, $y=-191$.

3. Continued Fractions[5]

We assume again for two relatively prime integers a and b

$$a>b>0 \tag{1}$$

and we apply the Euclidean algorithm as in (2), p. 1116. By using the abbreviations

$$\frac{a}{b}=x_0, \quad \frac{b}{r_1}=x_1 \ \ldots \ \frac{r_{k-1}}{r_k}=x_k \ \ldots \ \frac{r_{n-1}}{r_n}=x_n, \quad \frac{r_n}{1}=x_{n+1} \tag{18}$$

[5] From Lagrange (1736–1813) in an addition to Euler's Algebra (1807); application to diophantine equations in 1768.

the Euclidean algorithm takes now the form

$$\begin{aligned} x_0 &= q_0 + \frac{1}{x_1} \\ x_1 &= q_1 + \frac{1}{x_2} \\ &\vdots \\ x_{n-1} &= q_{n-1} + \frac{1}{x_n} \\ x_n &= q_n + \frac{1}{x_{n+1}} \\ x_{n+1} &= q_{n+1} \end{aligned} \tag{18a}$$

or, combined:

$$\frac{a}{b} = q_0 + \cfrac{1}{q_1 + \cfrac{1}{q_2 + \cfrac{\ddots}{+ \cfrac{1}{q_n + \cfrac{1}{q_{n+1}}}}}}. \tag{18b}$$

We say that (18a) or (18b) represent the expansion of a/b into a *continued fraction.* For the sake of convenience one writes instead of (18b)

$$\frac{a}{b} = (q_0, q_1, \ldots, q_n, q_{n+1}). \tag{18c}$$

Obviously we can also write

$$\frac{a}{b} = (q_0, q_1, \ldots, q_n, q_{n+1} - 1, 1) \tag{18d}$$

which means that the number of terms either in (18c) or in (18d) is even. Thus we may always assume that n is even.[6]

Theorem. If we define $n+2$ pairs of integers a_i and b_i by the recursive formulae

$$\begin{array}{lll} a_0 = 1, & a_1 = q_0, & a_{i+1} = q_i a_i + a_{i-1}, \\ b_0 = 0, & b_1 = 1, & b_{i+1} = q_i b_i + b_{i-1}, \end{array} \quad i = 1, 2, \ldots, n+1 \tag{19a}$$

then

$$\frac{a}{b} = \frac{a_i x_i + a_{i-1}}{b_i x_i + b_{i-1}} \quad \text{for } i = 1, 2, \ldots, n+1. \tag{19b}$$

Proof. From (18) and (18a) it follows, using (19a):

$$\frac{a}{b} = \frac{q_0 x_1 + 1}{x_1} = \frac{a_1 x_1 + a_0}{b_1 x_1 + b_0}$$

[6] Cf. above p. 1119.

which shows that (19b) is correct for $i=1$. We now show that the validity of (19b) for any $i \geqq 1$ implies its validity also for $i+1$. Indeed it follows from (18a) that

$$x_i = \frac{q_i x_{i+1} + 1}{x_{i+1}}$$

and therefore from (19b)

$$\frac{a}{b} = \frac{a_i(q_i x_{i+1} + 1) + a_{i-1} x_{i+1}}{b_i(q_i x_{i+1} + 1) + b_{i-1} x_{i+1}} = \frac{(a_i q_i + a_{i-1}) x_{i+1} + a_i}{(b_i q_i + b_{i-1}) x_{i+1} + b_i}$$

and with (19a)

$$\frac{a}{b} = \frac{a_{i+1} x_{i+1} + a_i}{b_{i+1} x_{i+1} + b_i}$$

q.e.d.

From the representation of the value a/b by means of the terms of a continued fraction one can derive an important conclusion concerning approximations of a given ratio.

It is clear from (18b) and (18c) that a continued fraction of given length can always be broken at an arbitrary point in two shorter fractions because it follows from (18a) that

$$x_0 = (q_0, q_1, \ldots, q_{i-1}, x_i)$$
$$x_i = (q_i, q_{i+1}, \ldots, q_{n+1}).$$

From (19b) and (18) it follows that

$$(q_0, q_1, \ldots, q_{i-1}, x_i) = \frac{a_i x_i + a_{i-1}}{b_i x_i + b_{i-1}}.$$

This is an identity in x_i. Consequently we may use for x_i the value q_i. Then we have because of (19a)

$$(q_0, q_1, \ldots, q_{i-1}, q_i) = \frac{a_i q_i + a_{i-1}}{b_i q_i + b_{i-1}} = \frac{a_{i+1}}{b_{i+1}}. \tag{20}$$

The expression $(q_0, q_1, \ldots, q_{i-1}, q_i)$ on the left-hand side is an approximation of a/b, obtained if one computes the continued fraction (18b) only to the term q_i. Thus we see from (20) that the consecutive rational numbers

$$\frac{a_1}{b_1}, \frac{a_2}{b_2}, \ldots, \frac{a_i}{b_i}, \ldots$$

are the successive approximations

$$q_0, (q_0, q_1), \ldots, (q_0, q_1, \ldots, q_{i-1}), \ldots$$

of a/b.

We now shall show that these successive approximations lie alternately below and above the value a/b and that the amount of the deviation is monotonically decreasing.

It follows from (19a) that

$$a_{i+1} b_i - a_i b_{i+1} = a_{i-1} b_i - a_i b_{i-1}$$

or, for $i=1, 2, \ldots, n+1$

$$(-1)^{i+1}(a_{i+1}b_i - a_i b_{i+1}) = (-1)^i(a_i b_{i-1} - a_{i-1}b_i).$$

Because of (19a) the right-hand side has the value

$$-(q_0 \cdot 0 - 1 \cdot 1) = +1.$$

Therefore in general

$$(-1)^{i+1}(a_{i+1}b_i - a_i b_{i+1}) = 1$$

or

$$a_{i+1}b_i - a_i b_{i+1} = (-1)^{i+1}. \tag{21}$$

For the differences of consecutive approximations we obtain with (21)

$$\frac{a_i}{b_i} - \frac{a_{i+1}}{b_{i+1}} = \frac{a_i b_{i+1} - a_{i+1} b_i}{b_i b_{i+1}} = \frac{(-1)^i}{b_i b_{i+1}}, \qquad i=1, 2, \ldots, n+1. \tag{22}$$

Since, because of (19a),

$$b_{i+1} > b_i$$

we see that the differences (22) decrease monotonically but are of alternating signs.

For the last step we find for $i=n+1$ from (19b), (18b), and (19a)

$$\frac{a}{b} = \frac{a_{n+1}x_{n+1} + a_n}{b_{n+1}x_{n+1} + b_n} = \frac{a_{n+1}q_{n+1} + a_n}{b_{n+1}q_{n+1} + b_n} = \frac{a_{n+2}}{b_{n+2}}.$$

Thus the sequence

$$\frac{a_1}{b_1} = q_0, \frac{a_2}{b_2}, \ldots, \frac{a_{n+1}}{b_{n+1}}, \frac{a_{n+2}}{b_{n+2}} = \frac{a}{b} \tag{23}$$

represents with increasing accuracy the value of a/b, being alternately below and above the final value a/b. It also follows from (21) and (23) that

$$x = b_{n+1}, \qquad y = a_{n+1} \tag{24a}$$

is a solution of the diophantine equation

$$ax - by = 1. \tag{24b}$$

Indeed, for even n, we have for $i=n+1$ from (21)

$$a_{n+2}b_{n+1} - a_{n+1}b_{n+2} = 1 \tag{25}$$

and from (23)

$$a_{n+2} = a, \qquad b_{n+2} = b \tag{26}$$

since (25) shows that a_{n+2} and b_{n+2} must be relatively prime. Thus (24a) satisfies (24b).

Finally it should be noted that the restriction to even values of n is not essential. It is easy to see that for odd n one obtains by (24a) a solution of $ax - by = -1$ which can be transformed into a solution of (24b) by changing x modulo b and y modulo a.

Example and Applications. For $a=242$, $b=223$ we know from p. 1117 that

$$\frac{242}{223}=(1, 11, 1, 2, 1, 4)=(1, 11, 1, 2, 1, 3, 1).$$

From (19a) we therefore obtain either

i	0	1	2	3	$4=n$	5	6
q_i	1	11	1	2	1	4	
a_i	1	1	12	13	38	51	$242=a$
b_i	0	1	11	12	35	47	$223=b$

or

i	...	4	$5=n$	6	7
q_i	...	1	3	1	
a_i	...	38	51	191	$242=a$
b_i	...	35	47	176	$223=b$

The fact that a and b must be the final result of the recursive process constitutes a convenient check for the correctness of the computation.[7]

Hence we have found the following sequence of successive approximations for 242/223

i	1	2	3	4	5	6
$\frac{a_i}{b_i}$	1	$\frac{12}{11}$	$\frac{13}{12}$	$\frac{38}{35}$	$\frac{51}{47}$	$\frac{191}{176}$

If we express these ratios by means of sexagesimal fractions we obtain the two following sequences

too great: 1;5,27, ... 1;5,8,17, ... 1;5,6,49, ...
too small: 1 1;5,0 1;5,6,23, ...

which both converge toward $242/223=1;5,6,43,\ldots$.

The possibility of obtaining by means of continued fractions successive approximations of a given ratio a/b is of great practical importance. The numbers a and b chosen in our examples come from the relation (known as the "*Saros*")

242 draconitic months = 223 synodic months.

Consequently we can say that eclipses can recur approximately after 11 or after 12 months, or after 35, or 47, or 176 months.[8]

[7] Note that we have for $n=5$ the solutions of the diophantine equation $ax-by=1$. Cf. p. 1120.
[8] These numbers concern eclipses related to the same node.

Another example may be taken from a ratio given for Jupiter by Āryabhaṭa[9]:

$$6,15,0 \text{ years} = 31,37 \text{ sidereal rotations.}$$

Since

$$\frac{a}{b} = \frac{6,15,0}{31,37} = (11, 1, 6, 5, 2, \ldots)$$

one finds the following approximations

i	0	1	2	3	4	5	...
q_i	11	1	6	5	2	...	...
a_i	1	11	12	83	427	937	...
b_i	0	1	1	7	36	79	...

Thus, beginning with $i=2$, we have the following relations

$$\begin{aligned} 12 \text{ years} &\approx 1 \text{ rotation} \\ 83 \text{ years} &\approx 7 \text{ rotations} \\ 427 \text{ years} &\approx 36 \text{ rotations, etc.} \end{aligned}$$

The first is the crude 12-year period of Jupiter, the second is used in the Babylonian "goal year texts",[10] the third is the basic relation for the Babylonian "procedure texts".[11]

[9] Aryabh. I. 1 (trsl. Clark, p. 9).
[10] Cf. p. 554 (1).
[11] Cf. p. 390 (10a).

§ 5. Tables

1. Sexagesimal Computations

Table 3 permits the change of sexagesimal integers to decimals, and vice versa. Examples:

by direct entry: $7,0,0 = 25\,200$
with addition: $28,0,0 = 90\,000 + 10\,800 = 100\,800$
inverse: $26\,000 = 25\,200 + 780 + 20 = 7,0,0 + 13,0 + 20 = 7,13,20$.

Table 4 gives the sexagesimal equivalents of the decimal fraction from 0.01 to 0.99. Table 5 is the basic sexagesimal table of multiplication up to 1,0,0.

The sexagesimal reciprocals of the numbers from 1 to 60 are listed in Table 6. None of the finite reciprocals can have more than 3 digits. If a three-digit number shows at the end a comma the full expansion requires infinitely many digits (of course periodic). Digits beyond the third place are not rounded but simply truncated; if the first omitted digit is ≥ 30 then this is indicated by a dot.

Table 3

n	n,0	n,0,0	n,0,0,0	n,0,0,0,0
1	60	3600	216000	12960000
2	120	7200	432	25920
3	180	10800	648	38880
4	240	14400	864	51840
5	300	18000	1080	64800
6	360	21600	1296000	77760000
7	420	25200	1512	90720
8	480	28800	1728	103680
9	540	32400	1944	116640
10	600	36000	2160	129600
15	900	54000	3240000	194400000
20	1200	72	4320	259200
25	1500	90	5400	324000
30	1800	108	6480	388800
35	2100	126	7560	453600
40	2400	144000	8640000	518400000
45	2700	162	9720	583200
50	3000	180	10800	648000
55	3300	198	11880	712800
60	3600	216	12960	777600

Table 4

0.00	0; 0. 0	0.20	0;12	0.40	0;24	0.60	0;36	0.80	0;48
01	0.36	21	12,36	41	24,36	61	36,36	81	48,36
02	1.12	22	13,12	42	25,12	62	37,12	82	49,12
03	1.48	23	13,48	43	25,48	63	37,48	83	49,48
04	2.24	24	14,24	44	26,24	64	38,24	84	50,24
0.05	0; 3	0.25	0;15	0.45	0;27	0.65	0;39	0.85	0;51
06	3.36	26	15,36	46	27,36	66	39,36	86	51,36
07	4.12	27	16,12	47	28,12	67	40,12	87	52,12
08	4.48	28	16,48	48	28,48	68	40,48	88	52,48
09	5.24	29	17,24	49	29,24	69	41,24	89	53,24
0.10	0; 6	0.30	0;18	0.50	0;30	0.70	0;42	0.90	0;54
11	6.36	31	18,36	51	30,36	71	42,36	91	54,36
12	7,12	32	19,12	52	31,12	72	43,12	92	55,12
13	7,48	33	19,48	53	31,48	73	43,48	93	55,48
14	8.24	34	20,24	54	32,24	74	44,24	94	56,24
0.15	0; 9	0.35	21	0.55	0;33	0.75	0;45	0.95	0;57
16	9,36	36	21,36	56	33,36	76	45,36	96	57,36
17	10,12	37	22,12	57	34,12	77	46,12	97	58,12
18	10,48	38	22,48	58	34,48	78	46,48	98	58,48
19	11,24	39	23,24	59	35,24	79	47,24	99	59,24

Table 5 see pp. 1128 and 1129.

Table 6

n	$\bar{n}$	n	$\bar{n}$	n	$\bar{n}$
1	1	21	2,51,25,·	41	1,27,48,
2	30	22	2,43,38,	42	1,25,42,·
3	20	23	2,36,31,	43	1,23,43,
4	15	24	2,30	44	1,21,49,
5	12	25	2,24	45 •	1,20
6	10	26	2,18,27,·	46	1,18,15,·
7 ·	8,34,17,	27	2,13,20	47	1,16,35,·
8	7,30	28	2, 8,34,	48	1,15
9	6,40	29	2, 4, 8	49	1,13,28,
10	6	30	2	50	1,12
11	5,27,16,	31	1,56, 7,·	51	1,10,35,
12	5	32	1,52,30	52	1, 9,13,·
13	4,36,55,	33	1,49, 5,	53	1, 7,55,
14	4,17, 8,·	34	1,45,52,·	54	1, 6,40
15	4	35	1,42,51,	55	1, 5,27,
16	3,45	36	1,40	56	1, 4,17,
17	3,31,45,·	37	1,37,17,·	57	1, 3, 9,
18	3,20	38	1,34,44,	58	1, 2, 4,
19	3, 9,28,	39	1,32,18,	59	1, 1, 1,
20	3	40	1,30	1	1

Table 5

1	59	58	57	56	55	54	53	52	51	50	49	48	47	46	45	44	43	42	41	40	39	38
2	1,58	1,56	1,54	1,52	1,50	1,48	1,46	1,44	1,42	1,40	1,38	1,36	1,34	1,32	1,30	1,28	1,26	1,24	1,22	1,20	1,18	1,16
3	2,57	2,54	2,51	2,48	2,45	2,42	2,39	2,36	2,33	2,30	2,27	2,24	2,21	2,18	2,15	2,12	2,9	2,6	2,3	2,0	1,57	1,54
4	3,56	3,52	3,48	3,44	3,40	3,36	3,32	3,28	3,24	3,20	3,16	3,12	3,8	3,4	3,0	2,56	2,52	2,48	2,44	2,40	2,36	2,32
5	4,55	4,50	4,45	4,40	4,35	4,30	4,25	4,20	4,15	4,10	4,5	4,0	3,55	3,50	3,45	3,40	3,35	3,30	3,25	3,20	3,15	3,10
6	5,54	5,48	5,42	5,36	5,30	5,24	5,18	5,12	5,6	5,0	4,54	4,48	4,42	4,36	4,30	4,24	4,18	4,12	4,6	4,0	3,54	3,48
7	6,53	6,46	6,39	6,32	6,25	6,18	6,11	6,4	5,57	5,50	5,43	5,36	5,29	5,22	5,15	5,8	5,1	4,54	4,47	4,40	4,33	4,26
8	7,52	7,44	7,36	7,28	7,20	7,12	7,4	6,56	6,48	6,40	6,32	6,24	6,16	6,8	6,0	5,52	5,44	5,36	5,28	5,20	5,12	5,4
9	8,51	8,42	8,33	8,24	8,15	8,6	7,57	7,48	7,39	7,30	7,21	7,12	7,3	6,54	6,45	6,36	6,27	6,18	6,9	6,0	5,51	5,42
10	9,50	9,40	9,30	9,20	9,10	9,0	8,50	8,40	8,30	8,20	8,10	8,0	7,50	7,40	7,30	7,20	7,10	7,0	6,50	6,40	6,30	6,20
11	10,49	10,38	10,27	10,16	10,5	9,54	9,43	9,32	9,21	9,10	8,59	8,48	8,37	8,26	8,15	8,4	7,53	7,42	7,31	7,20	7,9	6,58
12	11,48	11,36	11,24	11,12	11,0	10,48	10,36	10,24	10,12	10,0	9,48	9,36	9,24	9,12	9,0	8,48	8,36	8,24	8,12	8,0	7,48	7,36
13	12,47	12,34	12,21	12,8	11,55	11,42	11,29	11,16	11,3	10,50	10,37	10,24	10,11	9,58	9,45	9,32	9,19	9,6	8,53	8,40	8,27	8,14
14	13,46	13,32	13,18	13,4	12,50	12,36	12,22	12,8	11,54	11,40	11,26	11,12	10,58	10,44	10,30	10,16	10,2	9,48	9,34	9,20	9,6	8,52
15	14,45	14,30	14,15	14,0	13,45	13,30	13,15	13,0	12,45	12,30	12,15	12,0	11,45	11,30	11,15	11,0	10,45	10,30	10,15	10,0	9,45	9,30
16	15,44	15,28	15,12	14,56	14,40	14,24	14,8	13,52	13,36	13,20	13,4	12,48	12,32	12,16	12,0	11,44	11,28	11,12	10,56	10,40	10,24	10,8
17	16,43	16,26	16,9	15,52	15,35	15,18	15,1	14,44	14,27	14,10	13,53	13,36	13,19	13,2	12,45	12,28	12,11	11,54	11,37	11,20	11,3	10,46
18	17,42	17,24	17,6	16,48	16,30	16,12	15,54	15,36	15,18	15,0	14,42	14,24	14,6	13,48	13,30	13,12	12,54	12,36	12,18	12,0	11,42	11,24
19	18,41	18,22	18,3	17,44	17,25	17,6	16,47	16,28	16,9	15,50	15,31	15,12	14,53	14,34	14,15	13,56	13,37	13,18	12,59	12,40	12,21	12,2
20	19,40	19,20	19,0	18,40	18,20	18,0	17,40	17,20	17,0	16,40	16,20	16,0	15,40	15,20	15,0	14,40	14,20	14,0	13,40	13,20	13,0	12,40
21	20,39	20,18	19,57	19,36	19,15	18,54	18,33	18,12	17,51	17,30	17,9	16,48	16,27	16,6	15,45	15,24	15,3	14,42	14,21	14,0	13,39	13,18
22	21,38	21,16	20,54	20,32	20,10	19,48	19,26	19,4	18,42	18,20	17,58	17,36	17,14	16,52	16,30	16,8	15,46	15,24	15,2	14,40	14,18	13,56
23	22,37	22,14	21,51	21,28	21,5	20,42	20,19	19,56	19,33	19,10	18,47	18,24	18,1	17,38	17,15	16,52	16,29	16,6	15,43	15,20	14,57	14,34
24	23,36	23,12	22,48	22,24	22,0	21,36	21,12	20,48	20,24	20,0	19,36	19,12	18,48	18,24	18,0	17,36	17,12	16,48	16,24	16,0	15,36	15,12
25	24,35	24,10	23,45	23,20	22,55	22,30	22,5	21,40	21,15	20,50	20,25	20,0	19,35	19,10	18,45	18,20	17,55	17,30	17,5	16,40	16,15	15,50
26	25,34	25,8	24,42	24,16	23,50	23,24	22,58	22,32	22,6	21,40	21,14	20,48	20,22	19,56	19,30	19,4	18,38	18,12	17,46	17,20	16,54	16,28
27	26,33	26,6	25,39	25,12	24,45	24,18	23,51	23,24	22,57	22,30	22,3	21,36	21,9	20,42	20,15	19,48	19,21	18,54	18,27	18,0	17,33	17,6
28	27,32	27,4	26,36	26,8	25,40	25,12	24,44	24,16	23,48	23,20	22,52	22,24	21,56	21,28	21,0	20,32	20,4	19,36	19,8	18,40	18,12	17,44
29	28,31	28,2	27,33	27,4	26,35	26,6	25,37	25,8	24,39	24,10	23,41	23,12	22,43	22,14	21,45	21,16	20,47	20,18	19,49	19,20	18,51	18,22
30	29,30	29,0	28,30	28,0	27,30	27,0	26,30	26,0	25,30	25,0	24,30	24,0	23,30	23,0	22,30	22,0	21,30	21,0	20,30	20,0	19,30	19,0
31	30,29	29,58	29,27	28,56	28,25	27,54	27,23	26,52	26,21	25,50	25,19	24,48	24,17	23,46	23,15	22,44	22,13	21,42	21,11	20,40	20,9	19,38
32	31,28	30,56	30,24	29,52	29,20	28,48	28,16	27,44	27,12	26,40	26,8	25,36	25,4	24,32	24,0	23,28	22,56	22,24	21,52	21,20	20,48	20,16
33	32,27	31,54	31,21	30,48	30,15	29,42	29,9	28,36	28,3	27,30	26,57	26,24	25,51	25,18	24,45	24,12	23,39	23,6	22,33	22,0	21,27	20,54
34	33,26	32,52	32,18	31,44	31,10	30,36	30,2	29,28	28,54	28,20	27,46	27,12	26,38	26,4	25,30	24,56	24,22	23,48	23,14	22,40	22,6	21,32
35	34,25	33,50	33,15	32,40	32,5	31,30	30,55	30,20	29,45	29,10	28,35	28,0	27,25	26,50	26,15	25,40	25,5	24,30	23,55	23,20	22,45	22,10
36	35,24	34,48	34,12	33,36	33,0	32,24	31,48	31,12	30,36	30,0	29,24	28,48	28,12	27,36	27,0	26,24	25,48	25,12	24,36	24,0	23,24	22,48
37	36,23	35,46	35,9	34,32	33,55	33,18	32,41	32,4	31,27	30,50	30,13	29,36	28,59	28,22	27,45	27,8	26,31	25,54	25,17	24,40	24,3	23,26
38	37,22	36,44	36,6	35,28	34,50	34,12	33,34	32,56	32,18	31,40	31,2	30,24	29,46	29,8	28,30	27,52	27,14	26,36	25,58	25,20	24,42	24,4
39	38,21	37,42	37,3	36,24	35,45	35,6	34,27	33,48	33,9	32,30	31,51	31,12	30,33	29,54	29,15	28,36	27,57	27,18	26,39	26,0	25,21	
40	39,20	38,40	38,0	37,20	36,40	36,0	35,20	34,40	34,0	33,20	32,40	32,0	31,20	30,40	30,0	29,20	28,40	28,0	27,20	26,40		
41	40,19	39,38	38,57	38,16	37,35	36,54	36,13	35,32	34,51	34,10	33,29	32,48	32,7	31,26	30,45	30,4	29,23	28,42	28,1			
42	41,18	40,36	39,54	39,12	38,30	37,48	37,6	36,24	35,42	35,0	34,18	33,36	32,54	32,12	31,30	30,48	30,6	29,24				
43	42,17	41,34	40,51	40,8	39,25	38,42	37,59	37,16	36,33	35,50	35,7	34,24	33,41	32,58	32,15	31,32	30,49					
44	43,16	42,32	41,48	41,4	40,20	39,36	38,52	38,8	37,24	36,40	35,56	35,12	34,28	33,44	33,0	32,16						
45	44,15	43,30	42,45	42,0	41,15	40,30	39,45	39,0	38,15	37,30	36,45	36,0	35,15	34,30	33,45							
46	45,14	44,28	43,42	42,36	42,10	41,24	40,38	39,52	39,6	38,20	37,34	36,48	36,2	35,16								
47	46,13	45,26	44,39	43,52	43,5	42,18	41,31	40,44	39,57	39,10	38,23	37,36	36,49									
48	47,12	46,24	45,36	44,48	44,0	43,12	42,24	41,36	40,48	40,0	39,12	38,24										
49	48,11	47,22	46,33	45,44	44,55	44,6	43,17	42,28	41,39	40,50	40,1											
50	49,10	48,20	47,30	46,40	45,50	45,0	44,10	43,20	42,30	41,40												
51	50,9	49,18	48,27	47,36	46,45	45,54	45,3	44,12	43,21													
52	51,8	50,16	49,24	48,32	47,40	46,48	45,56	45,4														
53	52,7	51,14	50,21	49,28	48,35	47,42	46,49															
54	53,6	52,12	51,18	50,24	49,30	48,36																
55	54,5	53,10	52,15	51,20	50,25																	
56	55,4	54,8	53,12	52,16																		
57	56,3	55,6	54,9																			
58	57,2	56,4																				
59	58,1																					

Table 5

35	34	33	32	31	30	29	28	27	26	25	24	23	22	21	20	19	18	17	16	15	14	13	12	11	10	9	8
10	1,8	1,6	1,4	1,2	1,0	58	56	54	52	50	48	46	44	42	40	38	36	34	32	30	28	26	24	22	20	18	16
45	1,42	1,39	1,36	1,33	1,30	1,27	1,24	1,21	1,18	1,15	1,12	1,9	1,6	1,3	1,0	57	54	51	48	45	42	39	36	33	30	27	24
20	2,16	2,12	2,8	2,4	2,0	1,56	1,52	1,48	1,44	1,40	1,36	1,32	1,28	1,24	1,20	1,16	1,12	1,8	1,4	1,0	56	52	48	44	40	36	32
55	2,50	2,45	2,40	2,35	2,30	2,25	2,20	2,15	2,10	2,5	2,0	1,55	1,50	1,45	1,40	1,35	1,30	1,25	1,20	1,15	1,10	1,5	1,0	55	50	45	40
30	3,24	3,18	3,12	3,6	3,0	2,54	2,48	2,42	2,36	2,30	2,24	2,18	2,12	2,6	2,0	1,54	1,48	1,42	1,36	1,30	1,24	1,18	1,12	1,6	1,0	54	48
5	3,58	3,51	3,44	3,37	3,30	3,23	3,16	3,9	3,2	2,55	2,48	2,41	2,34	2,27	2,20	2,13	2,6	1,59	1,52	1,45	1,38	1,31	1,24	1,17	1,10	1,3	56
40	4,32	4,24	4,16	4.8	4,0	3,52	3,44	3,36	3,28	3,20	3,12	3,4	2,56	2,48	2,40	2,32	2,24	2,16	2,8	2,0	1,52	1,44	1,36	1,28	1,20	1,12	1,4
15	5,6	4,57	4,48	4,39	4,30	4,21	4,12	4,3	3,54	3,45	3,36	3,27	3,18	3,9	3,0	2,51	2,42	2,33	2,24	2,15	2,6	1,57	1,48	1,39	1,30	1,21	
50	5,40	5,30	5,20	5,10	5,0	4,50	4,40	4,30	4,20	4,10	4,0	3,50	3,40	3,30	3,20	3,10	3,0	2,50	2,40	2,30	2,20	2,10	2,0	1,50	1,40		
25	6,14	6,3	5,52	5,41	5,30	5,19	5,8	4,57	4,46	4,35	4,24	4,13	4,2	3,51	3,40	3,29	3,18	3,7	2,56	2,45	2,34	2,23	2,12	2,1			
0	6,48	6,36	6,24	6,12	6,0	5,48	5,36	5,24	5,12	5,0	4,48	4,36	4,24	4,12	4,0	3,48	3,36	3,24	3,12	3,0	2,48	2,36	2,24				
35	7,22	7,9	6,56	6,43	6,30	6,17	6,4	5,51	5,38	5,25	5,12	4,59	4,46	4,33	4,20	4,7	3,54	3,41	3,28	3,15	3,2	2,49					
10	7,56	7,42	7,28	7,14	7,0	6,46	6,32	6,18	6,4	5,50	5,36	5,22	5,8	4,54	4,40	4,26	4,12	3,58	3,44	3,30	3,16						
45	8,30	8,15	8,0	7,45	7,30	7,15	7,0	6,45	6,30	6,15	6,0	5,45	5,30	5,15	5,0	4,45	4,30	4,15	4,0	3,45							
20	9,4	8,48	8,32	8,16	8,0	7,44	7,28	7,12	6,56	6,40	6,24	6,8	5,52	5,36	5,20	5,4	4,48	4,32	4,16								
55	9,38	9,21	9,4	8,47	8,30	8,13	7,56	7,39	7,22	7,5	6,48	6,31	6,14	5,57	5,40	5,23	5,6	4,49									
30	10,12	9,54	9,36	9,18	9,0	8,42	8,24	8,6	7,48	7,30	7,12	6,54	6,36	6,18	6,0	5,42	5,24										
5	10,46	10,27	10,8	9,49	9,30	9,11	8,52	8,33	8,14	7,55	7,36	7,17	6,58	6,39	6,20	6,1											
40	11,20	11,0	10,40	10,20	10,0	9,40	9,20	9,0	8,40	8,20	8,0	7,40	7,20	7,0	6,40												
15	11,54	11,33	11,12	10,51	10,30	10,9	9,48	9,27	9,6	8,45	8,24	8,3	7,42	7,21													
50	12,28	12,6	11,44	11,22	11,0	10,38	10,16	9,54	9,32	9,10	8,48	8,26	8,4														
25	13,2	12,39	12,16	11,53	11,30	11,7	10,44	10,21	9,58	9,35	9,12	8,49															
0	13,36	13,12	12,48	12,24	12,0	11,36	11,12	10,48	10,24	10,0	9,36																
35	14,10	13,45	13,20	12,55	12,30	12,5	11,40	11,15	10,50	10,25																	
10	14,44	14,18	13,52	13,26	13,0	12,34	12,8	11,42	11,16																		
45	15,18	14,51	14,24	13,57	13,30	13,3	12,36	12,9																			
20	15,32	15,24	14,56	14,28	14,0	13,32	13,4																				
55	16,26	15,57	15,28	14,59	14,30	14,1																					
30	17.0	16,30	16,0	15,30	15,0																						
5	17,34	17,3	16,32	16,1																							
40	18,8	17,36	17,4																								
15	18,42	18,9																									
50	19,16																										
25																											

2. Trigonometric Functions

Table 7 is an excerpt from a table computed by H. F. Trotter of the Computer Center of Princeton University, ranging in steps of single minutes from 0° to 89;59° for all four trigonometric functions. I have here given only Sin α and Tan α for $R=60$; from the latter table I have derived Tan α for $R=12$ because of the interest of this norm for shadow lengths. Progress in steps of 0;30° will suffice for many historical applications.

Table 8 gives Crd α for $R=57;18=3438'$ in steps of 7;30° for α.[1] The convenience of this norm for R lies in the fact that the circumference c of the circle is then measured in the same units as the radius. Thus the basic idea is the same that underlies the concept of "radians" but the units are chosen such that R and c are measured in "degrees", i.e. such that $c=2\pi R=360$.

For the connection of this table with the Indian table of sines and with the table of chords of Hipparchus cf. above p. 299f.

[1] The same table is also found in Toomer [1973], p. 8.

Table 7

	$R=60$		$R=12$		$R=60$		$R=12$
α	Sin α	Tan α	Tan α	α	Sin α	Tan α	Tan α
0;30°	0;31,25	0;31,25	0; 6,13	20	20;31,16	21;50,18	4;22, 4
1	1; 2,50	1; 2,50	0;12,34	20;30	21; 0,45	22;25,59	4;29,12
1;30	1;34,14	1;34,16	0;18,51	21	21;30, 7	23; 1,55	4;36,23
2	2; 5,38	2; 5,43	0;25, 9	21;30	21;59,24	23;38, 5	4;43,37
2;30	2;37, 2	2;37,11	0;31,26	22	22;28,35	24;14,30	4;50,54
3	3; 8,25	3; 8,40	0;37,44	22;30	22;57,40	24;51,10	4;58,14
3;30	3;39,46	3;40,11	0;44, 2	23	23;26,38	25;28, 7	5; 5,37
4	4;11, 7	4;11,44	0;50,21	23;30	23;55,30	26; 5,19	5;13, 4
4;30	4;42,27	4;43,20	0;56,40	24	24;24,15	26;42,49	5;20,34
5	5;13,46	5;14,58	1; 3, 0	24;30	24;52,54	27;20,37	5;28, 7
5;30	5;45, 3	5;46,38	1; 9,20	25	25;21,26	27;58,42	5;35,44
6	6;16,18	6;18,23	1;15,41	25;30°	25;49,50	28;37, 7	5;43,25
6;30	6;47,32	6;50,10	1;22, 2	26	26;18, 8	29;15,50	5;51,10
7	7;18,44	7;22, 1	1;28,24	26;30	26;46,19	29;54,54	5;58,59
7;30	7;49,54	7;53,57	1;34,47	27	27;14,22	30;34,17	6; 6,51
8	8;21, 1	8;25,57	1;41,11	27;30	27;42,18	31;14, 2	6;14,48
8;30	8;52, 7	8;58, 1	1;47,24	28	28;10, 6	31;54, 9	6;22,50
9	9;23,10	9;30,11	1;54, 2	28;30	28;37,46	32;34,38	6;30,56
9;30	9;54,10	10; 2,26	2; 0,29	29	29; 5,19	33;15,31	6;39, 6
				29;30	29;32,43	33;56,47	6;47,21
10	10;25, 8	10;34,47	2; 6,57	30	30; 0, 0	34;38,28	6;55,42
10;30	10;56, 3	11; 7,13	2;13,27	30;30	30;27, 8	35;20,34	7; 4, 7
11	11;26,55	11;39,46	2;19,57	31	30;54, 8	36; 3, 6	7;12,37
11;30	11;57,43	12;12,26	2;26,29	31;30	31;21, 0	36;46, 5	7;21,13
12	12;28,29	12;45,12	2;33, 2	32	31;47,43	37;29,32	7;29,54
12;30	12;59,11	13;18, 6	2;39,37	32;30	32;14,17	38;13,27	7;38,41
13	13;29,49	13;51, 8	2;46,14	33	32;40,42	38;57,52	7;59,34
13;30	14; 0,24	14;24,17	2;52,51	33;30	33; 6,58	39;42,47	7;56,33
14	14;30,55	14;57,35	2;59,31	34	33;33, 6	40;28,14	8; 5,39
14;30	15; 1,22	15;31, 1	3; 6,12	34;30	33;59, 4	41;14,13	8;14,51
15	15;31,45	16; 4,37	3;12,55	35	34;24,53	42; 0,45	8;24, 9
15;30	16; 2, 3	16;38,22	3;19,40	35;30	34;50,32	42;47,51	8;33,36
16	16;32,18	17;12,17	3;26,27	36	35;16, 2	43;35,33	8;43, 7
16;30	17; 2,27	17;46,22	3;33,28	36;30	35;41,22	44;23,52	8;52,46
17	17;32,32	18;20,38	3;40, 8	37	36; 6,32	45;12,48	9; 2,34
17;30	18; 2,32	18;55, 5	3;47, 1	37;30	36;31,32	46, 2,23	9;12,29
18	18;32,28	19;29,43	3;53,57	38	36;56,23	46;52,38	9;22,32
18;30	19; 2,18	20; 4,33	4; 0,55	38;30	37;21, 3	47;43,34	9;32,43
19	19;32, 3	20;39,35	4; 7,55	39	37;45,33	48;35,13	9;43, 3
19;30	20; 1,42	21;14,50	4;14,58	39;30	38; 9,53	49;27,37	9;53,31

Table 7 (continued)

α	R = 60 Sin α	R = 60 Tan α	R = 12 Tan α	α	R = 60 Sin α	R = 60 Tan α	R = 12 Tan α
40	38;34, 2	50;20,46	10; 4, 9	60	51;57,41	1,43;55,23	20;47, 5
40;30	38;58, 1	51;14,41	10;14,56	60;30	52;13,17	1;46, 2,59	21;12,36
41	39;21,49	52; 9,26	10;25,53	61	52;28,38	1,48;14,34	21;38,55
41;30	39;45,26	53, 5, 1	10;37, 0	61;30	52;43,44	1,50;30,23	22; 6, 5
42	40; 8,52	54; 1,27	10;48,17	62	52;58,37	1.52;50,37	22;34, 7
42;30	40;32, 7	54;58,48	10;59,46	62;30	53;13,14	1,55;15,32	23; 3, 6
43	40;55,12	55;57, 3	11;11,25	63	53;27,37	1,57;45,24	23;33, 5
43;30	41;18, 5	56;56,16	11;23,15	63;30	53;41,46	2, 0;20,29	24; 5, 6
44	41;40,46	57;56,29	11;35,18	64	53;35,40	2, 3; 1, 6	24;36,13
44;30	42; 3,16	58;57,43	11;47,33	64;30	54; 9,18	2, 5;47,33	25; 9,31
45	42;25,35	1, 0; 0, 0	12; 0, 0	65	54;22,42	2, 8;40,13	25;44, 3
45;30	42;47,42	1, 1; 3,23	12;12,41	65;30	54;35,52	2,11;39,29	26;19,54
46	43; 9,37	1, 2; 7,55	12;25,35	66	54;48,46	2,14;45,44	26;57, 9
46;30	43;31,21	1, 3;13,37	12;38,43	66;30	55; 1,25	2,17;59,26	27;35,58
47	43;52,52	1, 4;20,32	12;52, 6	67	55;13,49	2,21;21, 4	28;16,13
47;30	44;14,12	1, 5;28,43	13; 5,57	67;30	55;25,58	2,24;51,10	28;58,14
48	44;35,19	1, 6;38,12	13;19,38	68	55;37,52	2,28;30,19	29;42, 4
48;30	44;56,14	1, 7;49, 4	13;33,49	68;30	55;49,30	2,32;19, 8	30;27,50
49	45;16,57	1, 9; 1,20	13;48,16	69	56; 0,53	2,36;18,19	31;15,40
49;30	45;37,28	1,10;15, 4	14; 3, 1	69;30	56;12, 1	2,40;28,38	32; 5,44
50°	45;57,46	1,11;30,19	14;18, 4	70°	56;22,54	2,44;50,55	32;10,11
50;30	46;17,51	1,12;47, 9	14;33,26	70;30	56;33,31	2,49;26, 5	33;53,13
51	46;37,44	1,14; 5,38	14;49, 8	71	56;43,52	2,54;15,10	34;51, 2
51;30	46;57,23	1,15;25,49	15; 5,10	71;30	56;53,58	2,59;19,16	35;51,51
52	47;16,50	1,16;47,47	15;21,33	72	57; 3,48	3, 4;39,40	36; 7,56
52;30	47;36, 4	1,18;11,37	15;38,19	72;30	57;13,23	3,10;17,44	38; 3,33
53	47;55, 5	1,19;37,22	15;55,28	73	57;22,42	3,16;15, 4	39;15, 1
53;30	48;13,53	1,21; 5, 7	16;13, 1	73;30	57;31,45	3,22;33,24	40;30,41
54	48;32,28	1,22;34,59	16;31, 0	74	57;40,33	3,29;14,42	41;50,56
54;30	48;50,49	1,24; 7, 1	16;49,24	74;30	57;49, 4	3,36;21,11	43;16,14
55	49; 8,57	1,25;41,20	17; 8,16	75	57;57,20	3,43;55,23	44;47, 5
55;30	49;26,51	1,27;18, 2	17;27,36	75;30	58; 5,20	3,52; 0,10	46;24, 2
56	49;44,32	1,28;57,13	17;47,27	76	58;13, 4	4, 0;38,49	48; 7,46
56;30	50; 1,59	1,30;39, 0	18; 7,48	76;30	58;20,32	4, 9;55, 5	49;59, 1
57	50;19,13	1,32;23,31	18;28,42	77	58;27,44	4,19;53,19	51;58,40
57;30	50;36,46	1,34;10,52	18;50,10	77;30	58;34,40	4,30;38,33	54; 7,43
58	50;52,58	1,36; 1,12	19;12,14	78	58;41,20	4,42;16,40	56;27,20
58;30	51; 9,30	1,37;54,40	19;34,56	78;30	58;47,44	4,54;54,34	58;58,55
59	51;25,40	1,39;51,24	19;58,29	79	58;53,51	5, 8;40,24	1, 1;44, 5
59;30	51;41,52	1,41;51,35	20;22,19	79;30	58;59,43	5,23;43,52	1, 4;44,46

Table 7 (continued)

α	R = 60 Sin α	R = 60 Tan α	R = 12 Tan α
80	59; 5,18	5,40;16,37	1, 8; 3,19
80;30	59;10,38	5,58;32,45	1,11;42,33
81	59;15,41	6,18;49,30	1,15;45,54
81;30	59;20,27	6,41;28,10	1,20;17,50
82	59;24,58	7, 6;55,20	1,25;23, 4
82;30	59;29,12	7,35;44,43	1,31; 8,57
83	59;33,10	8, 8;39,39	1,37;43,56
83;30	59;36,52	8,46;36,48	1,45;19,22
84	59;40,17	9,30;51,43	1,54;10,21
84;30	59;43,26	10,23; 7,26	2, 4;37,29
85	59;46,18	11,25;48,11	2,17; 9,50
85;30	59;48,54	12,42;22,20	2,32;28,28
86	59;51,14	14,18; 2,24	2,51;36,29
86;30	59;53,17	16,20;59,29	3,16;11,54
87	59;55, 4	19, 4;52, 5	3,48;58,25
87;30	59;56,34	22,54;13,33	4,34;50,43
88	59;57,48	28,38;10,31	5,43;38, 6
88;30	59;58,46	38,11;18,27	7,38;15,41
89	59;59,27	57,17;23,52	11,27;28,46
89;30	59;59,52	1,54,35;19	22,55; 4

Table 8

$R = 57;18° = 3438'$

α	Crd α		α	Crd α	
7;30°	7;30°	450′	97;30°	86; 9°	5169′
15	14;58	898	105	90;55	5455
22;30	22;21	1341	112;30	95;17	5717
30	29;40	1780	120	99;15	5945
37;30	36;50	2210	127;30	102;47	6167
45	43;51	2631	135	105;53	6353
52;30	50;41	3041	142;30	108;31	6511
60	57;18	3438	150	110;42	6642
67;30	63;38	3818	157;30	112;24	6744
75	69;46	4186	165	113;37	6817
82;30	75;34	4534	172;30	114;21	6861
90	81; 2	4862	180	114;36	6876

D. Indices

§ 1. Subject Index

References to sections without page numbers indicate that the whole section concerns the topic in question

§ 2. Bibliographical Abbreviations

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Aratus Comm.: *see* Hipparchus

Aratus: *see also* Schott

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Z. f. Math. u. Phys.: Zeitschrift für Mathematik und Physik
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§ 3. Notations and Symbols

I have tried, by and large, to maintain some uniformity of notation in formulae and diagrams. Nevertheless the number of required letters is so great that ambiguities are unavoidable. Even in the same area of investigation, e.g. lunar theory, notations cannot be kept unchanged regardless of the fundamental differences in approach within different historical periods. It would have also been impossible to apply the fairly standardized modern terminology to historical discussions since, e.g., planetary theory based on Keplerian models has eliminated the basic concepts of the Ptolemaic model. I could only try to adopt in each section a notation reasonably near to modern usage by avoiding rigid principles for mere consistency's sake.

1. Calendar, Chronology

A.E. Arsacid era
A.H. years of the Hijra
S.E. Seleucid era
Ś.E. Śaka era

w day of the week: $w=1$ Sunday
e epact

2. Spherical Astronomy

α right ascension. $\alpha'=90+\alpha$ "normed" right ascension
δ declination
ε obliquity of ecliptic
η ortive amplitude
i orbital inclination toward ecliptic
λ, β ecliptic coordinates (cf. also next section)
m, b polar longitude/latitude (*mediatio, basis latitudinis*)
v angle between ecliptic and horizon
n ascensional difference
ρ oblique ascension
$\sigma_1, \sigma_2, \ldots, \sigma_{12}$ rising times of zodiacal signs for sphaera recta
$\tau_1, \tau_2, \ldots, \tau_{12}$ rising times of zodiacal signs for sphaera obliqua
t hour angle
φ geographical latitude, $\bar{\varphi}=90-\varphi$ colatitude

g length of gnomon
h altitude, $\bar{h}=90-h$ zenith distance

p, p_λ, p_β parallax and its components
p_0 horizontal parallax
p', p'_0, etc. lunar − solar parallax ("adjusted" parallax)

H rising point of the ecliptic, ascendant, horoscope
Δ setting point of the ecliptic
M culminating point of the ecliptic, $\overline{\mathrm{M}} = \mathrm{M} + 180$ "lower midheaven"
C culminating point of the equator
V or G highest point of the ecliptic, H + 90°, "nonagesimal"
Z zenith
$r_☾, r_\odot, u$ (or s) apparent semi-diameters of moon, sun, and earth's shadow
$r_e, d_e; r_m, d_m; r_s, d_s; r_u, d_u$ actual radii and diameters of earth, moon, sun, and shadow.

zodiacal signs:

♈ Aries	♋ Cancer	♎ Libra	♑ Capricorn
♉ Taurus	♌ Leo	♏ Scorpio	♒ Aquarius
♊ Gemini	♍ Virgo	♐ Sagittarius	♓ Pisces

3. Lunar and Planetary Motion

$\bar{\ }$ mean motions, mean longitudes, etc.
α epicyclic anomaly
β geocentric latitude
η elongation from the sun
i inclination of orbit toward the ecliptic
$\kappa = \lambda - \lambda_A$ "normed" longitude, i.e. eccentric anomaly (κέντρον)
λ geocentric longitude; $\lambda = 0°$ vernal point ($= ♈ 0°$); λ_A longitude of apogee
ω argument of latitude, counted from ascending node; $\omega' = \omega - 90°$ "normed" argument of latitude, counted from β_{max} of orbit ("northern limiting point")

c equation of center; cf. also θ
e eccentricity (with respect to the center M of the deferent)
θ epicyclic equation
i inclination of orbit toward the ecliptic
l heliocentric longitude
r radius of epicycle, R radius of deferent

A apogee, or aphelium
E equant
M midpoint of deferent
O observer (earth)
P, or S planet, also sun or moon

planetary symbols:

⊙ sun	♄ Saturn	♀ Venus
☾ moon	♃ Jupiter	☿ Mercury
	♂ Mars	

4. Planetary and Fixed Star Phases

γ arcus visionis

Outer Planets (♄ ♃ ♂):

Γ first appearance
Φ first station
Θ opposition
Ψ second station
Ω last appearance
C conjunction

Inner Planets (♀ ☿):

Γ first appearance, Φ max. elongation, Σ last appearance — as morning star
Ξ first appearance, Ψ max. elongation, Ω last appearance — as evening star
C_i inferior conjunction
C_s superior conjunction

Fixed Stars:

Γ heliacal rising
Θ_1 last evening rising
Θ_2 last morning setting
Ω acronychal setting

§ 4. Greek Glossary

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ἀνάβασις 672
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ἀνατολή, θερινή and ἰσημερινή 295 n. 25
ἄνεμος 954 n. 24
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ἄνωθεν 758
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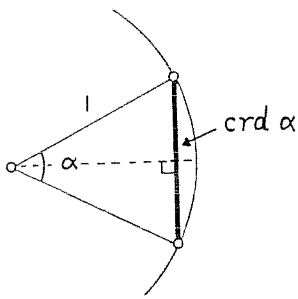


Fig. 1

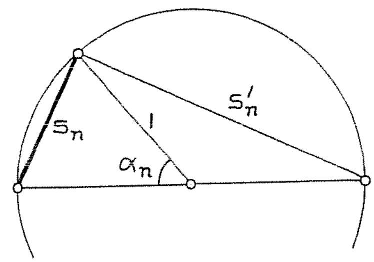
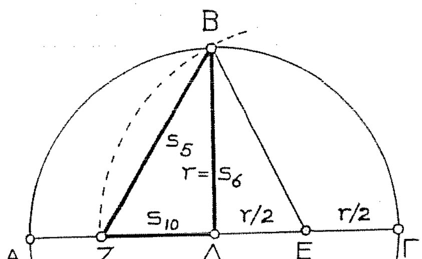


Fig. 2



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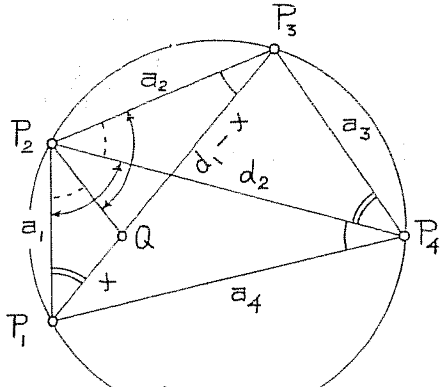

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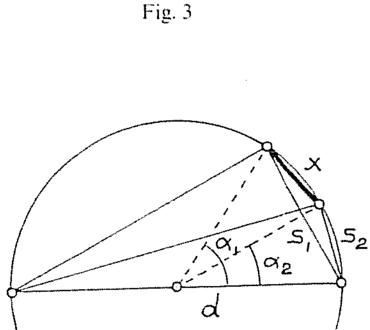
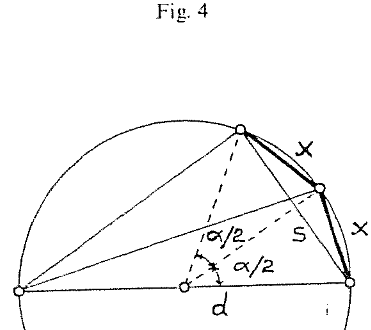


Fig. 5



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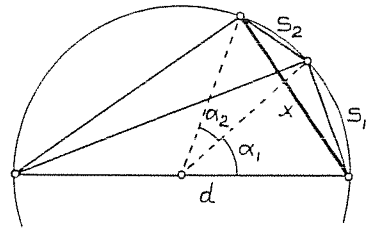

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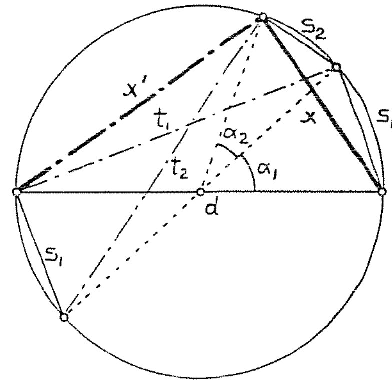

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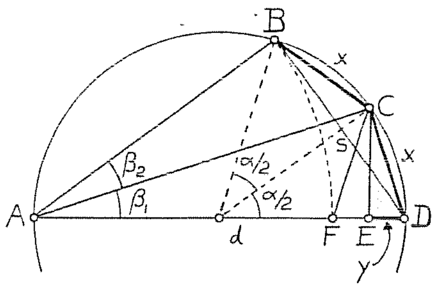

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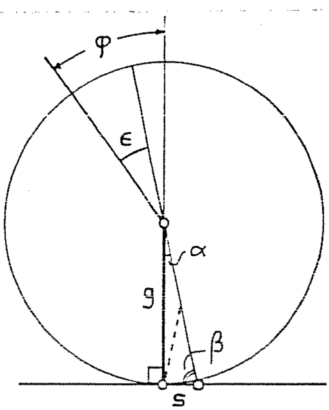

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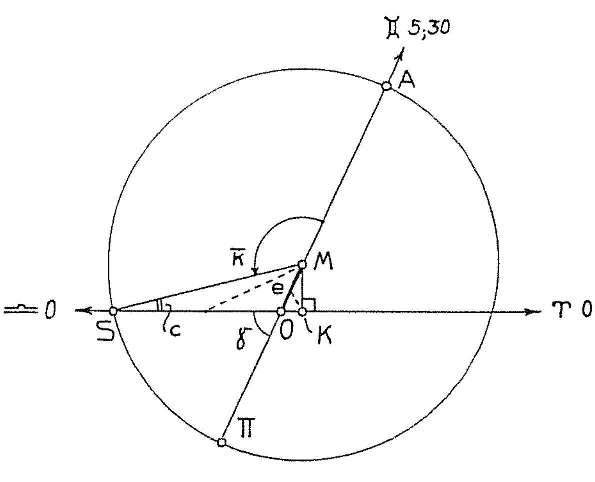

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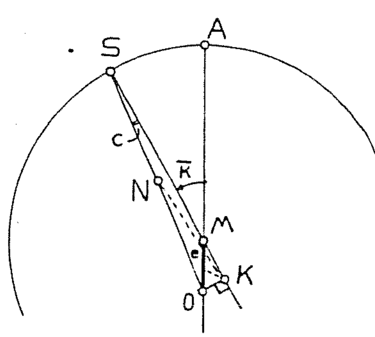

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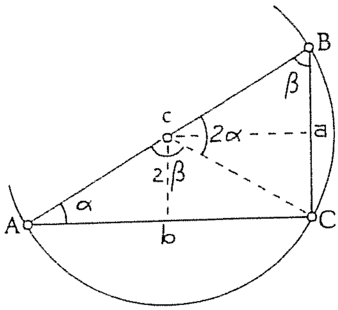

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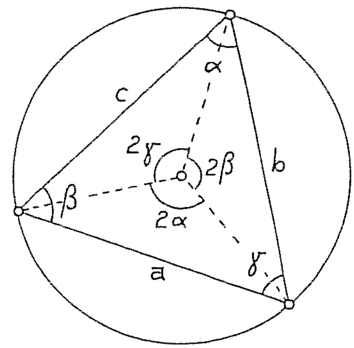

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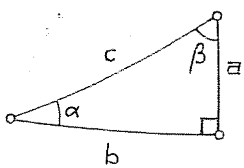

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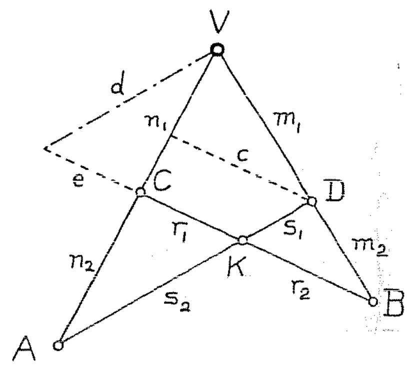

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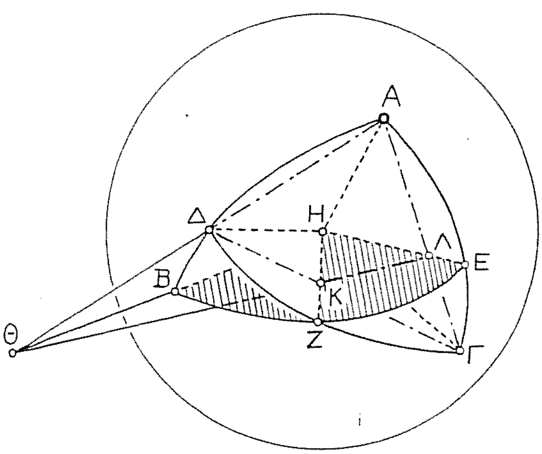

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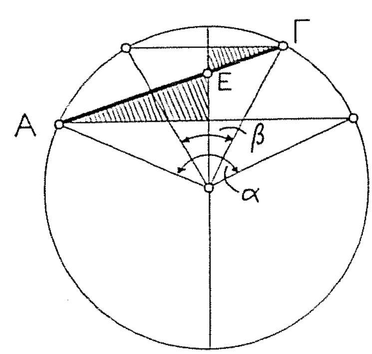

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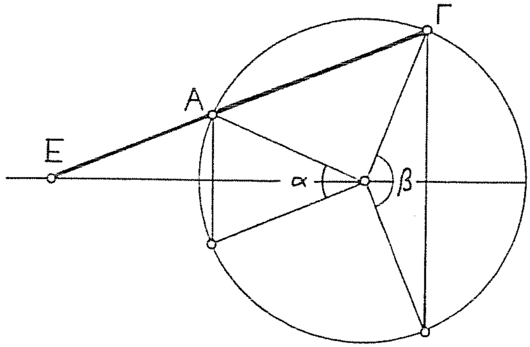

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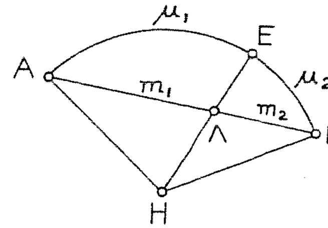

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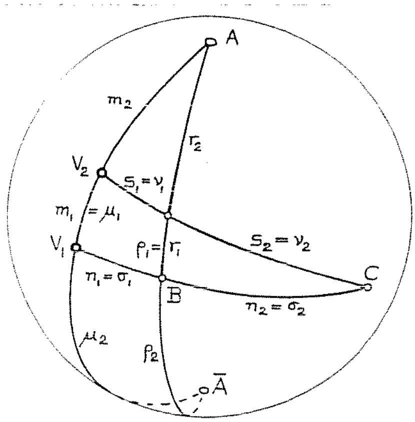

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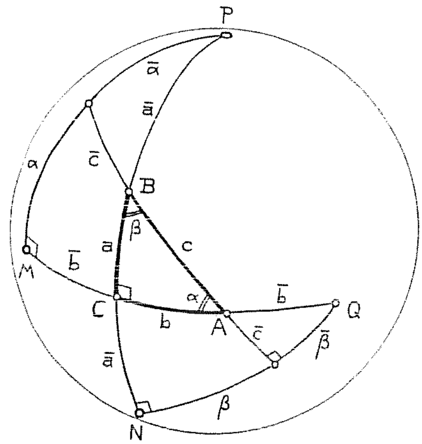

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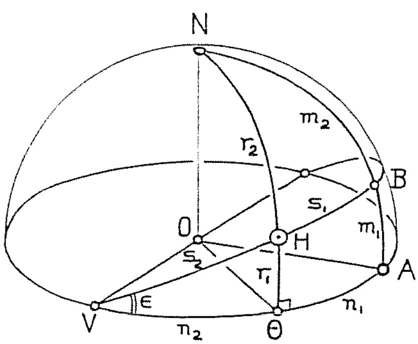

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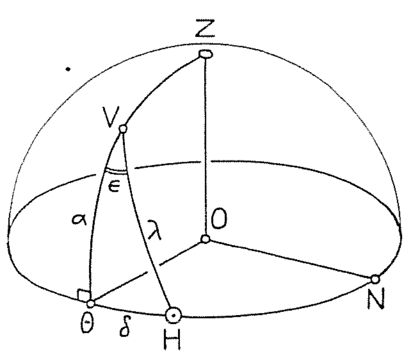

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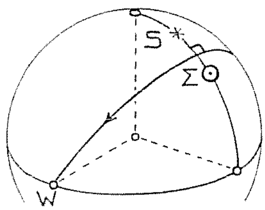

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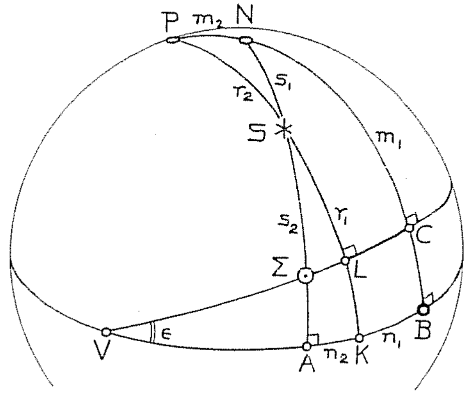


Fig. 26

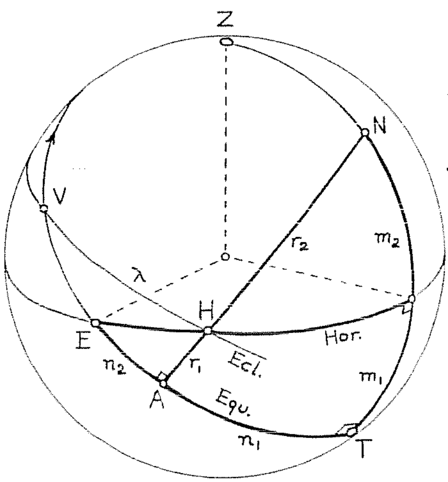


Fig. 27

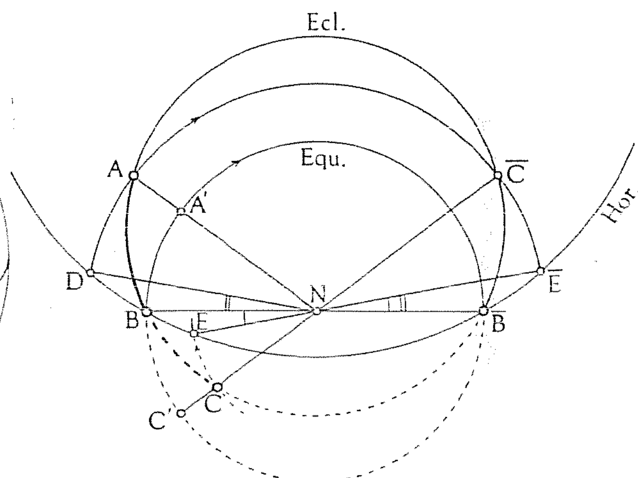


Fig. 28

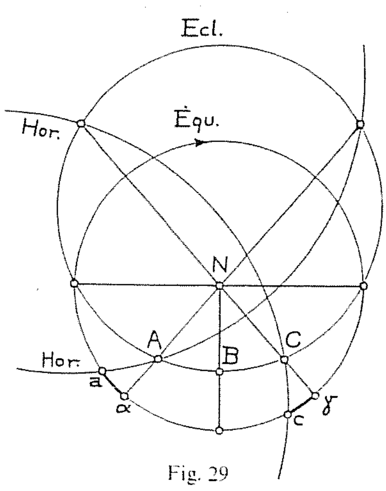


Fig. 29

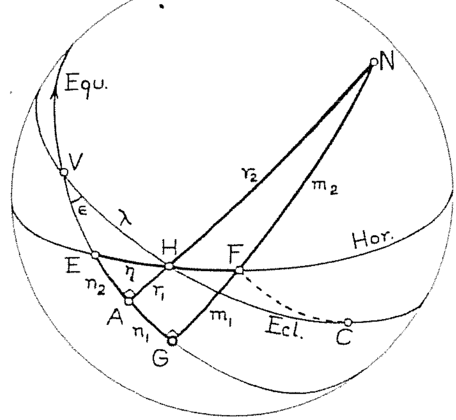
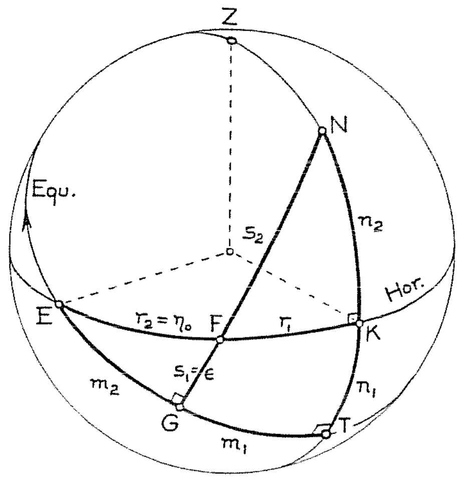
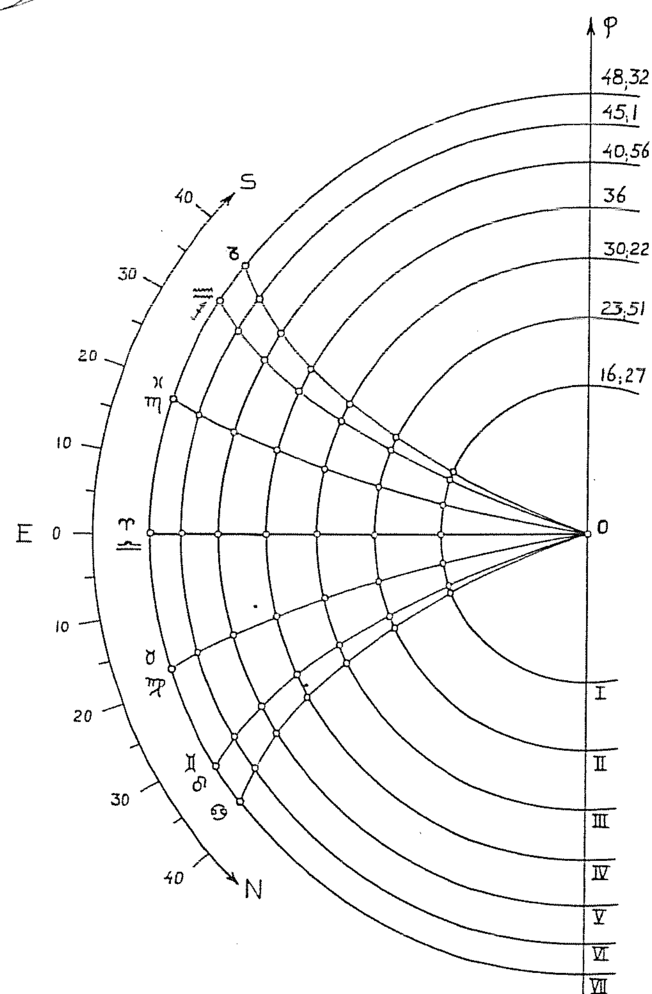


Fig. 30



Z
N
Equ.
s_2
n_2
Hor.
E
$r_2 = \eta_0$
F
r_1
K
$s_1 = \epsilon$
m_2
n_1
G
T
m_1

Fig. 31



φ
48;32
45;1
40;56
36
30;22
23;51
16;27
S
40
30
20
10
E
0
10
20
30
40
N
O
I
II
III
IV
V
VI
VII

Fig. 32

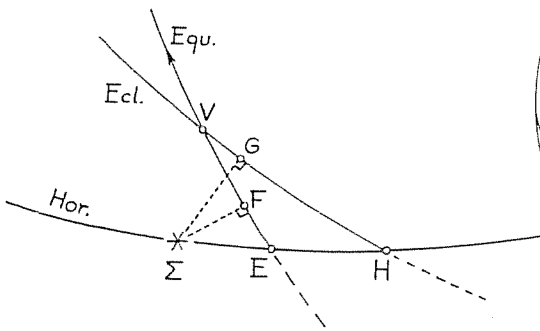


Fig. 33

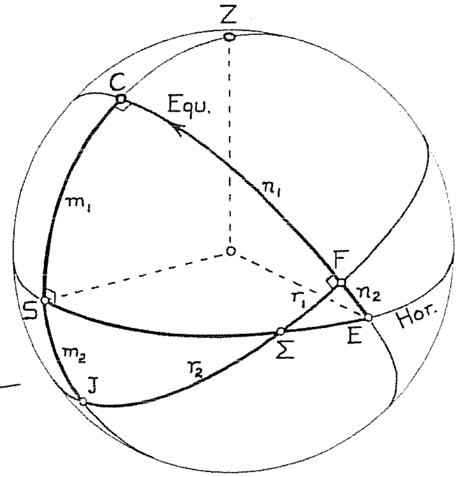


Fig. 34

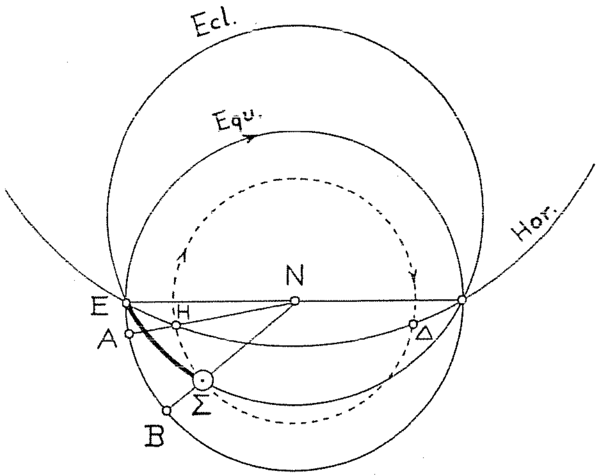


Fig. 35

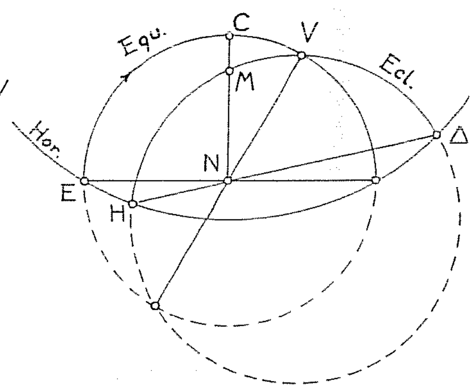


Fig. 36

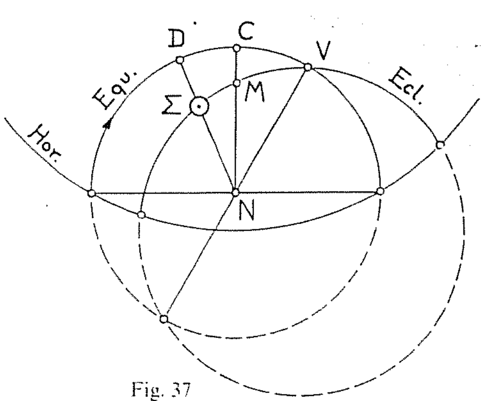


Fig. 37

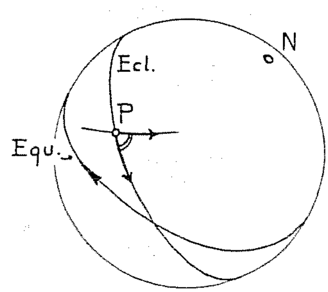


Fig. 38

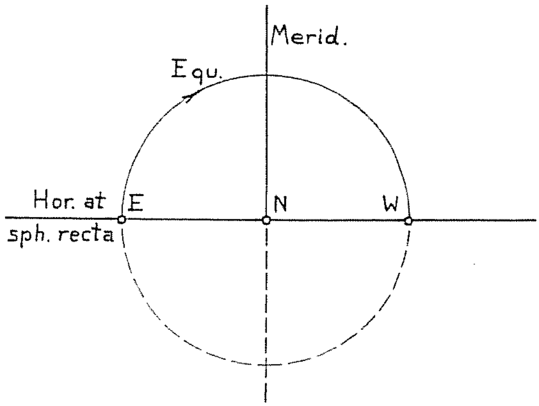


Fig. 39

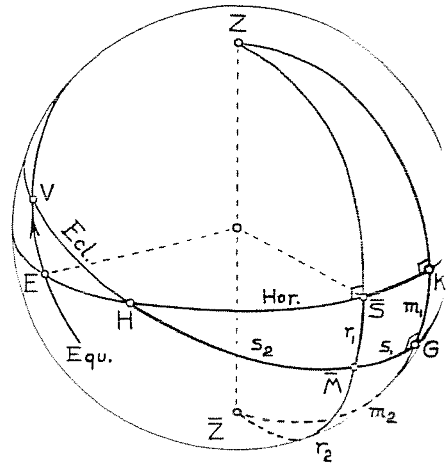


Fig. 40

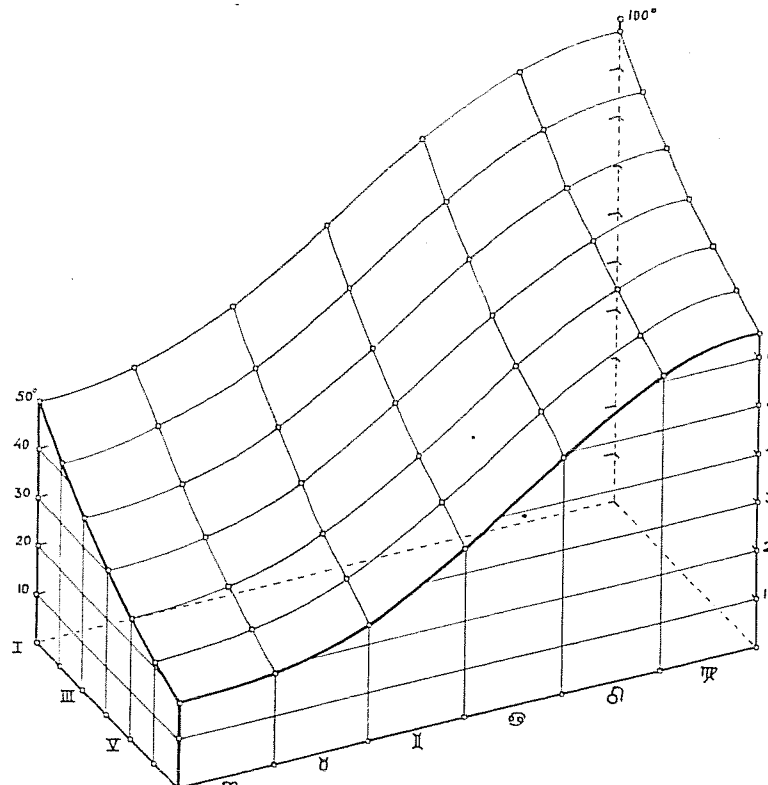


Fig. 41

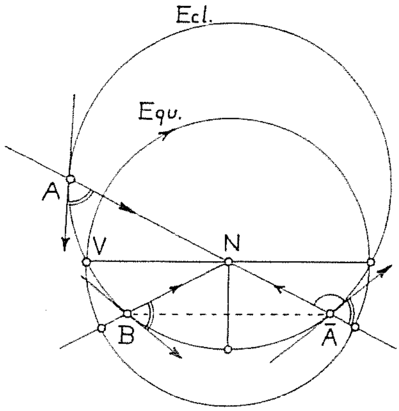


Fig. 42

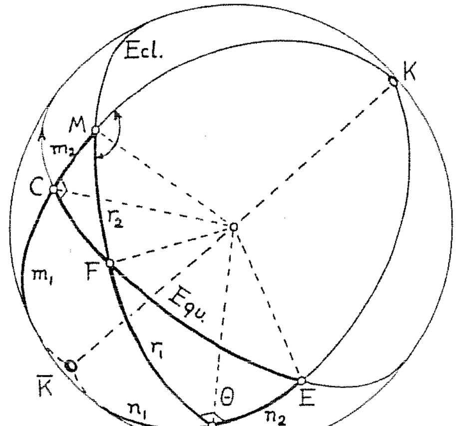


Fig. 43

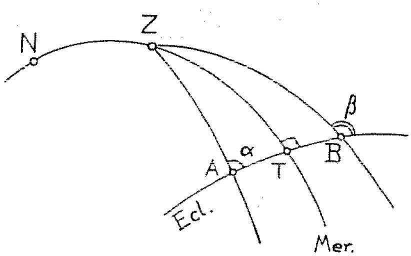


Fig. 44

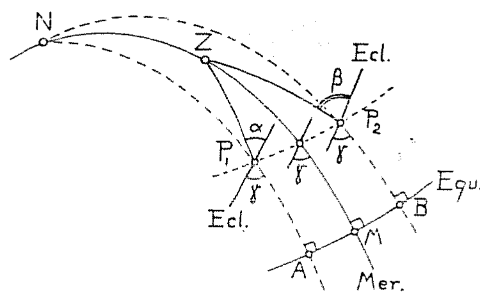


Fig. 45

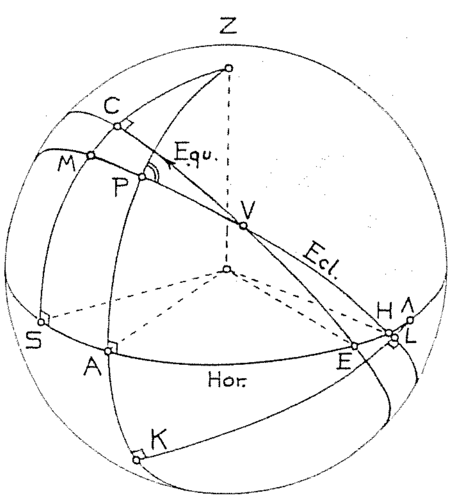


Fig. 46

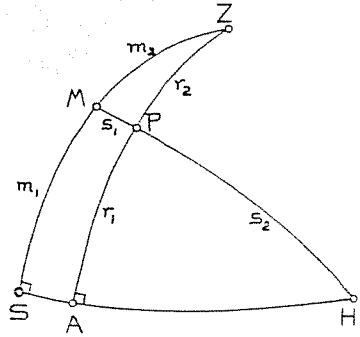


Fig. 47

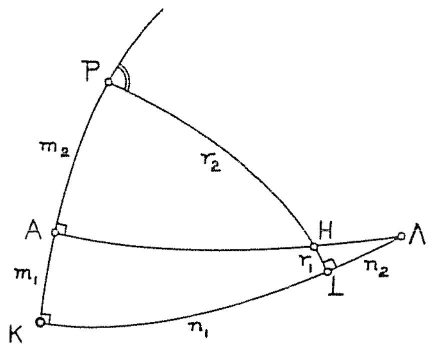


Fig. 48

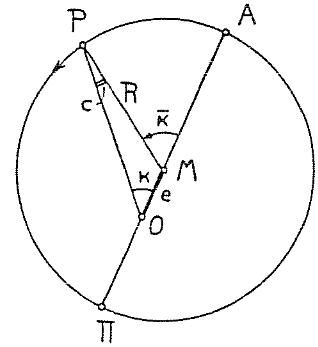


Fig. 49

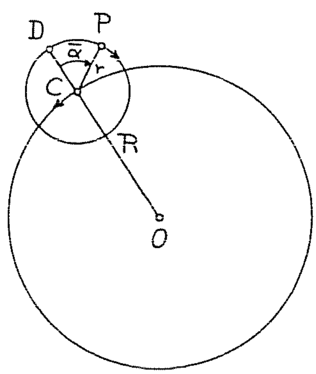


Fig. 50

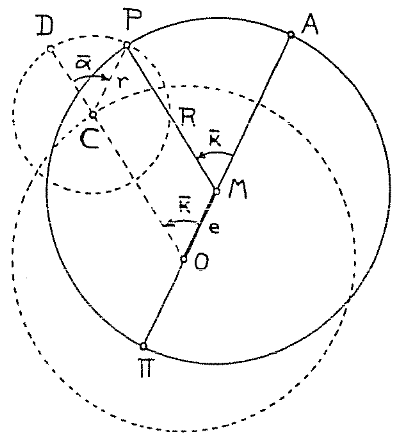


Fig. 51

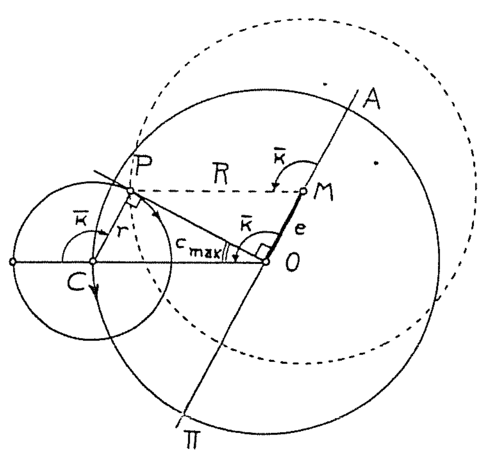


Fig. 52

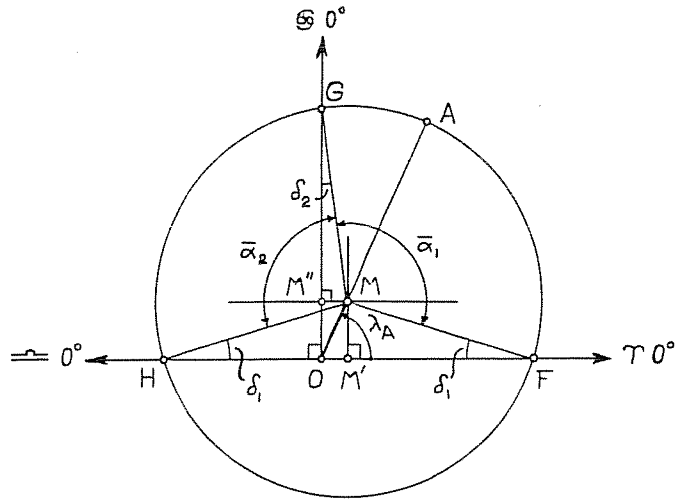


Fig. 53

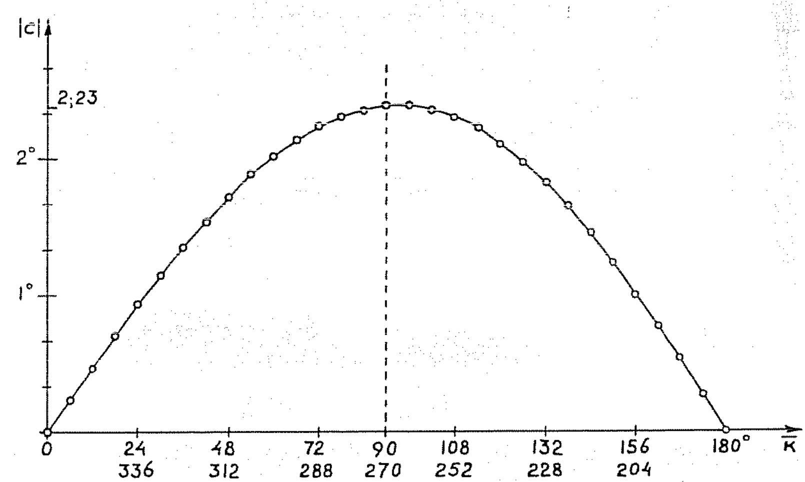


Fig. 54

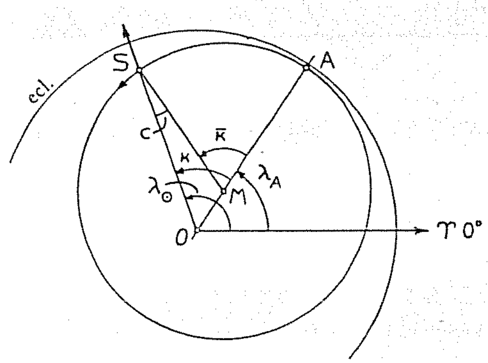


Fig. 55

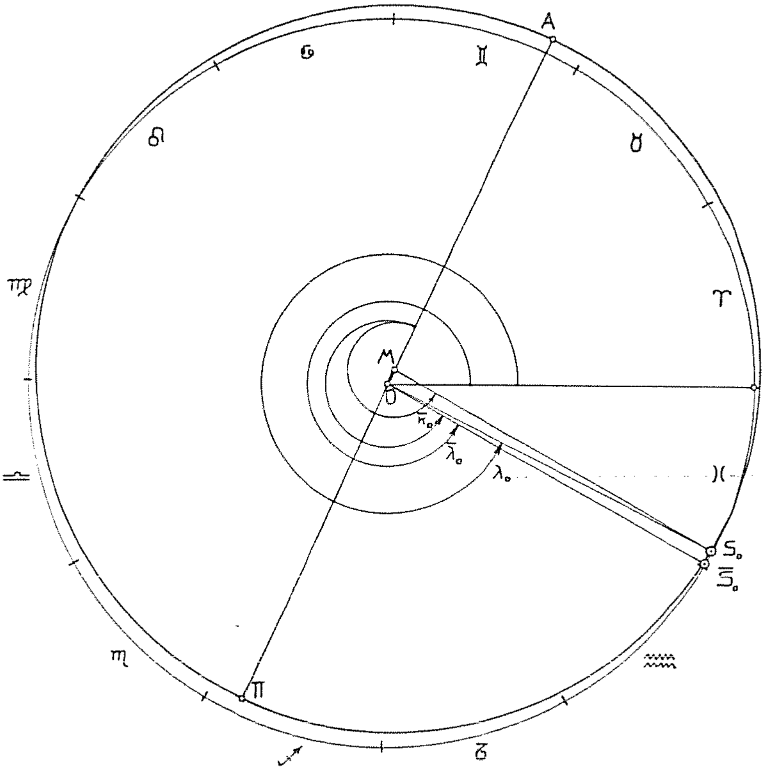

A
M
O
Π
S_o
$\bar{S}_o$
$\bar{\pi}_o$
$\bar{\lambda}_o$
λ_o

Fig. 56

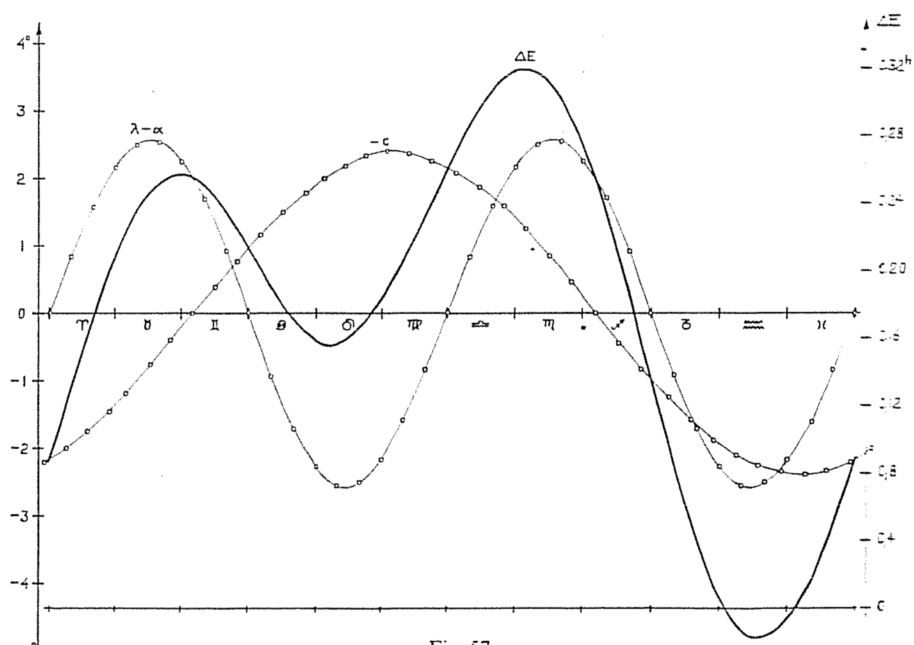

4°
3
2
1
0
-1
-2
-3
-4
-5°
λ−α
−c
ΔE
ΔE
0.32ʰ
0.28
0.24
0.20
0.16
0.12
0.08
0.04
0

Fig. 57

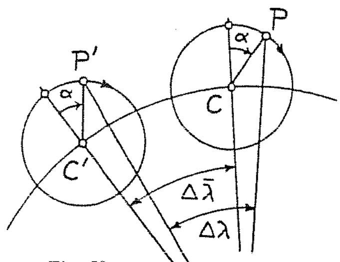


Fig. 58

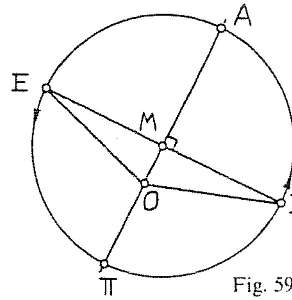


Fig. 59

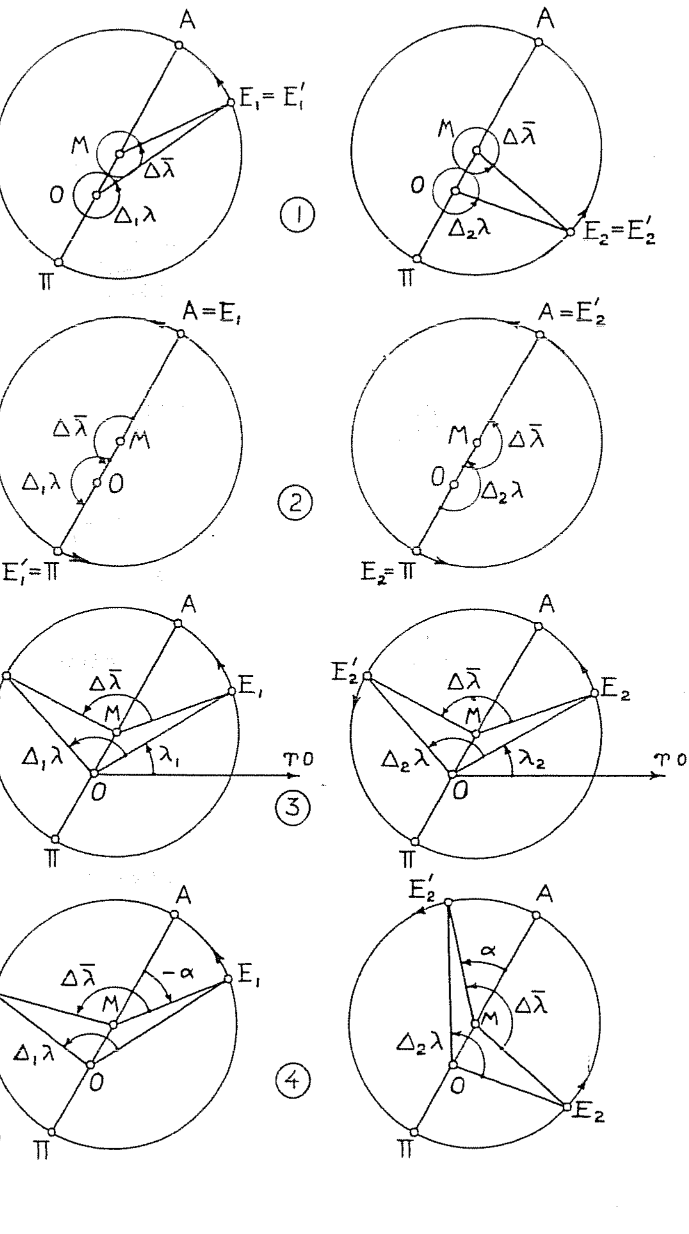


Fig. 60

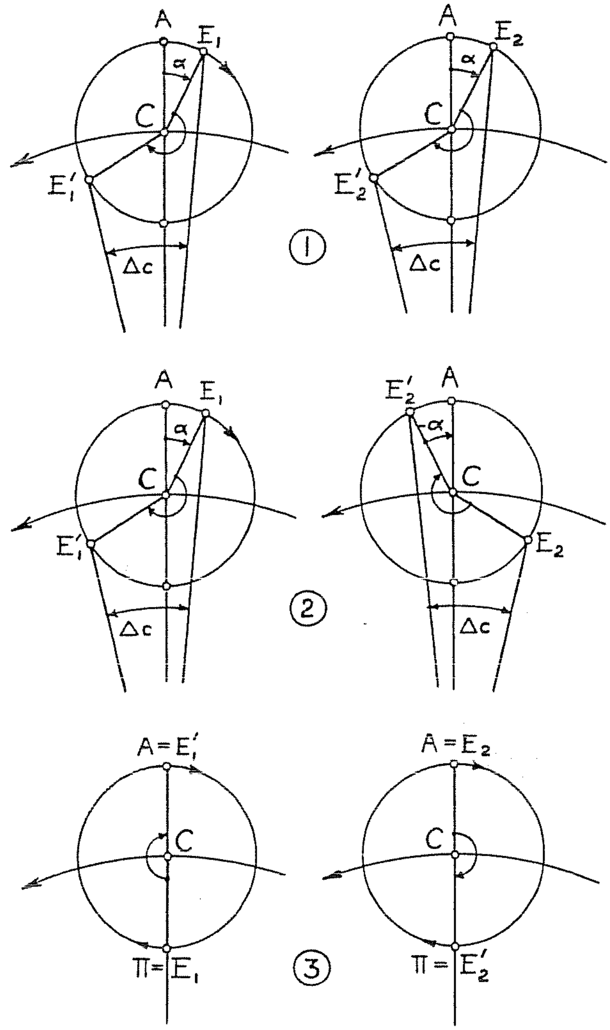

A
E_1
α
C
E_1'
Δc
A
E_2
α
C
E_2'
Δc
1
A
E_1
α
C
E_1'
Δc
E_2'
A
$-\alpha$
C
E_2
Δc
2
$A = E_1'$
C
$\Pi = E_1$
$A = E_2$
C
$\Pi = E_2'$
3

Fig. 61

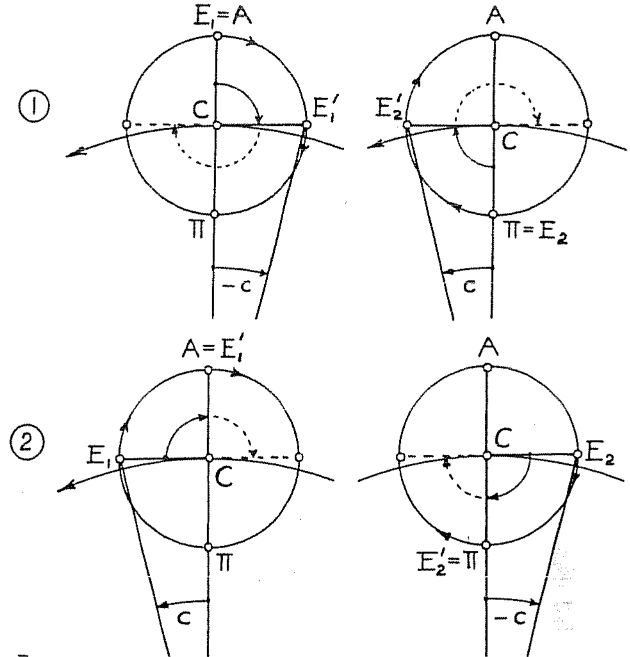

E₁=A
A
①
C
E₁′
E₂′
C
Π
Π=E₂
−c
c
A=E₁′
A
②
E₁
C
C
E₂
Π
E₂′=Π
c
−c

Fig. 62

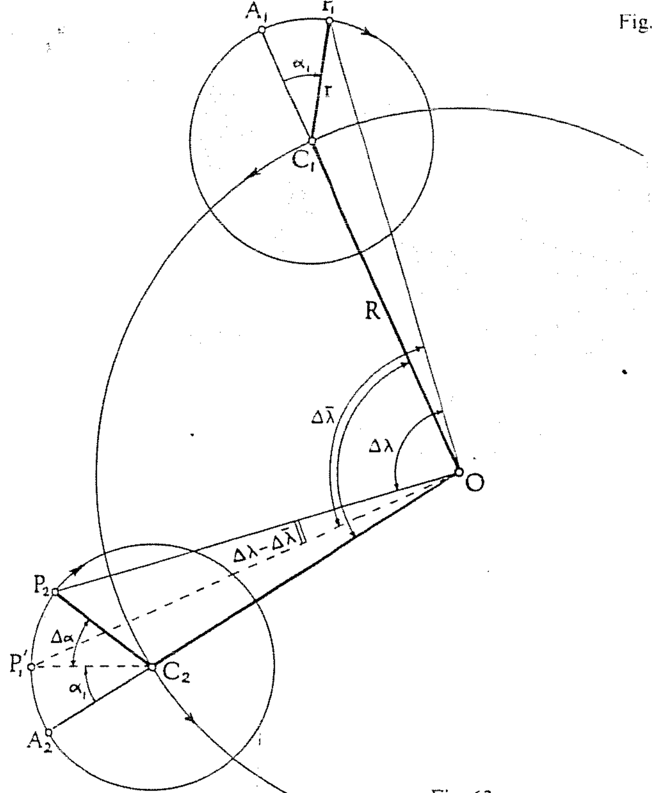

A₁
P₁
α₁
r
C₁
R
Δλ̄
Δλ
O
Δλ-Δλ̄
P₂
Δα
P₁′
C₂
α₁
A₂

Fig. 63

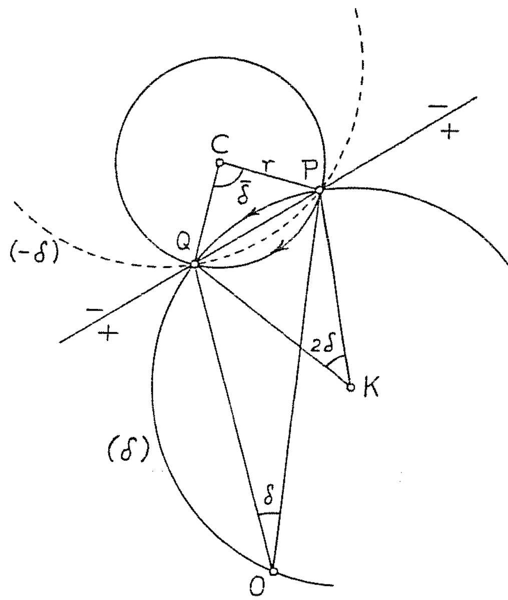

C
r
P
$\bar{\delta}$
Q
$(-\delta)$
K
2δ
(δ)
δ
O
−
+
−
+

Fig. 64

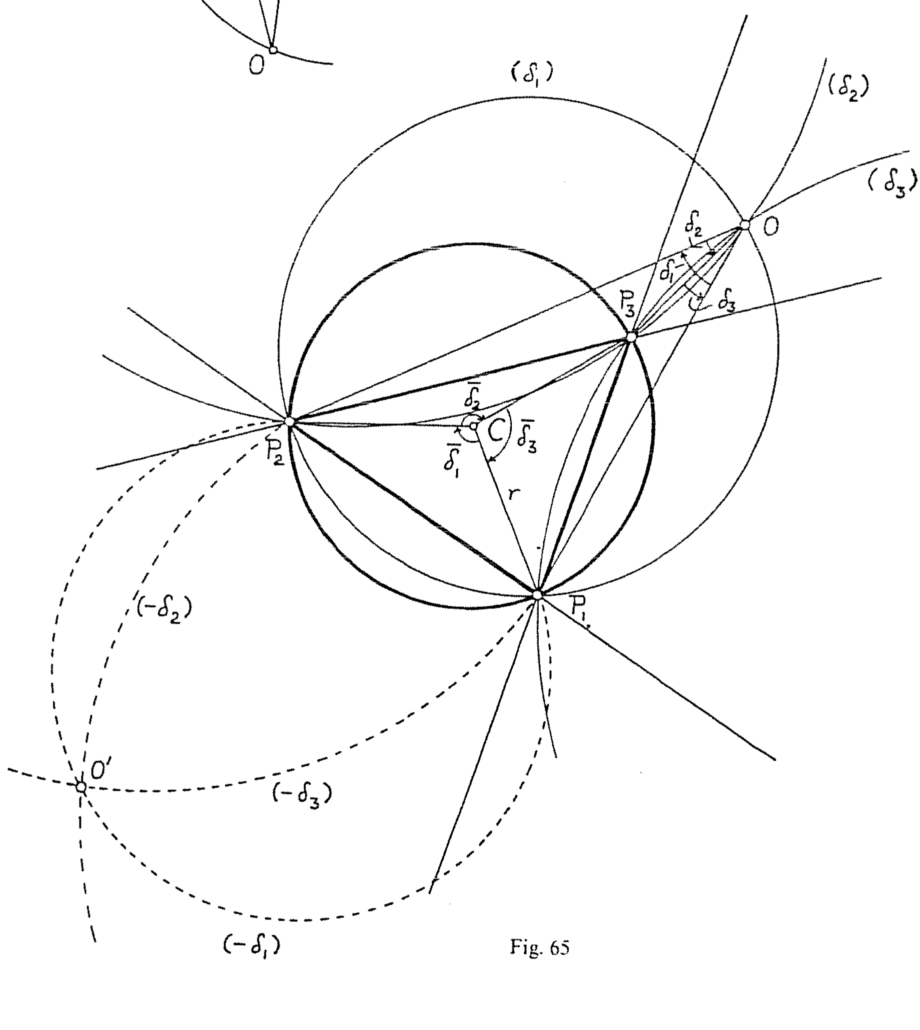

(δ_1)
(δ_2)
(δ_3)
O
δ_2
δ_1
δ_3
P_3
$\bar{\delta}_2$
C
$\bar{\delta}_3$
$\bar{\delta}_1$
P_2
r
P_1
$(-\delta_2)$
O'
$(-\delta_3)$
$(-\delta_1)$

Fig. 65

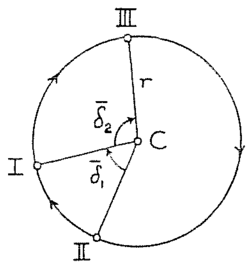


Fig. 66

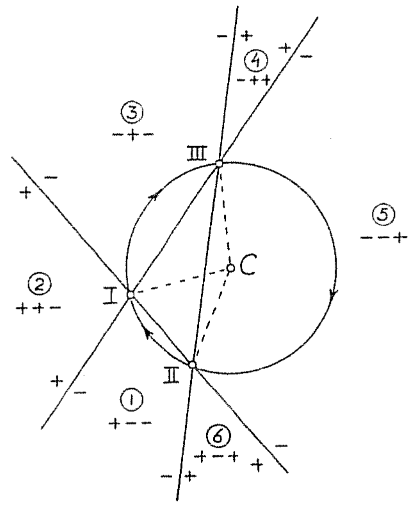


Fig. 67

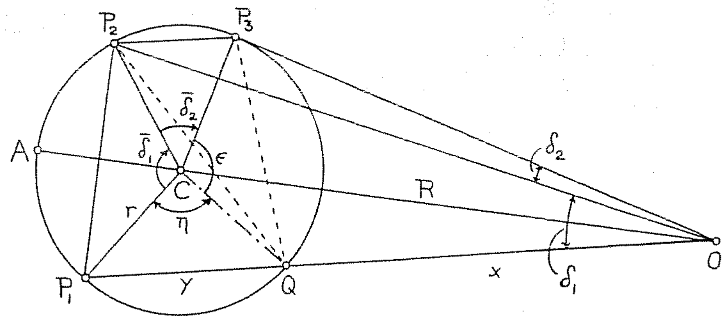


Fig. 68

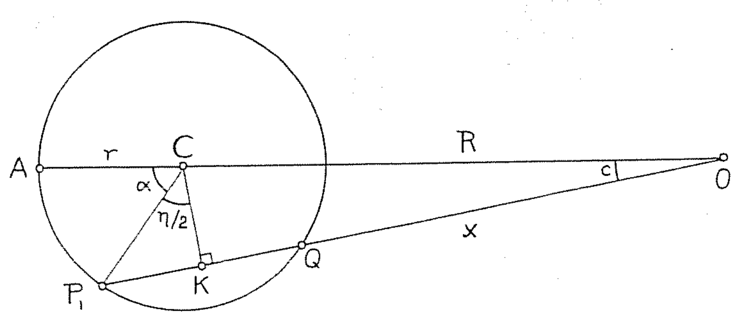


Fig. 69

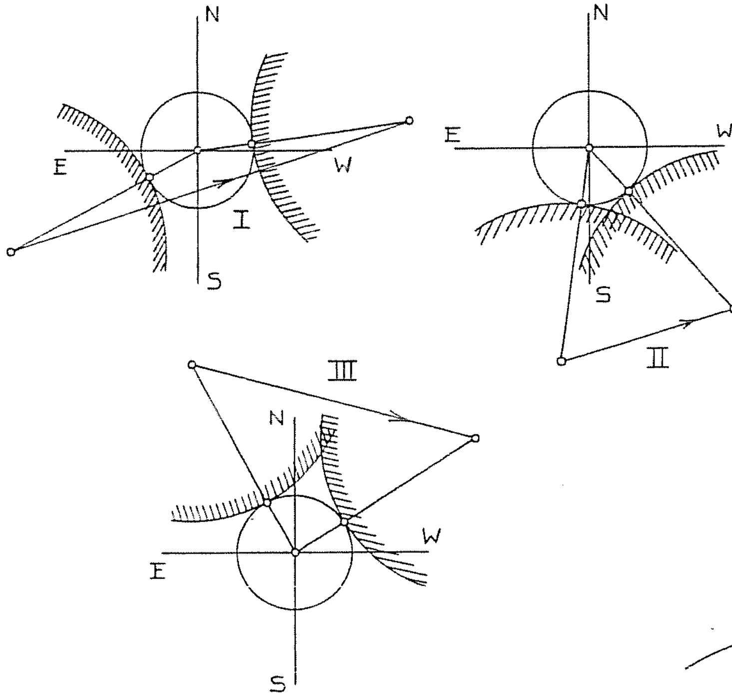


Fig. 70

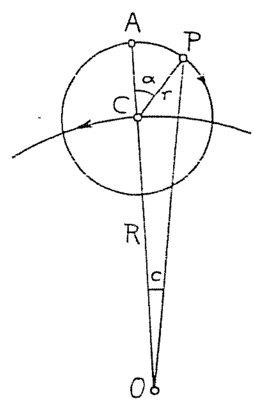


Fig. 71

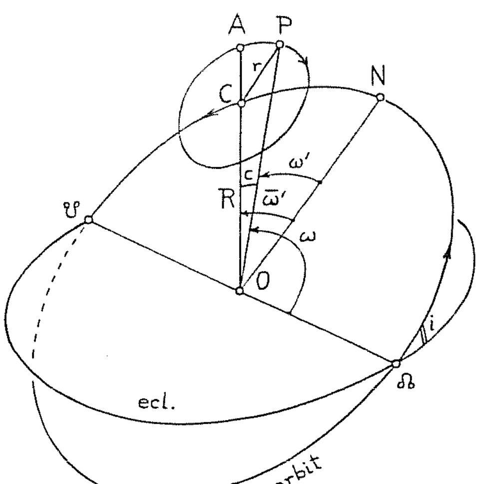


Fig. 72

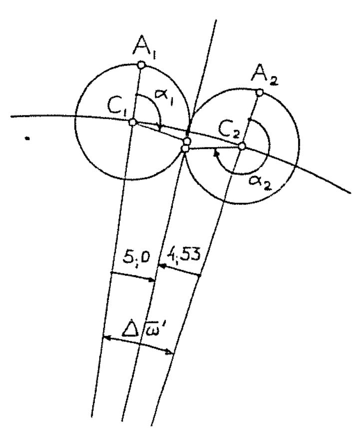


Fig. 73

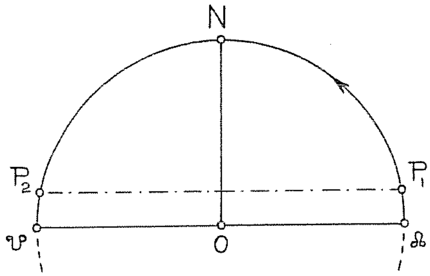


Fig. 74

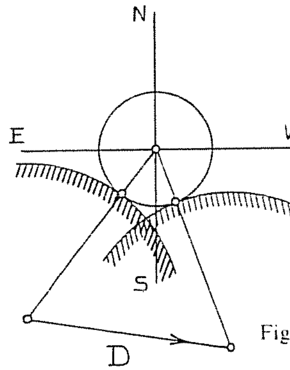


Fig. 75

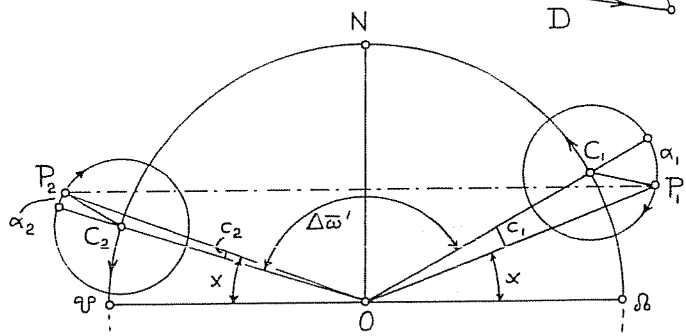


Fig. 76

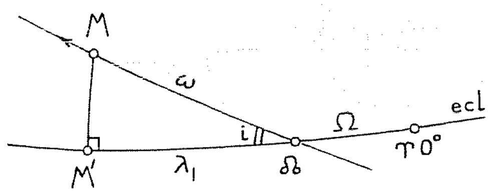


Fig. 77

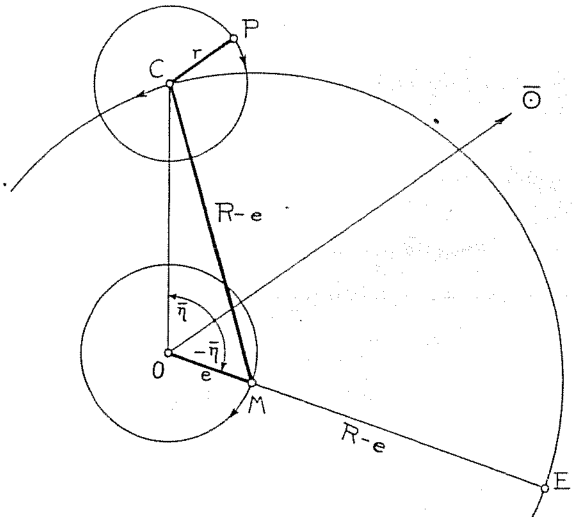


Fig. 78

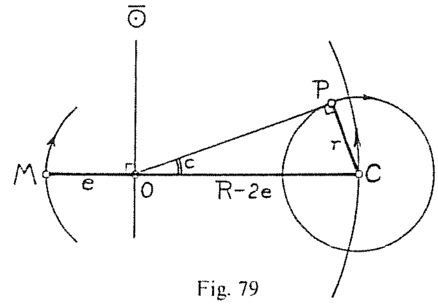


Fig. 79

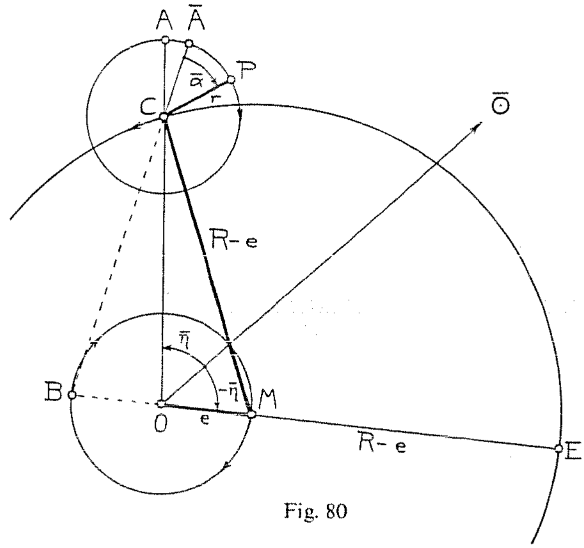


Fig. 80

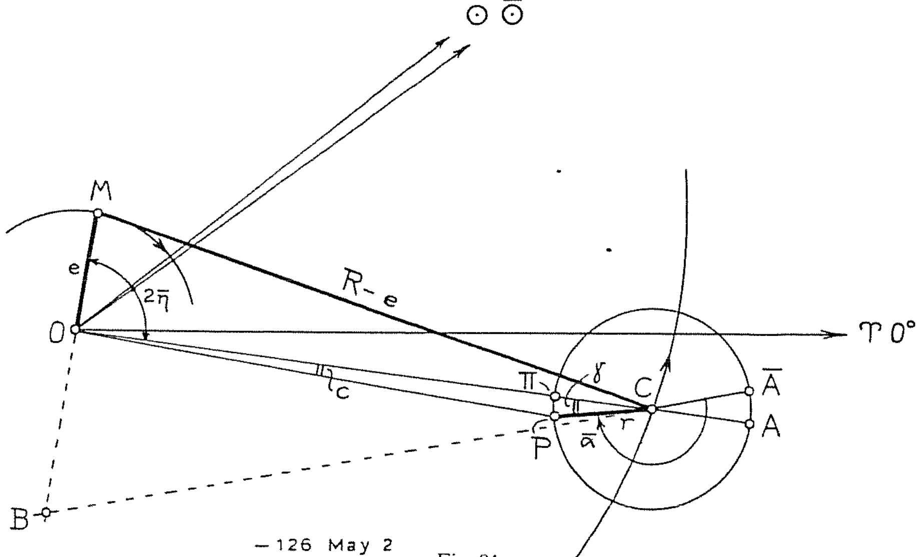


Fig. 81

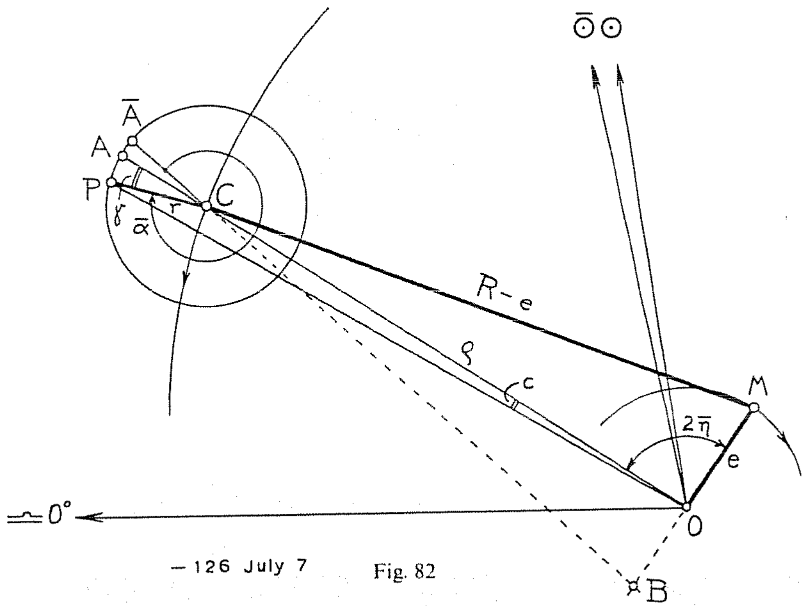


Fig. 82

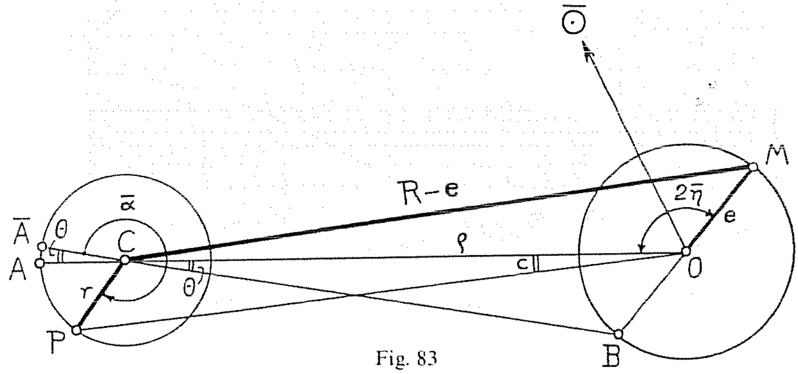


Fig. 83

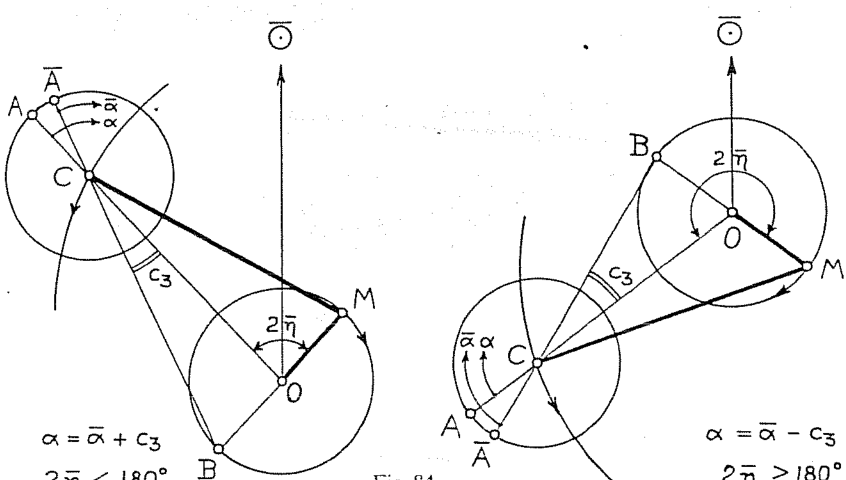


Fig. 84

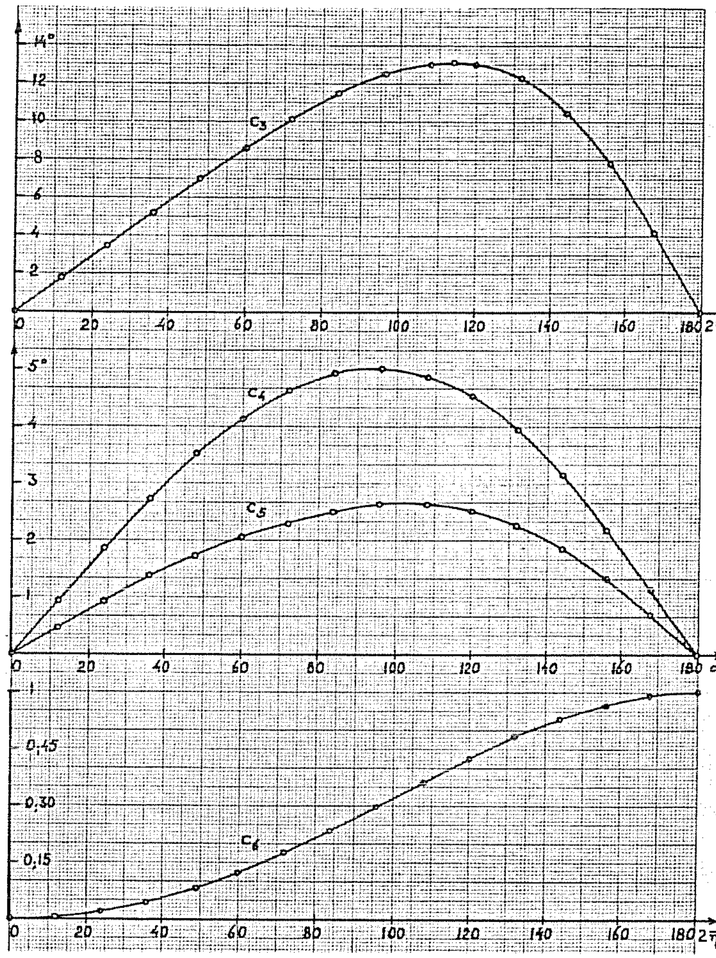


Fig. 85

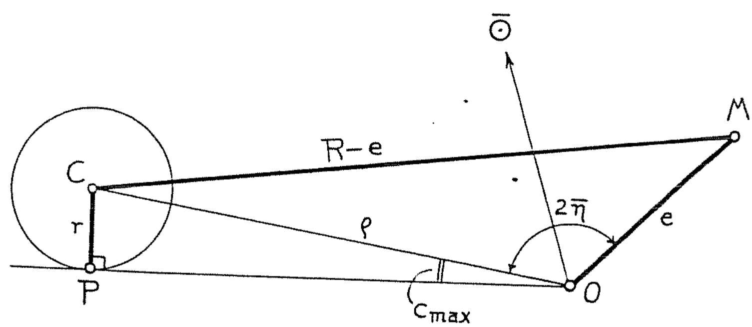


Fig. 86

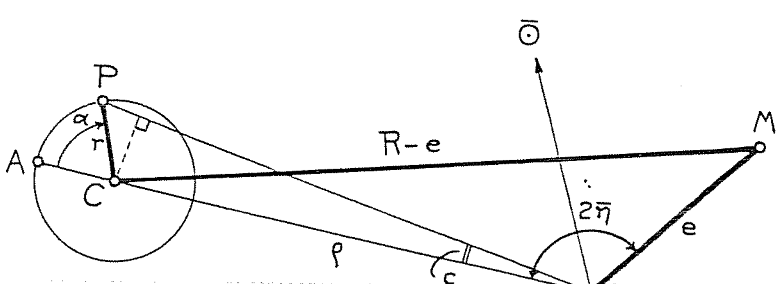


Fig. 87

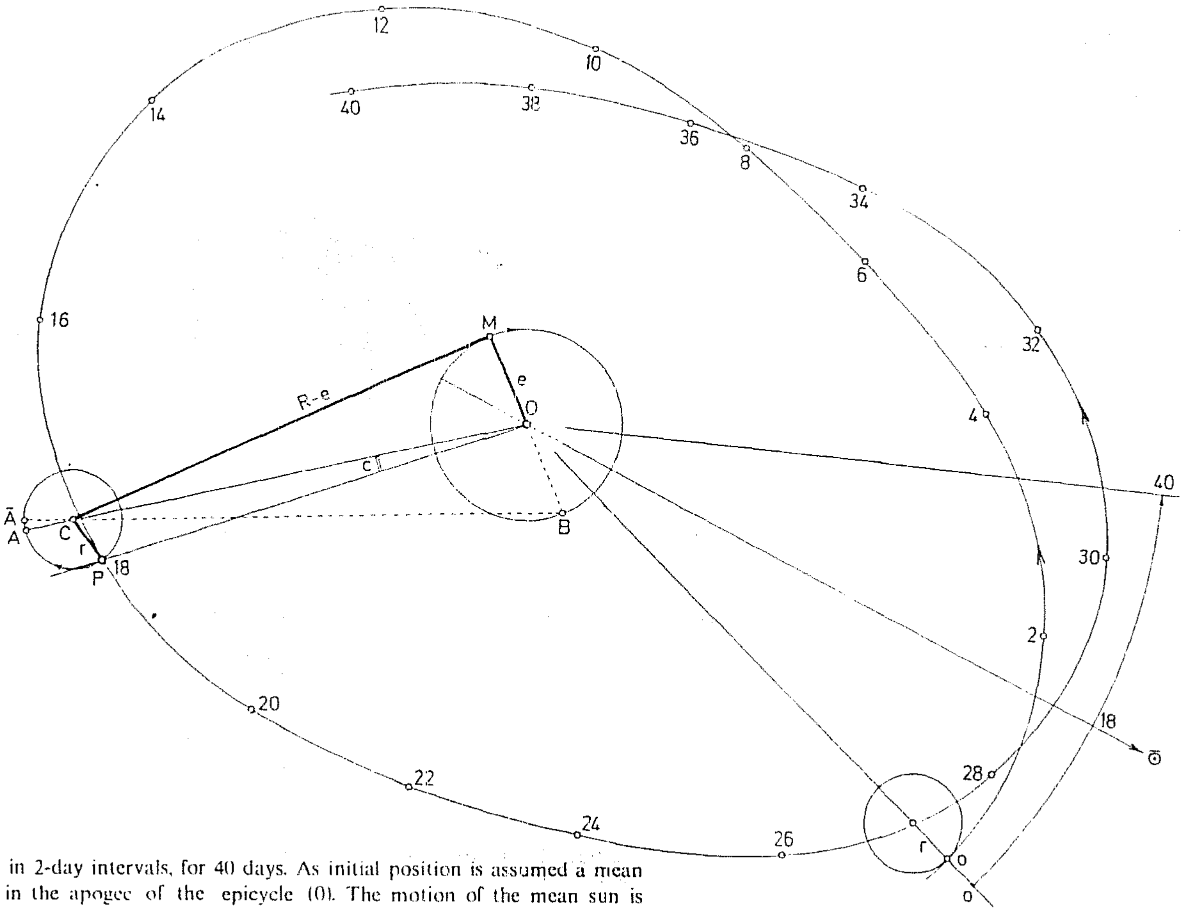


Fig. 88. Positions of the moon in 2-day intervals, for 40 days. As initial position is assumed a mean conjunction, the moon being in the apogee of the epicycle (0). The motion of the mean sun is indicated by the arrow from 0 over 18 to 40. The elongation for day 18 is 219 1/2°. Drawn to scale

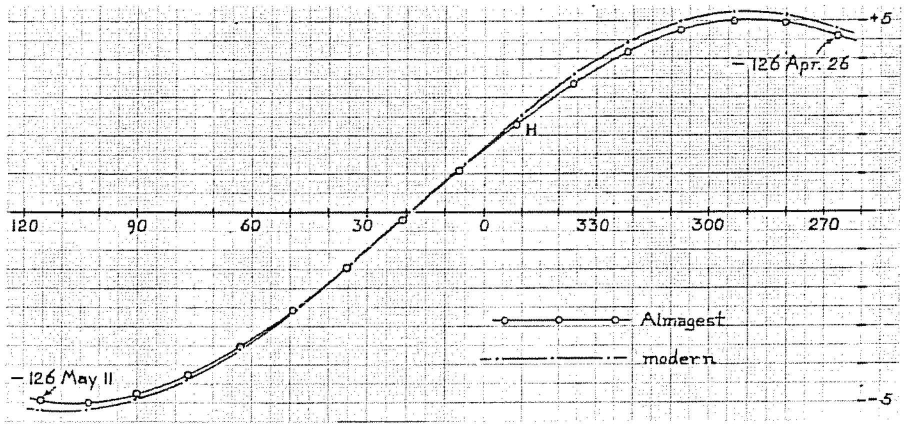


Fig. 89

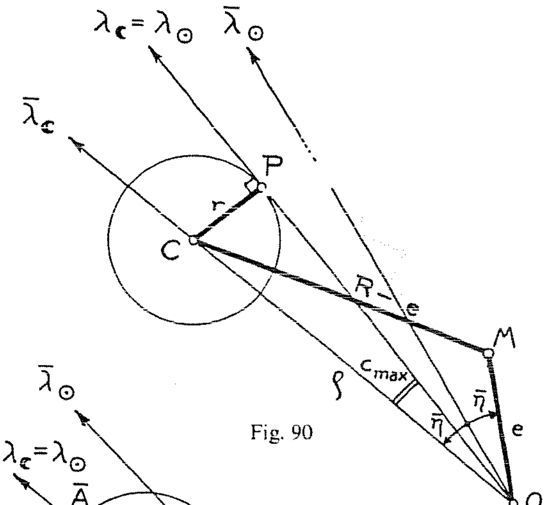


Fig. 90

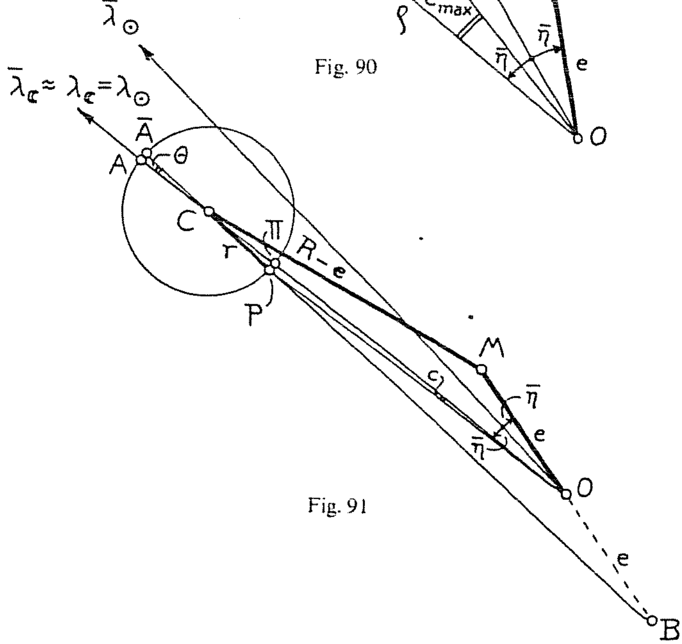


Fig. 91

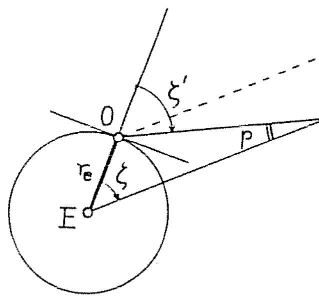


Fig. 92

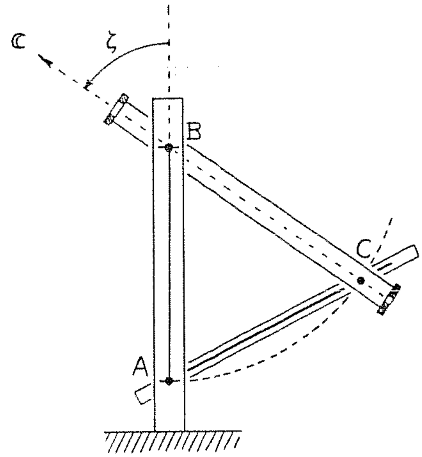


Fig. 93

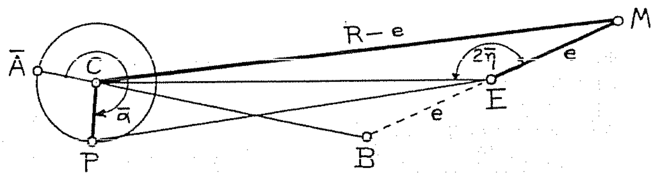
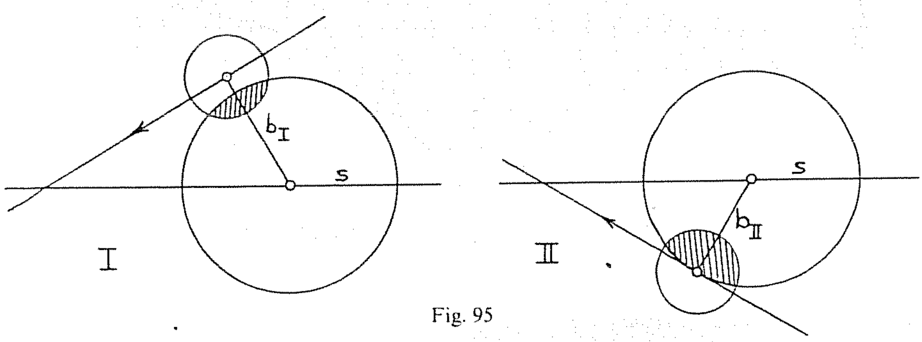


Fig. 94



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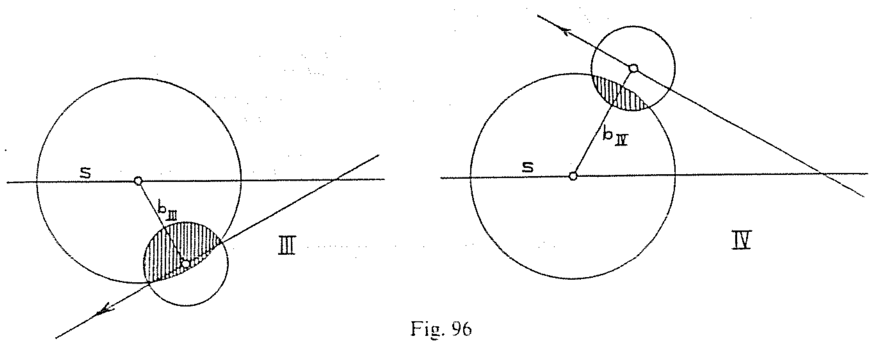

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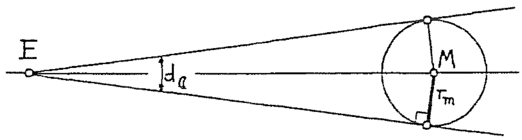

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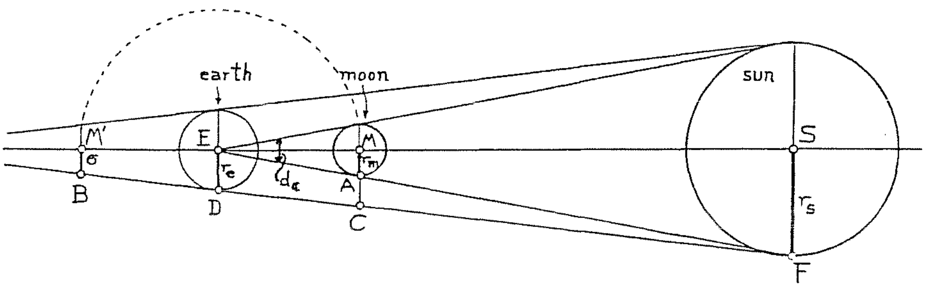

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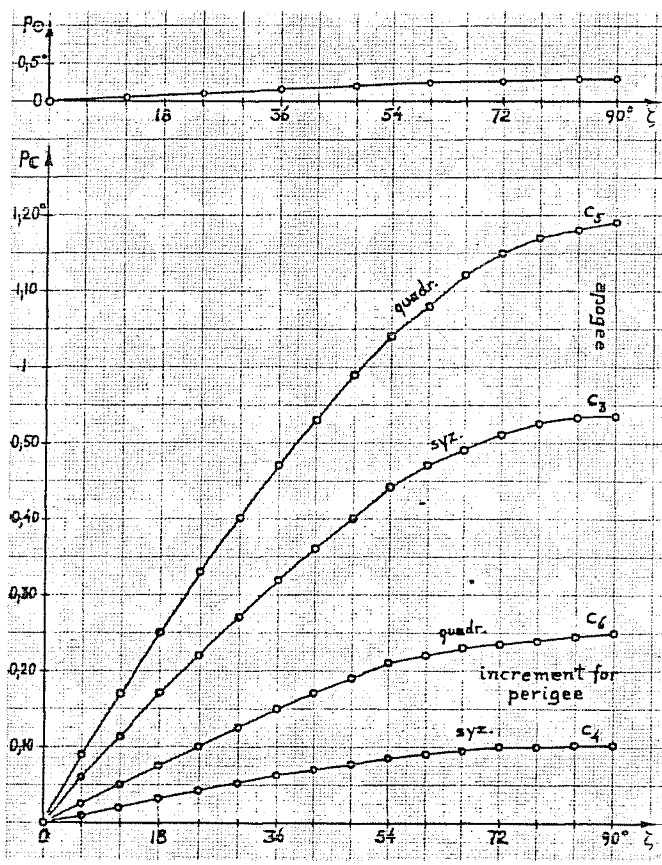

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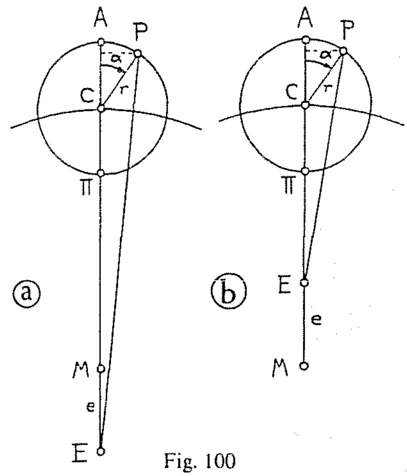

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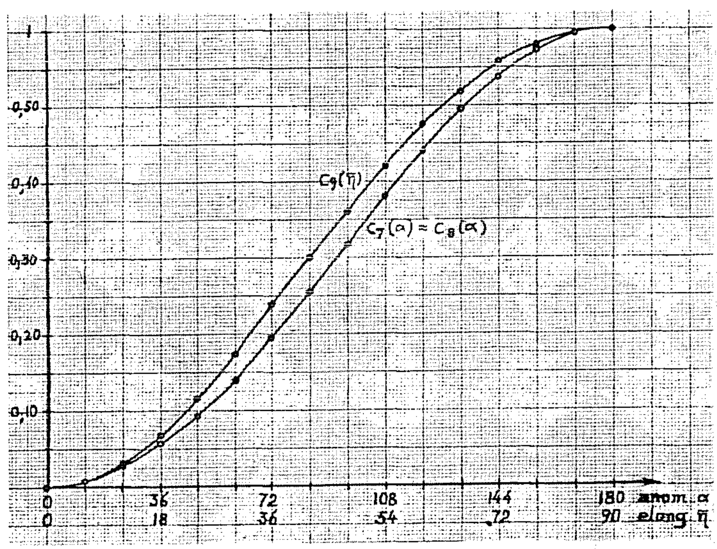

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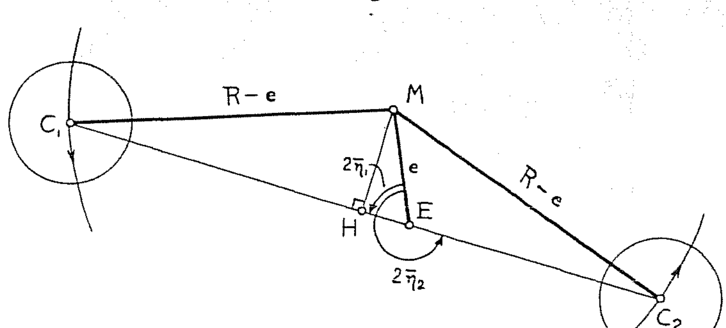

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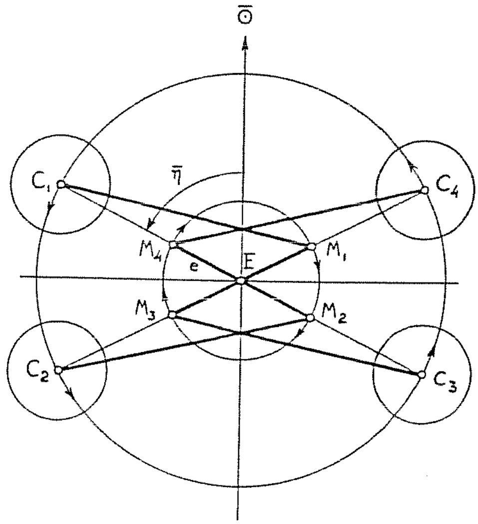

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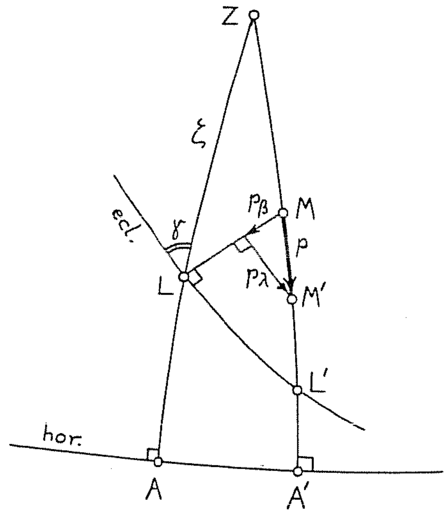

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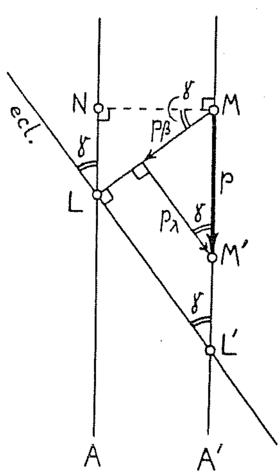

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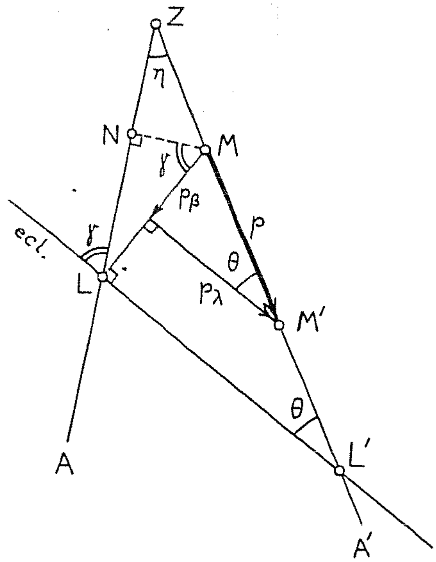

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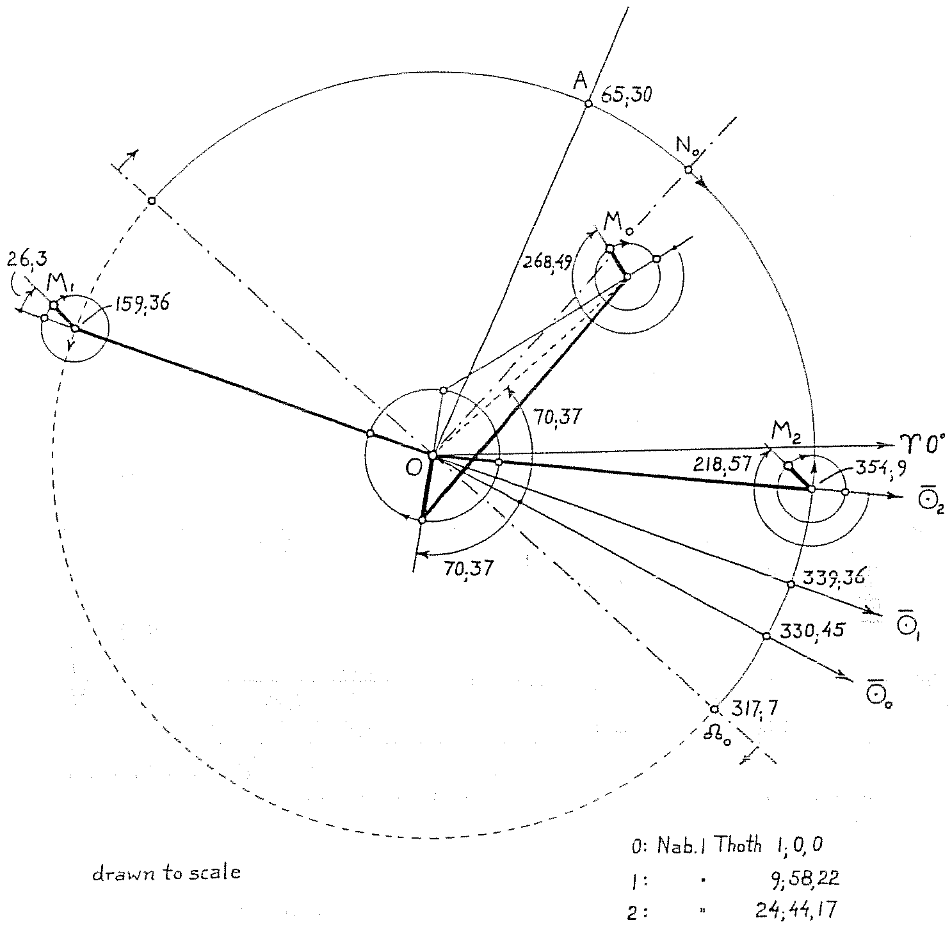

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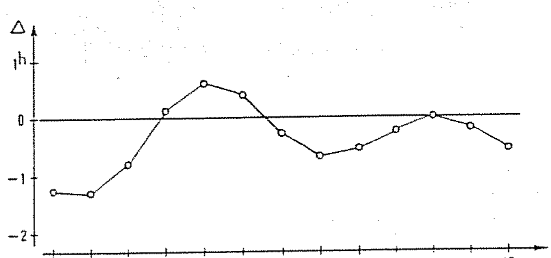

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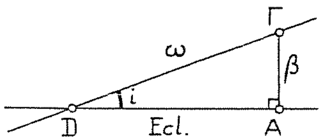

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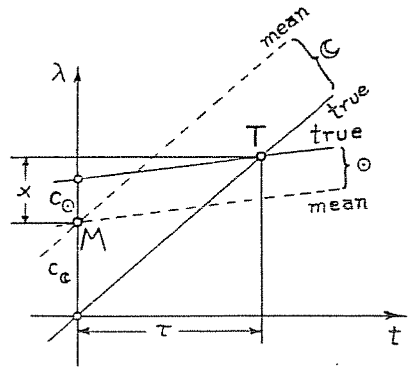

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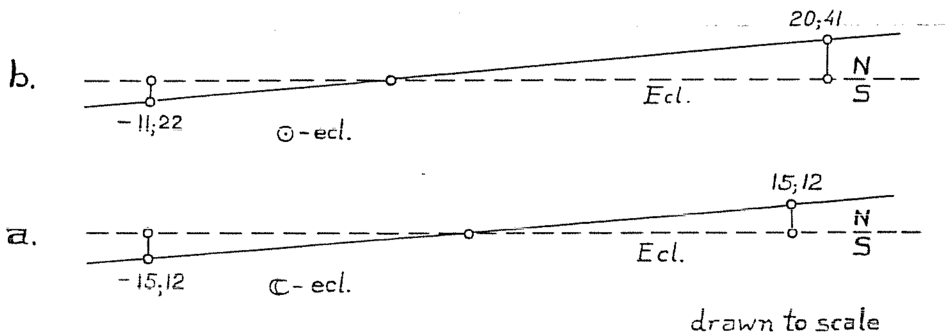

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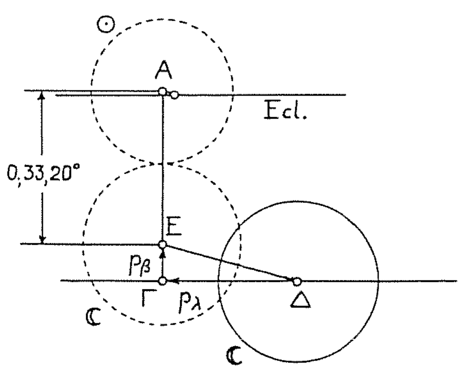

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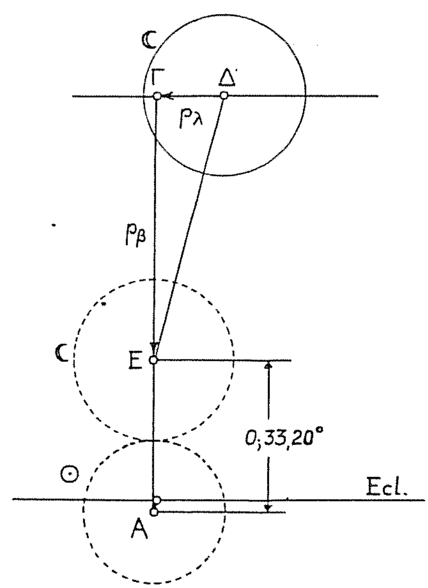

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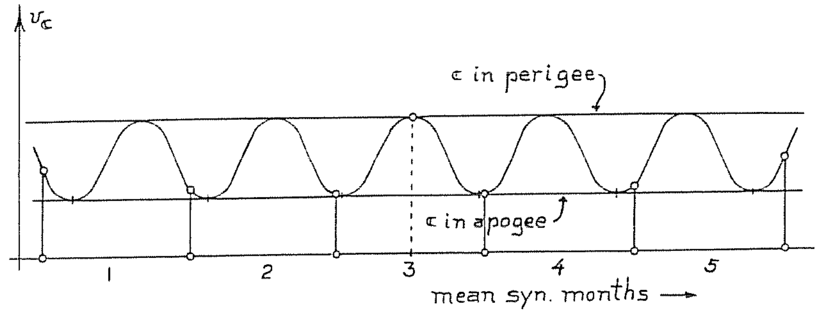

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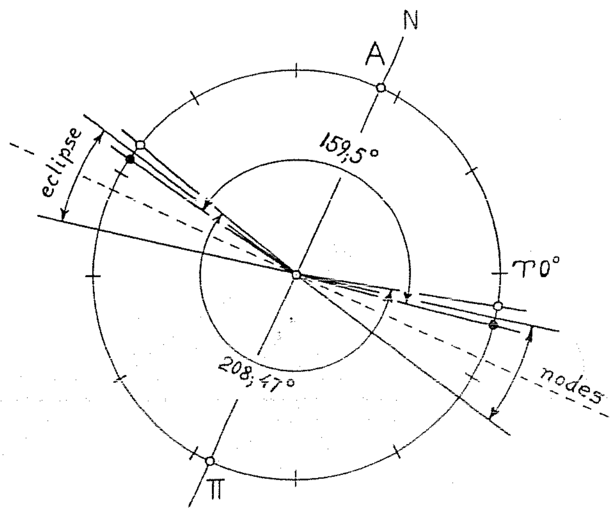

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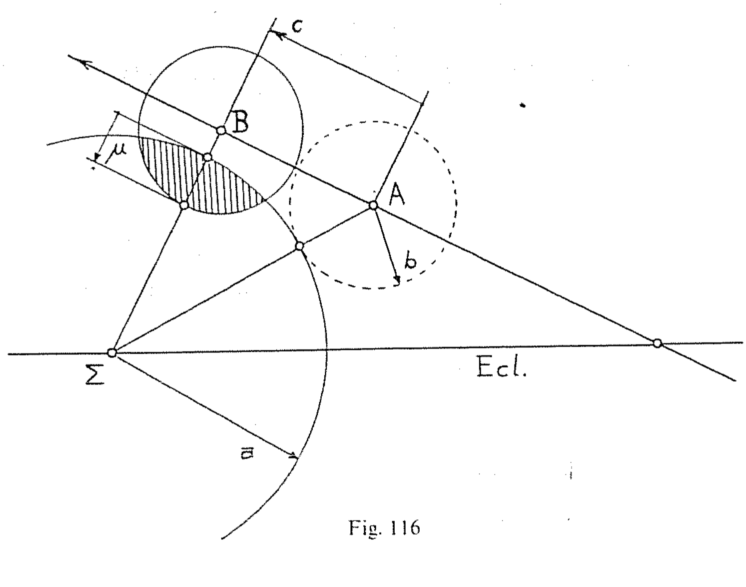

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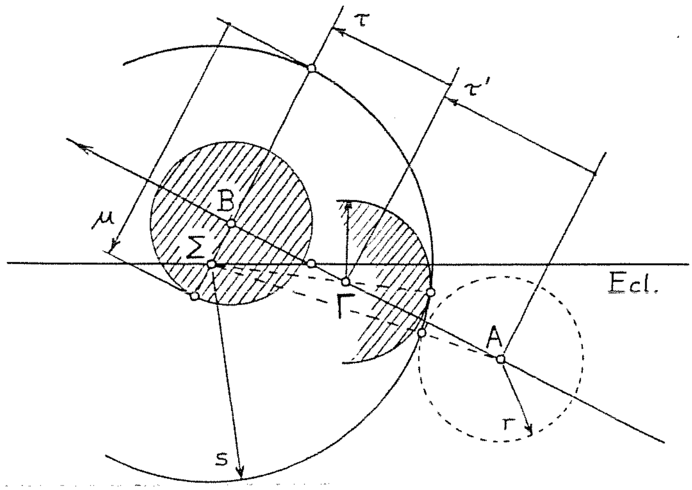

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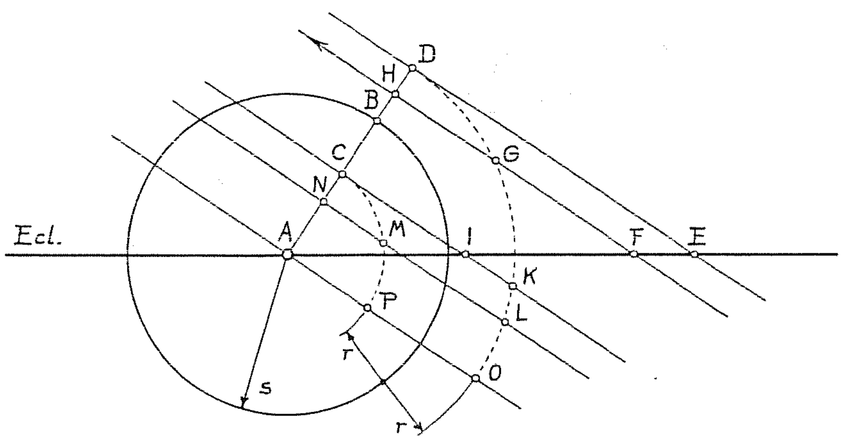

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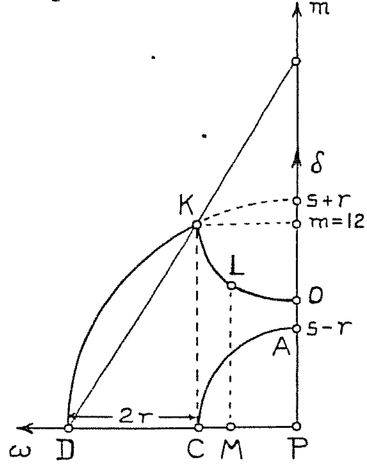

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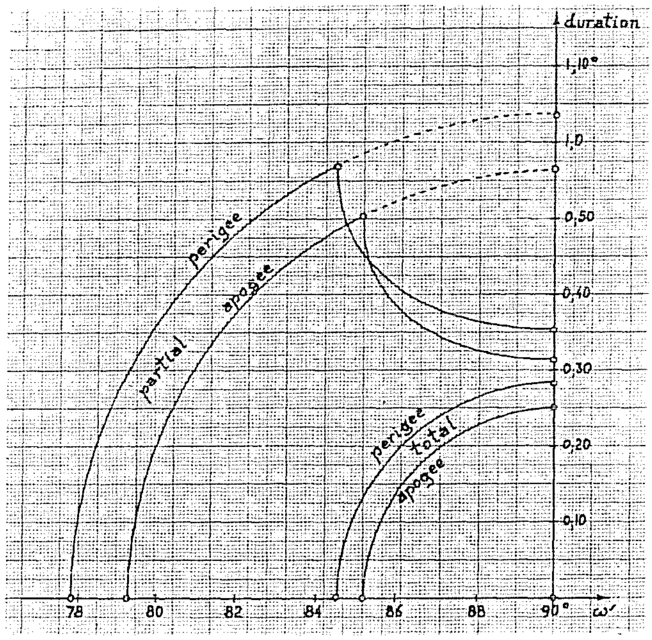

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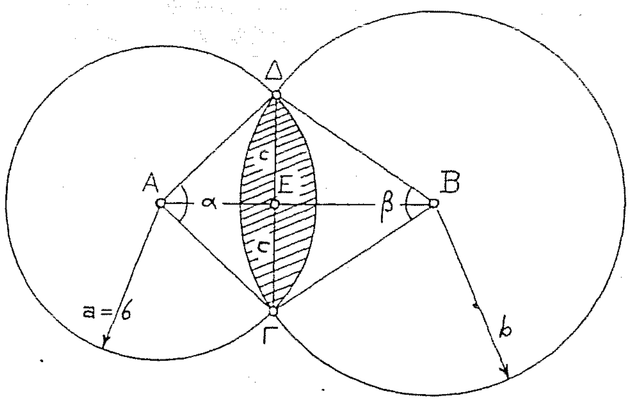

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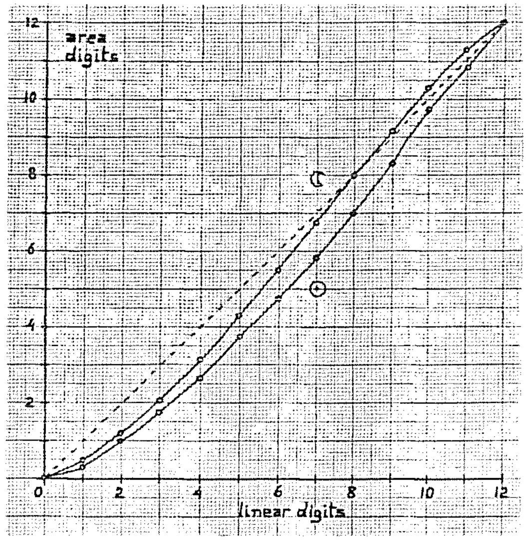

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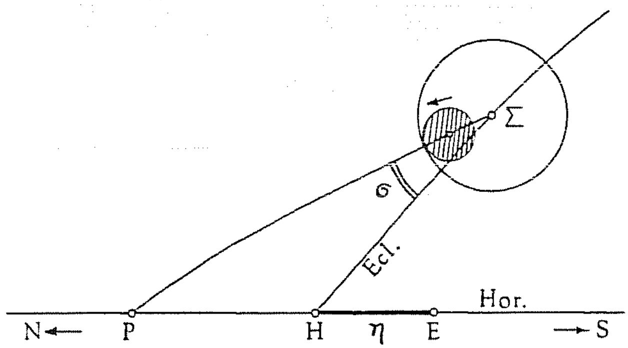

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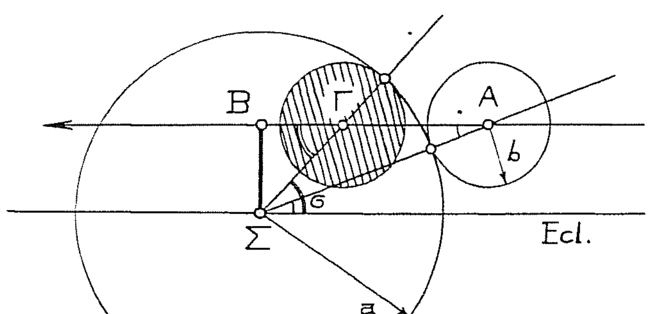

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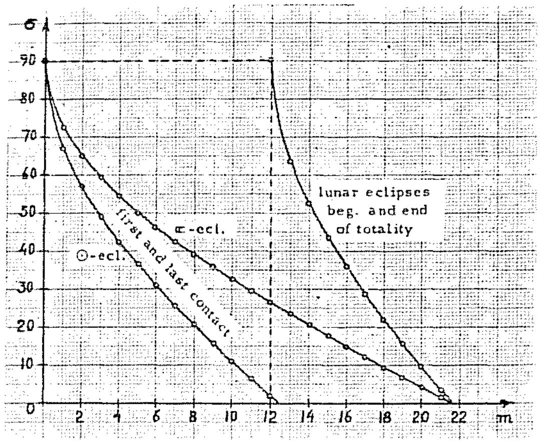

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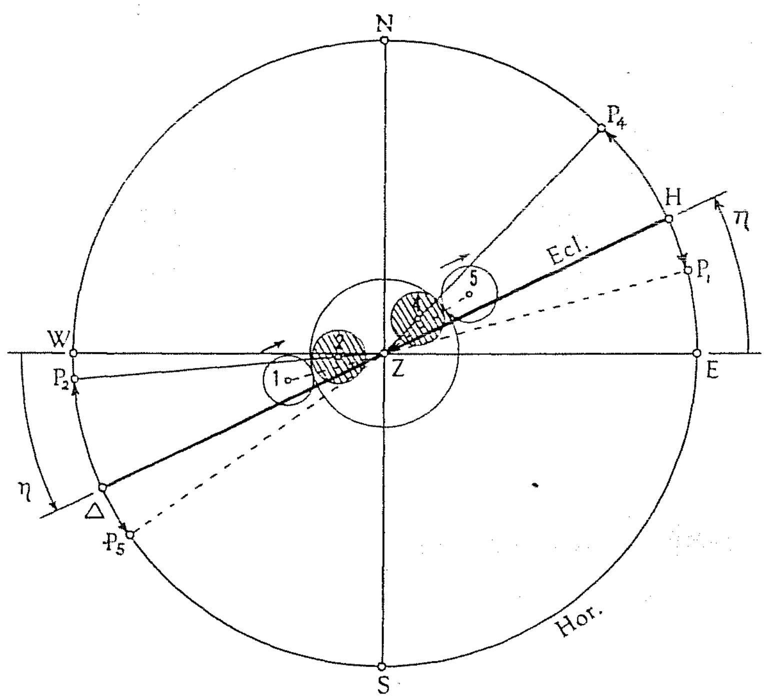

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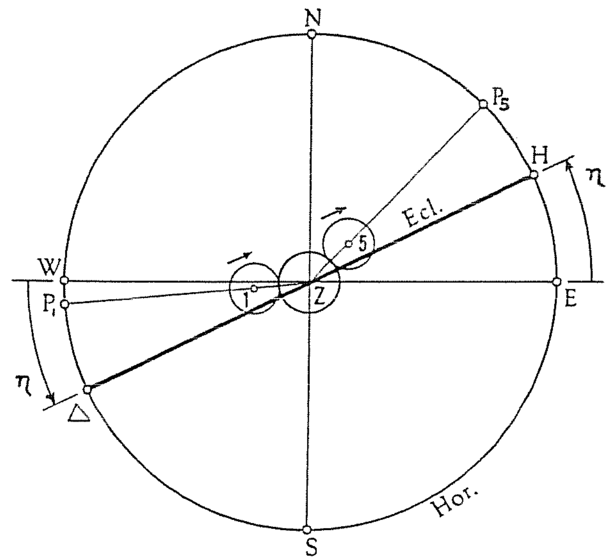

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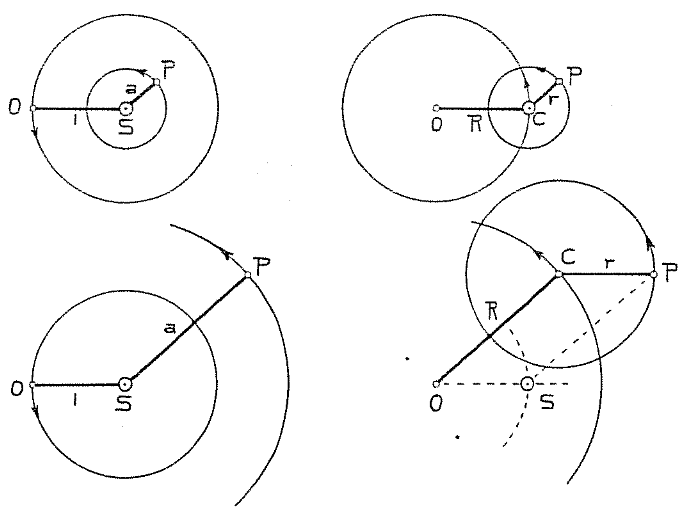

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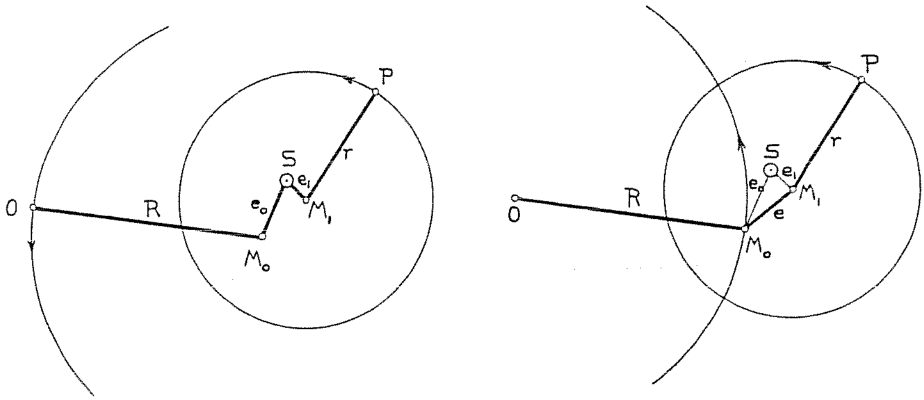

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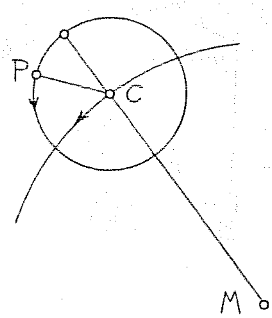

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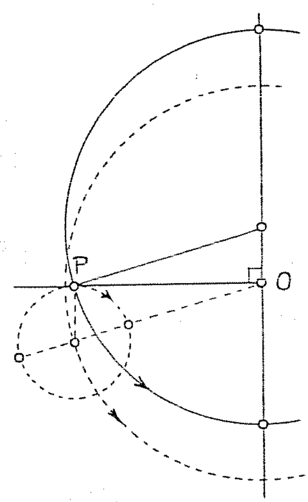

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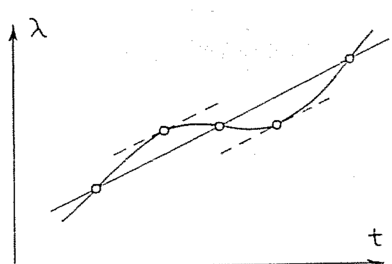

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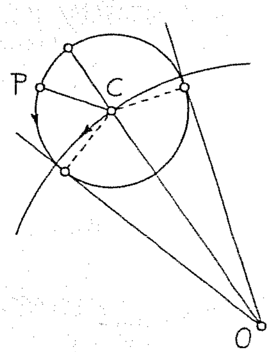

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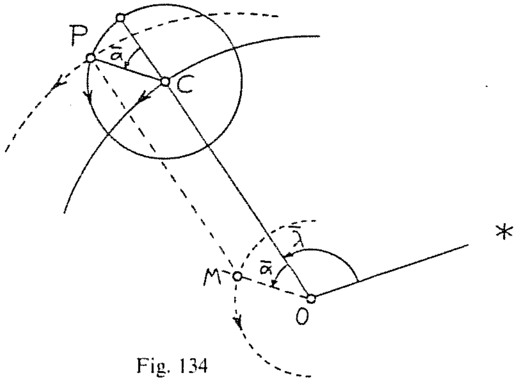

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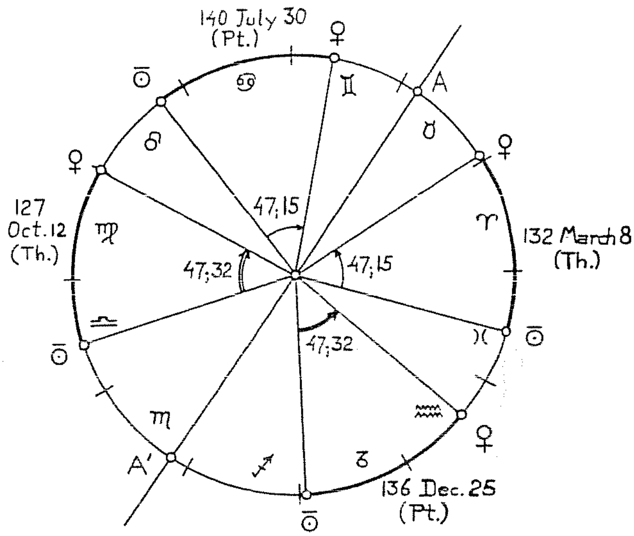

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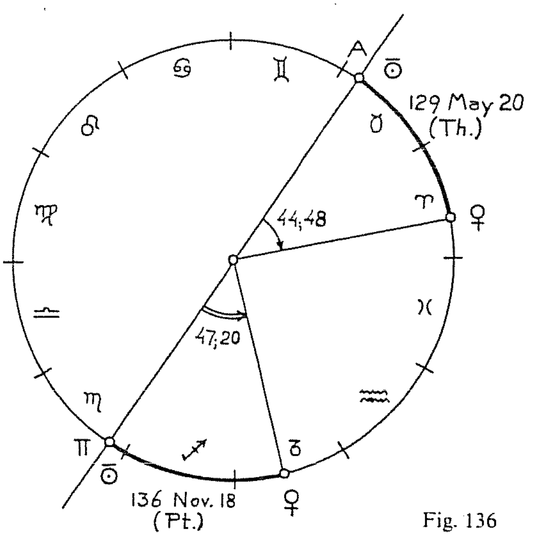

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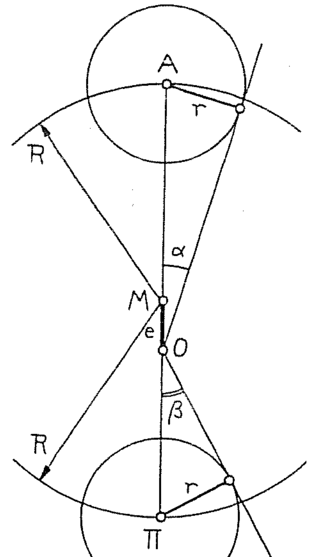

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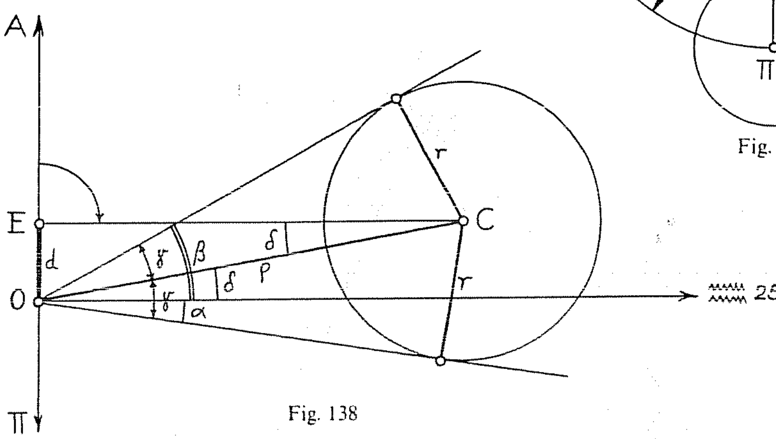

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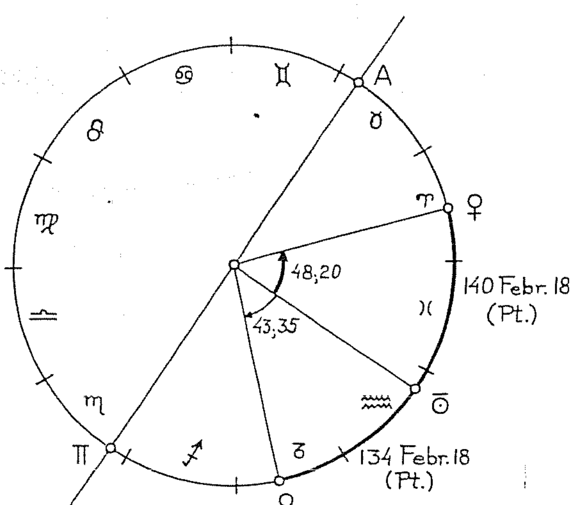

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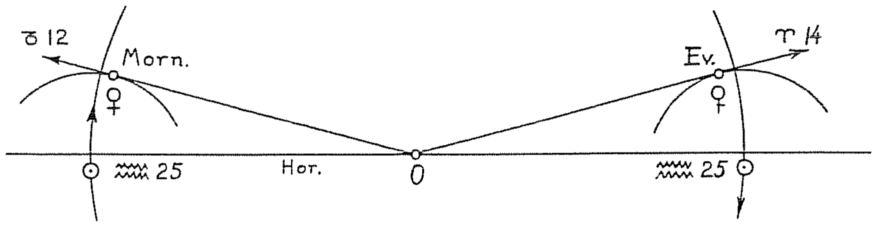

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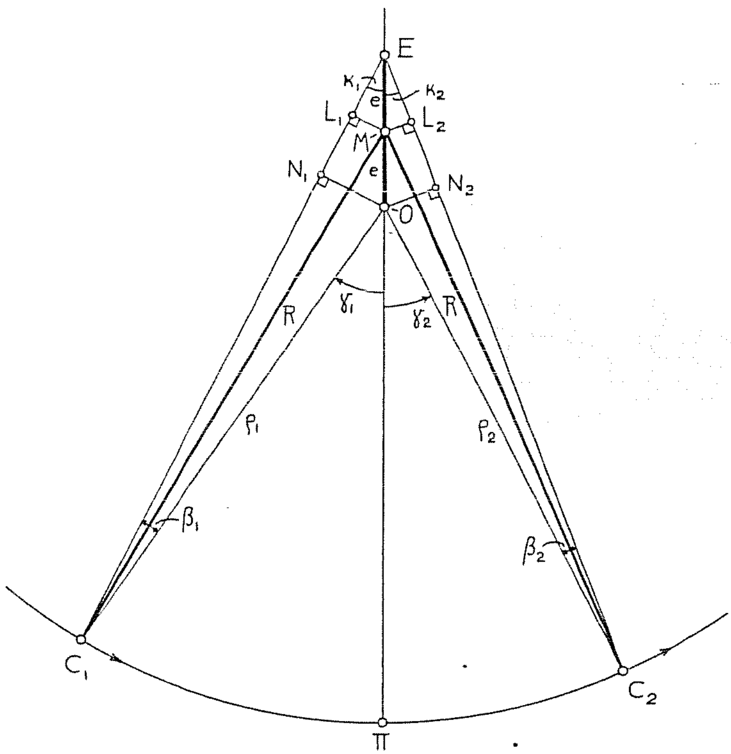

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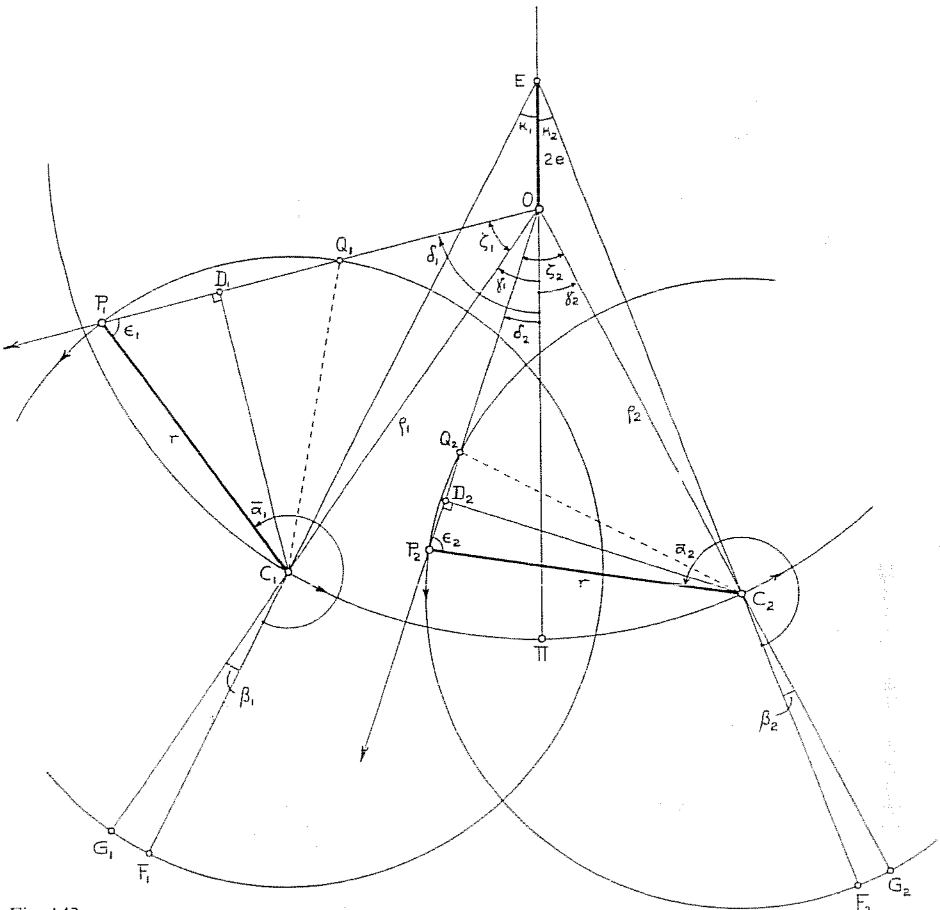

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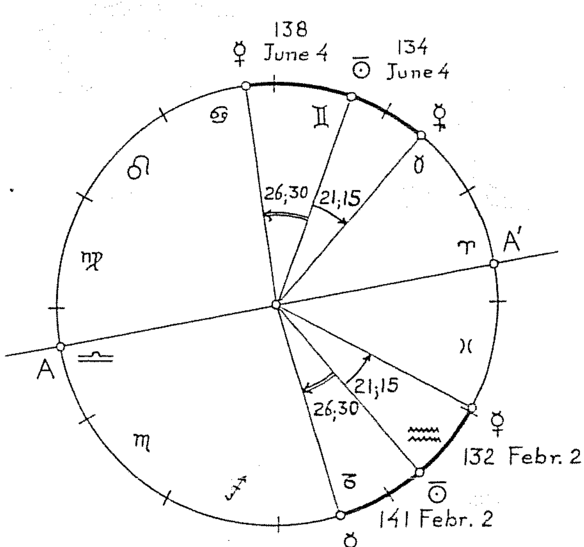

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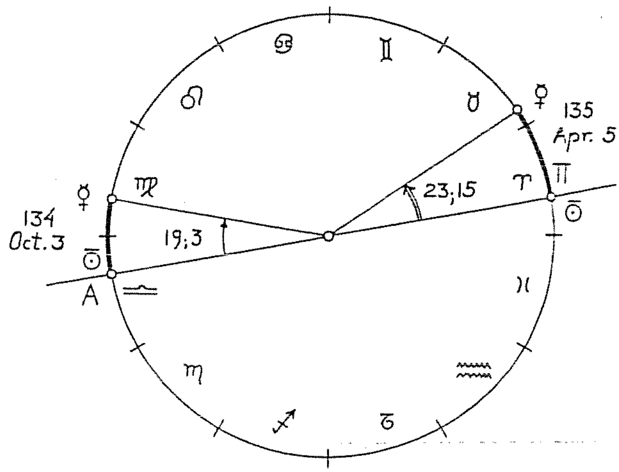

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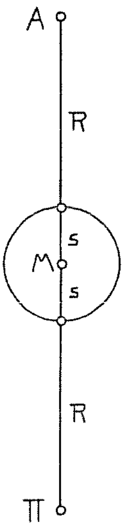

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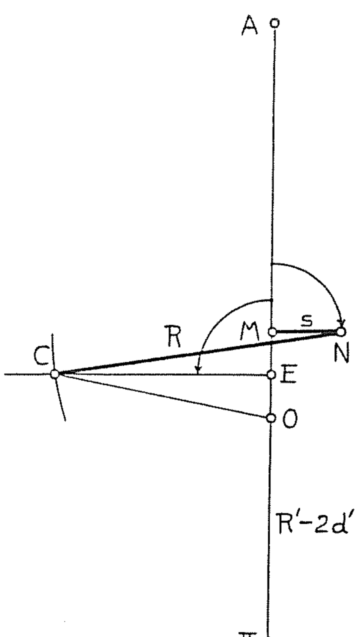

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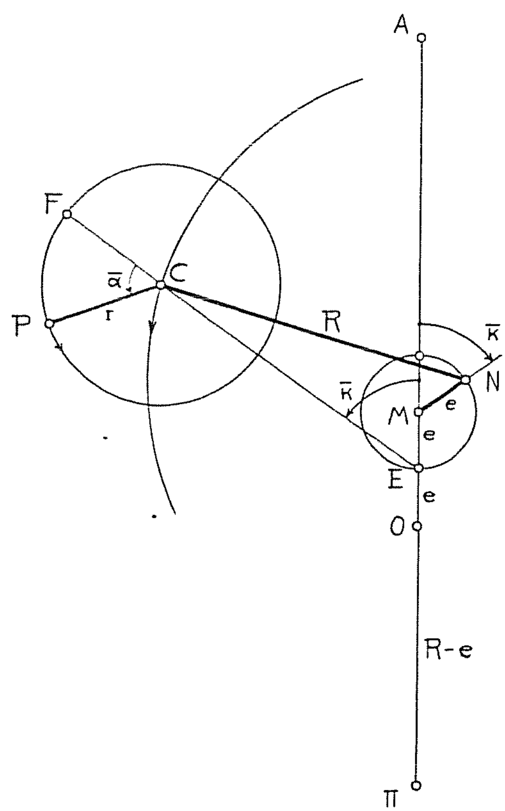

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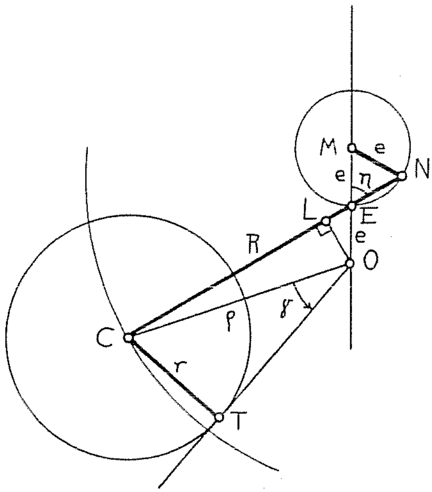

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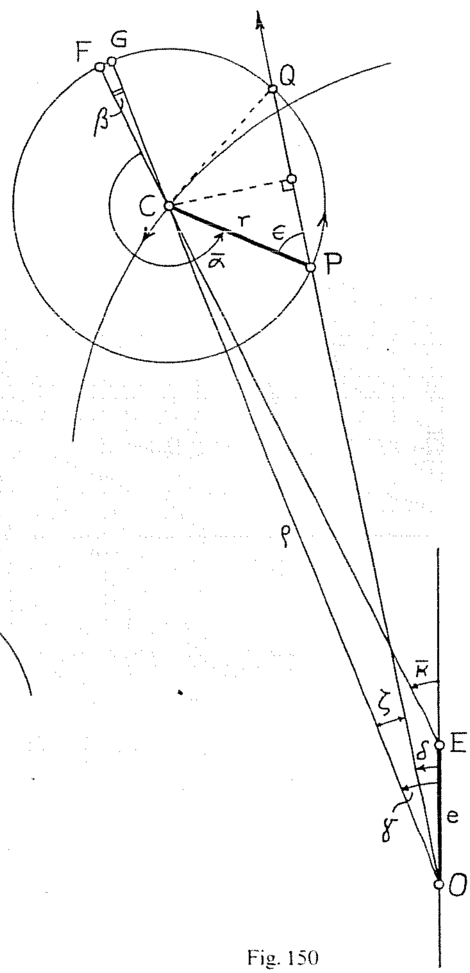


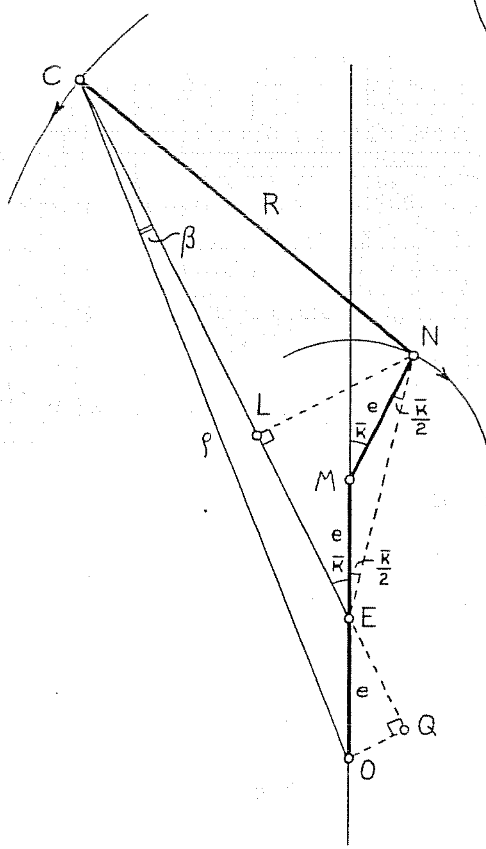
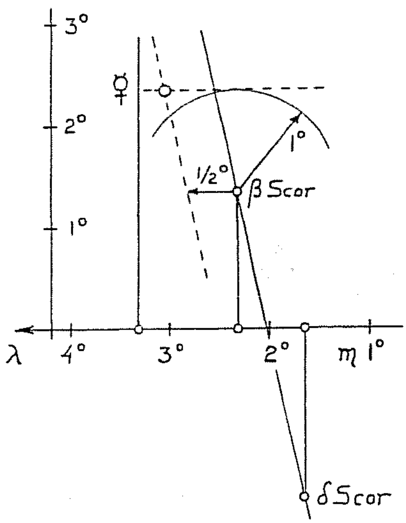


Fig. 149

Fig. 150



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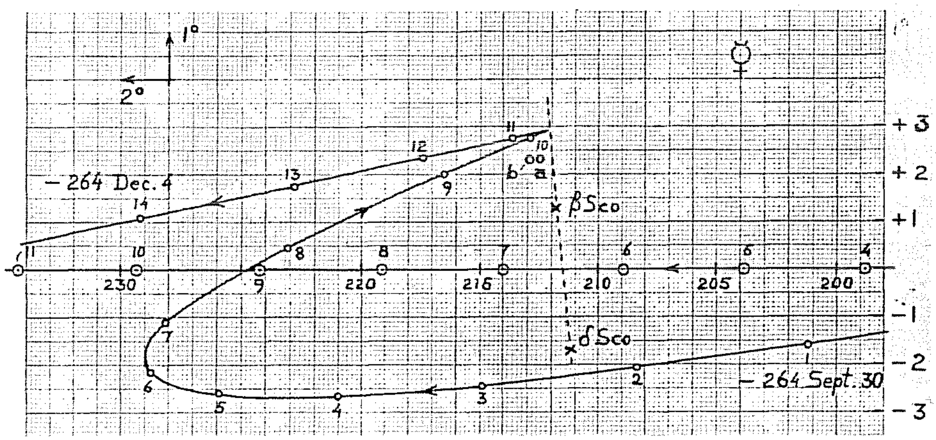

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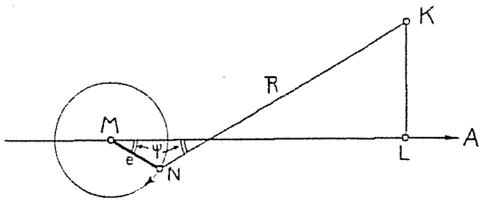

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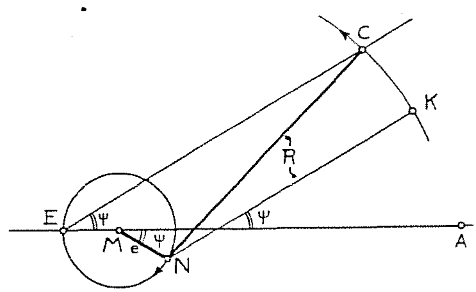
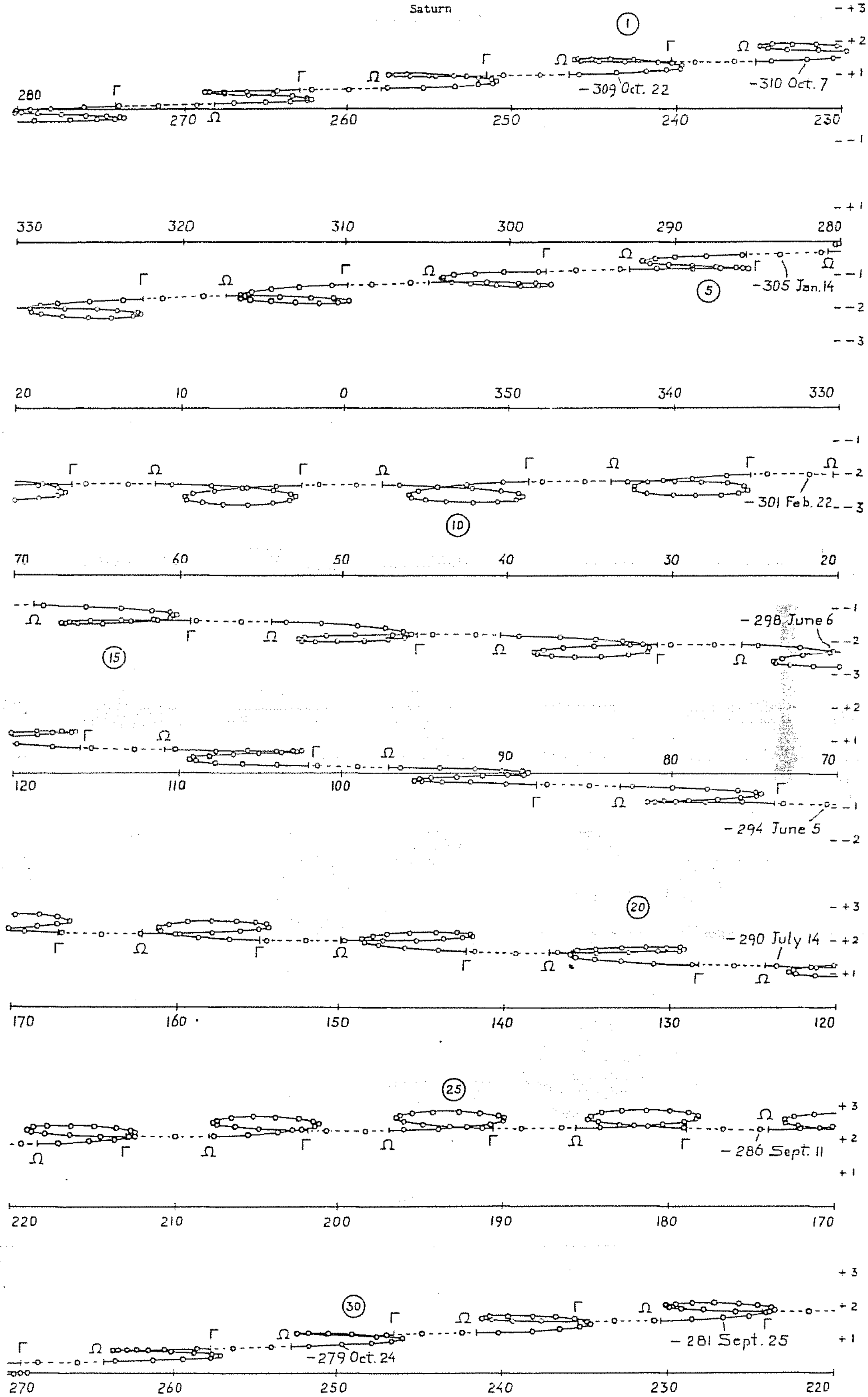


Fig. 154

Saturn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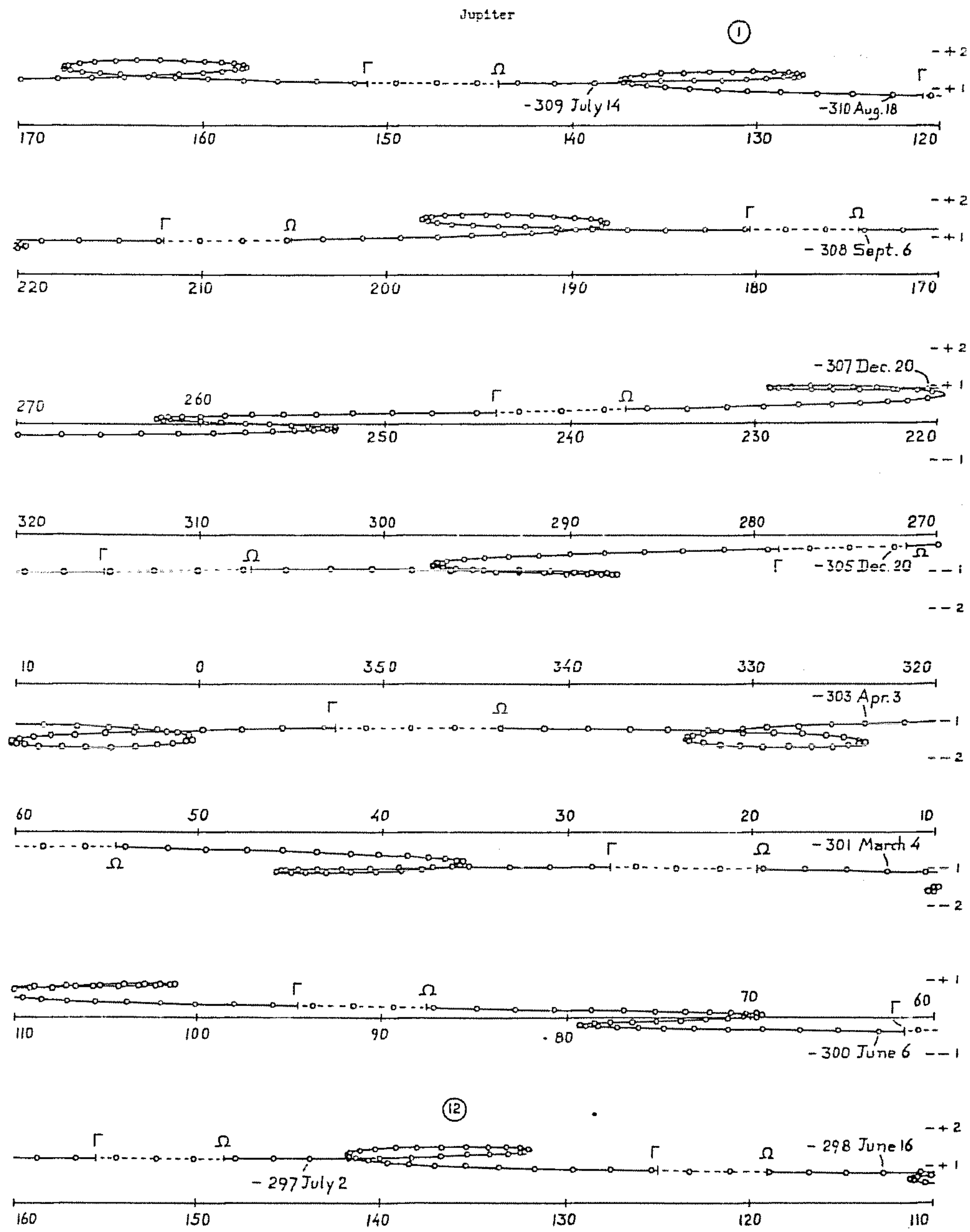
Jupiter
1
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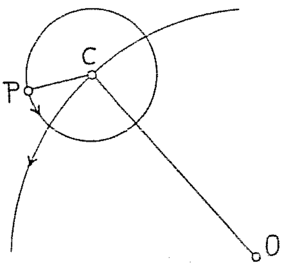
Fig. 156



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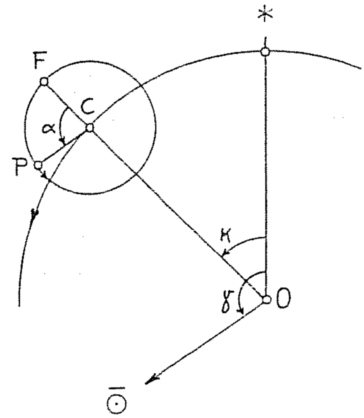

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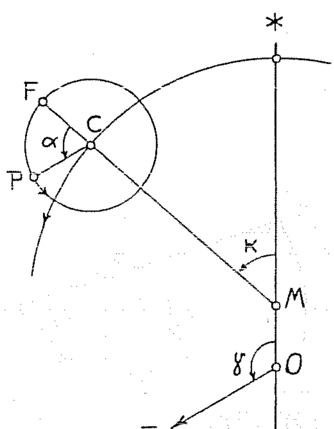

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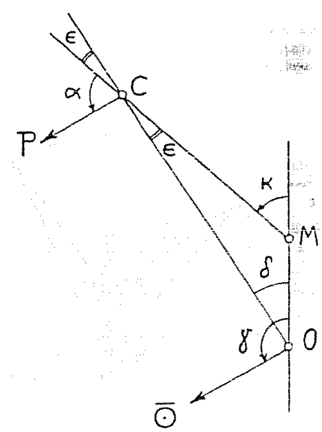

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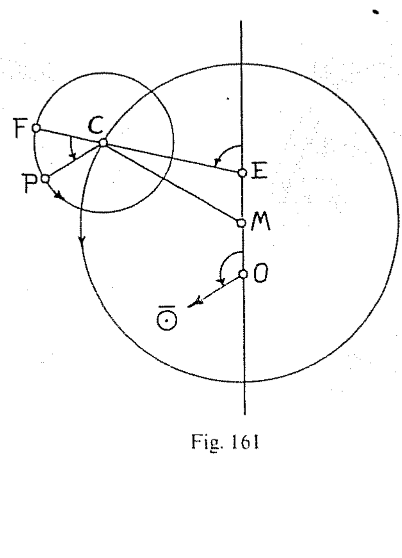

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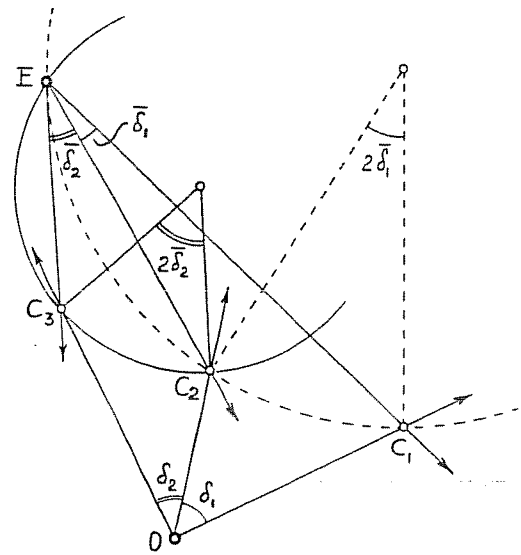

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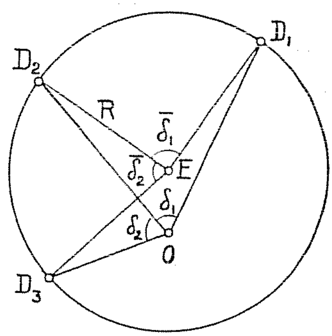

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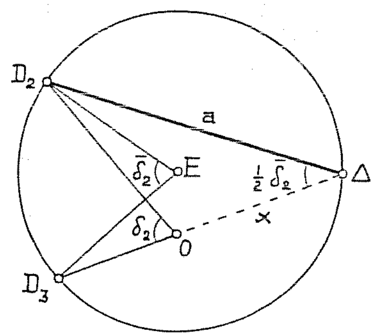

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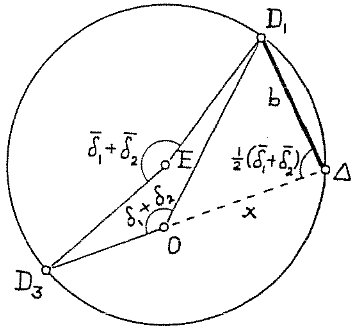


Fig. 165

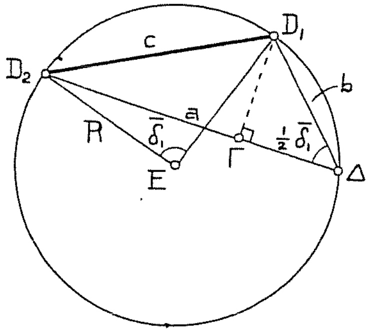


Fig. 166

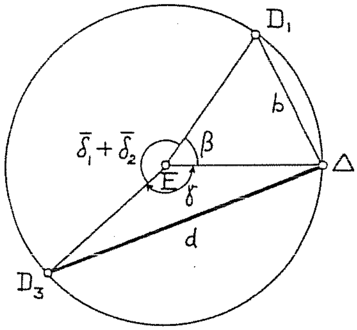


Fig. 167

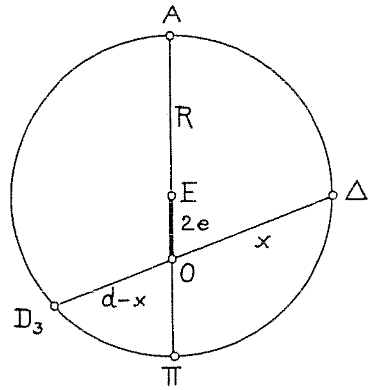
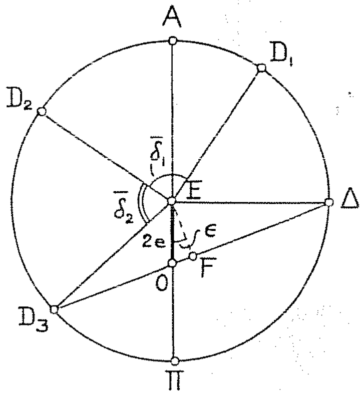


Fig. 168



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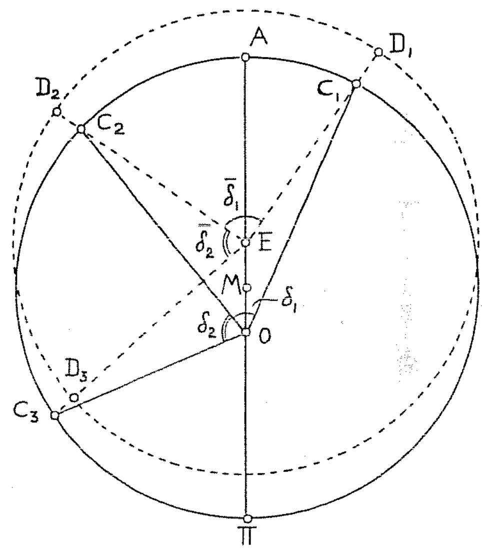

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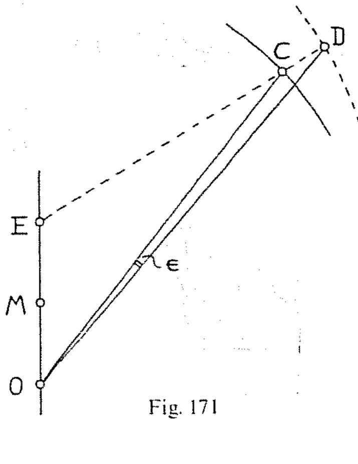

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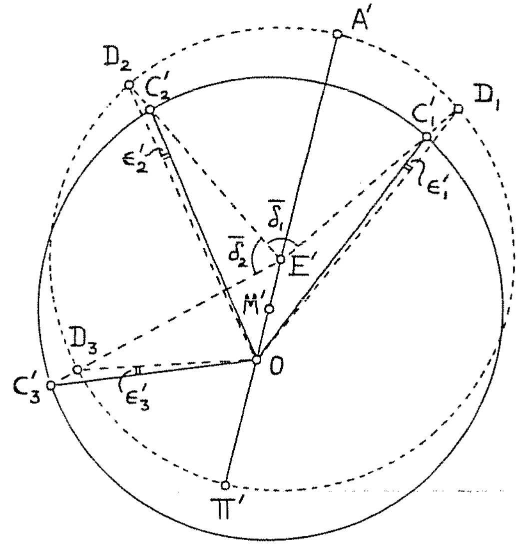

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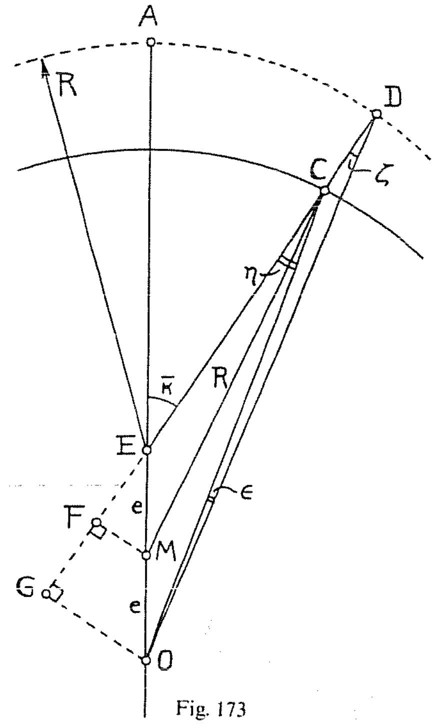

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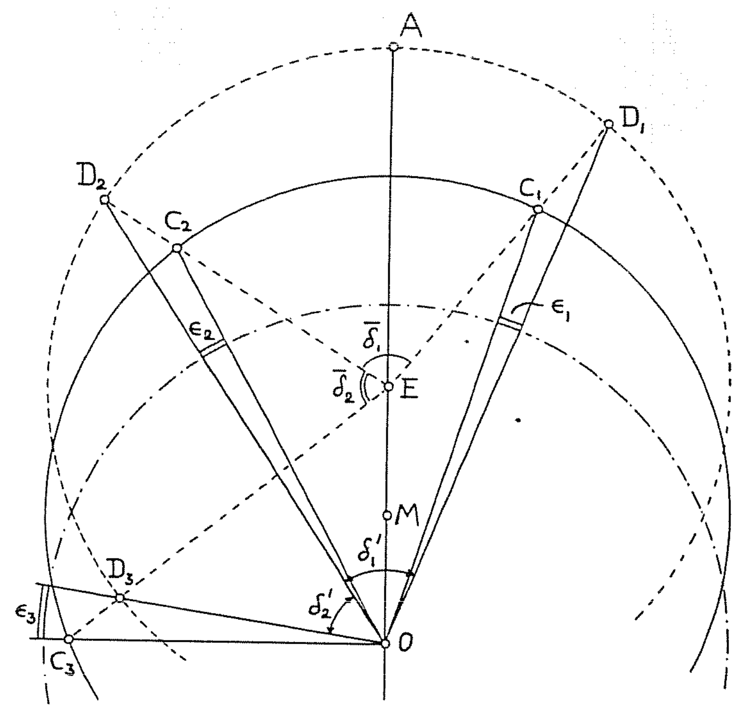

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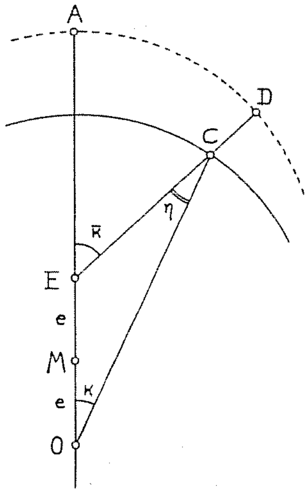

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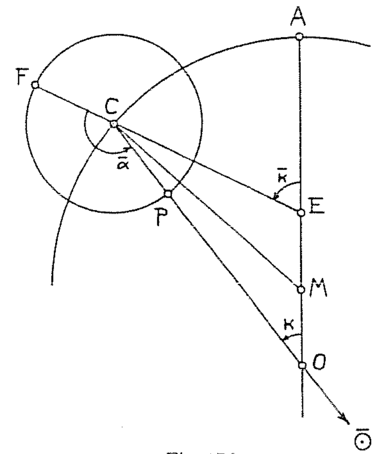

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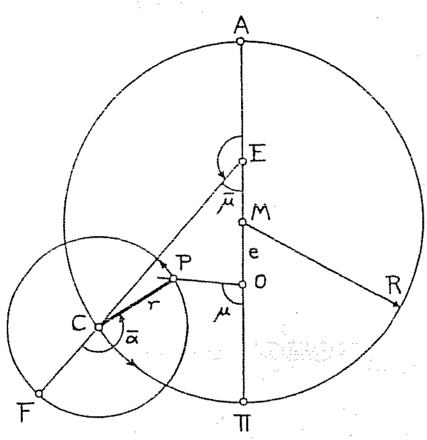

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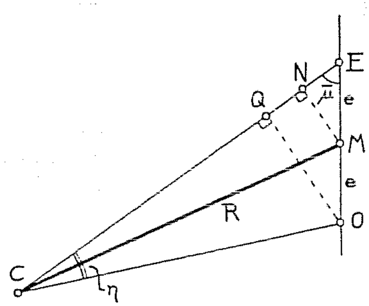

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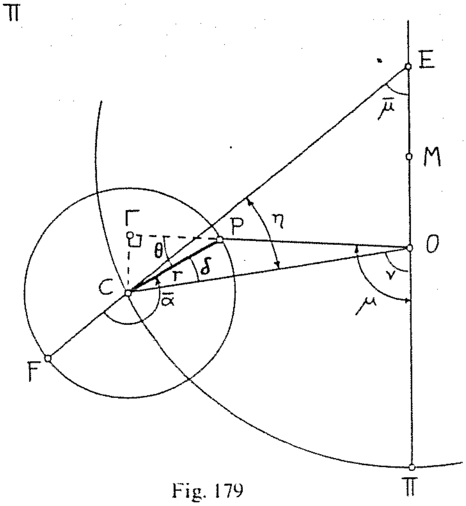

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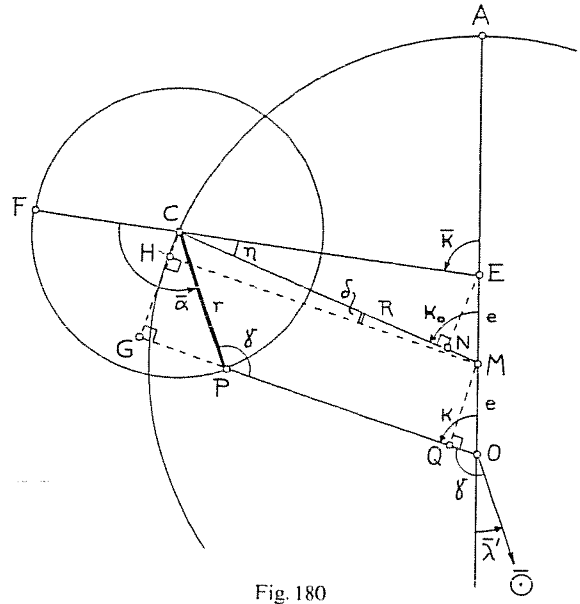

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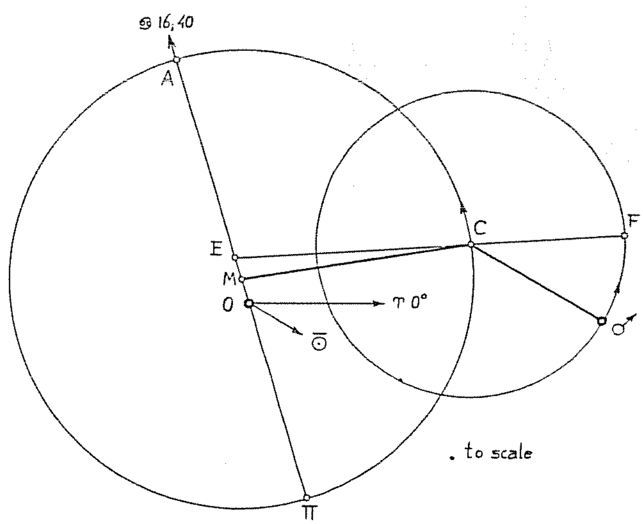

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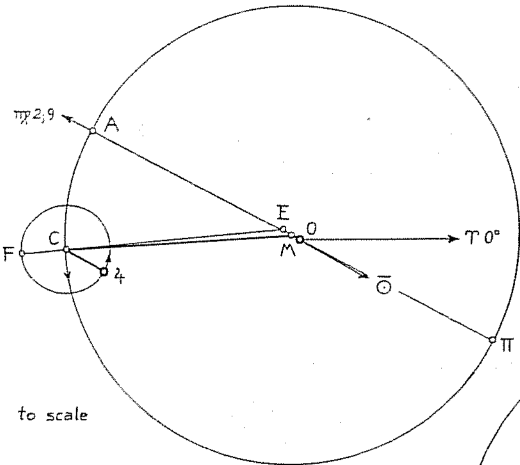

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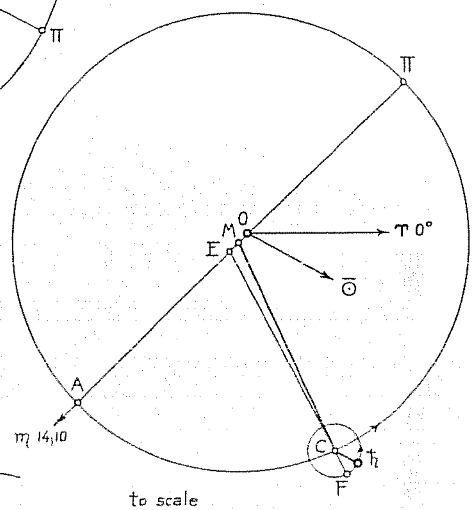
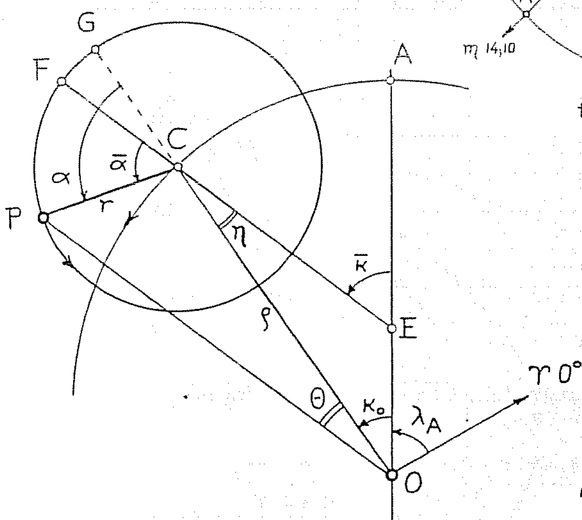


Fig. 183



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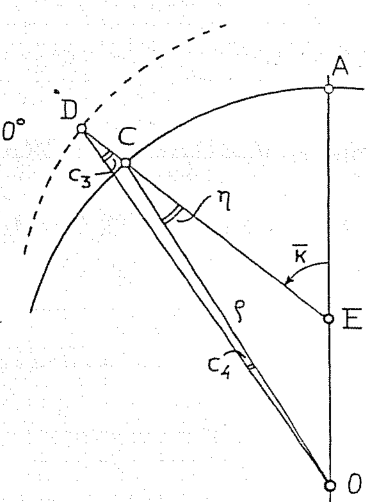

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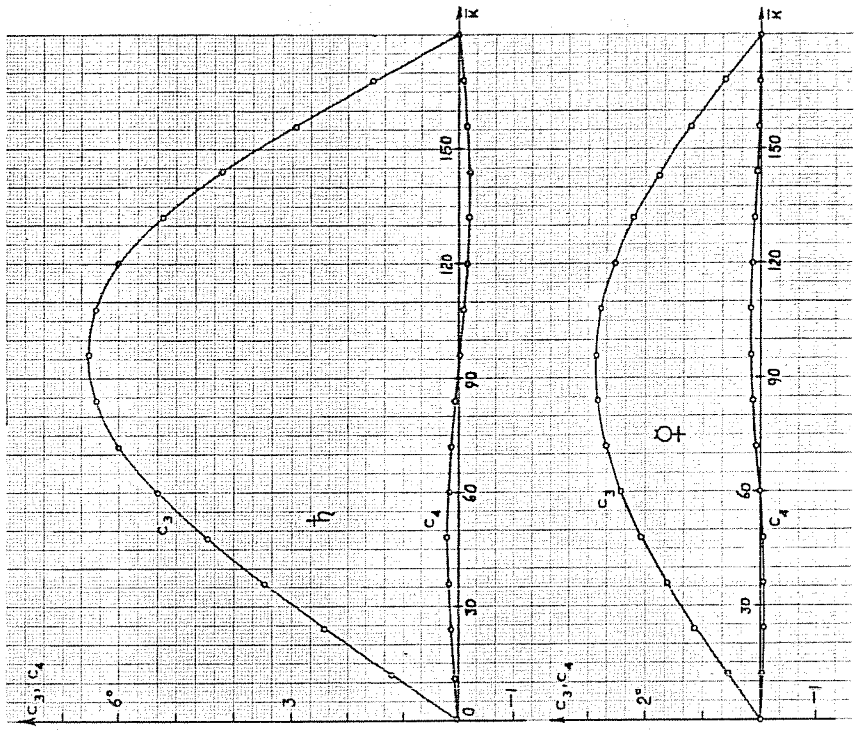
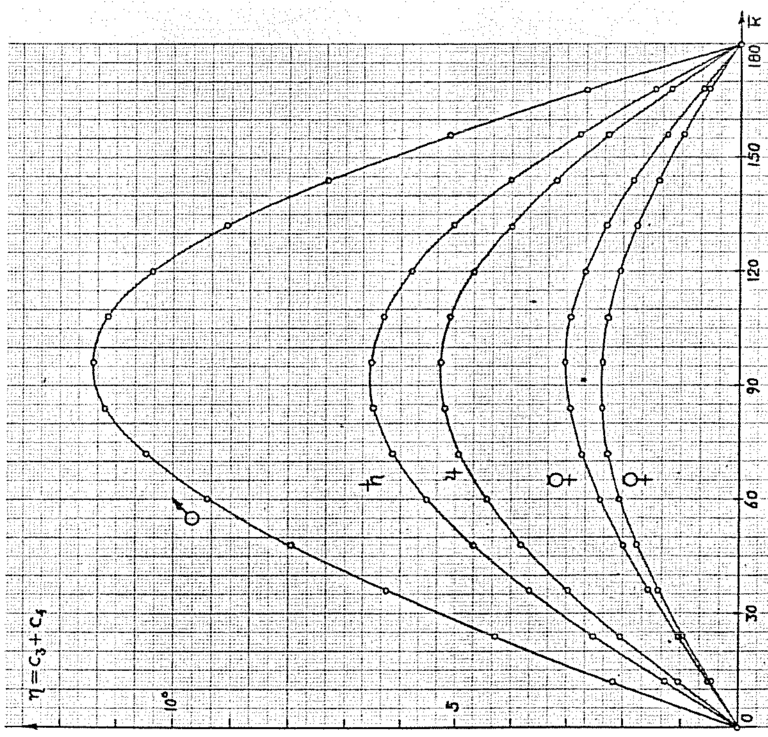


Fig. 187



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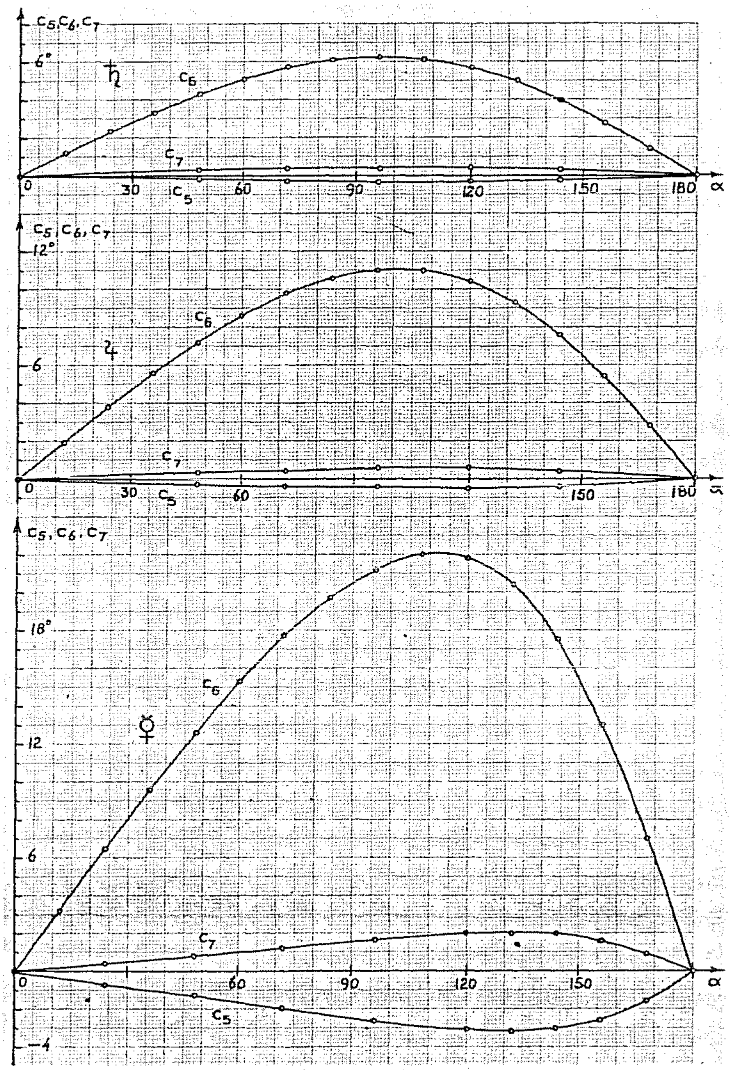
c5, c6, c7
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c7
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30
c5
60
150
180
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c5, c6, c7
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c5
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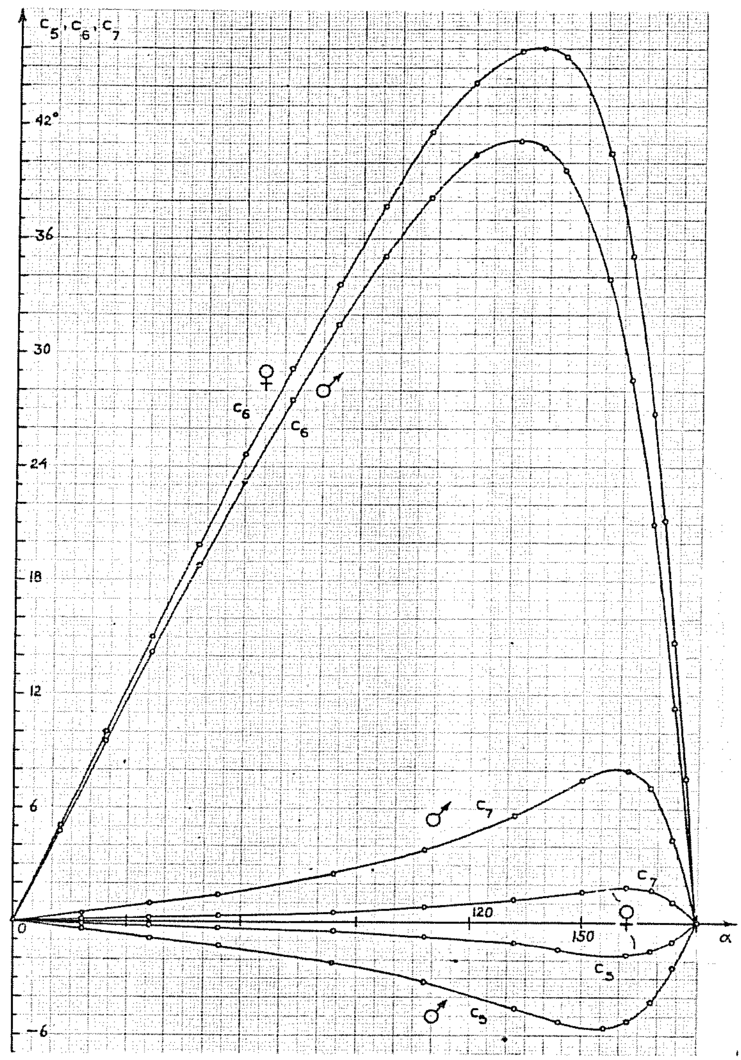
Fig. 188



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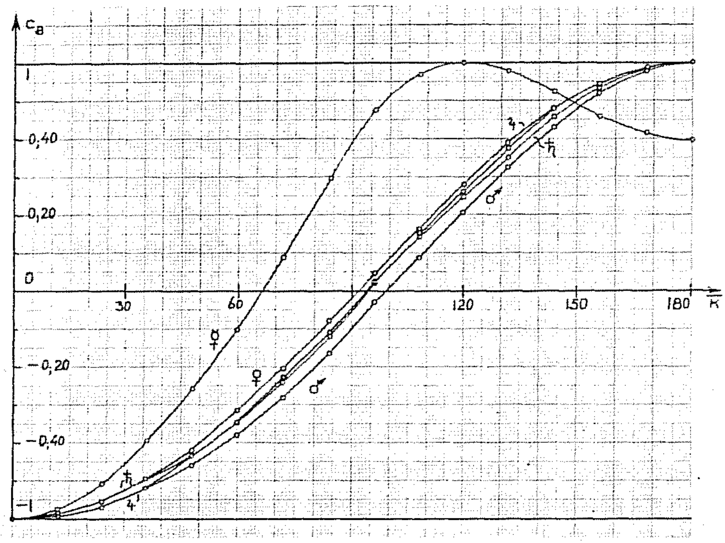

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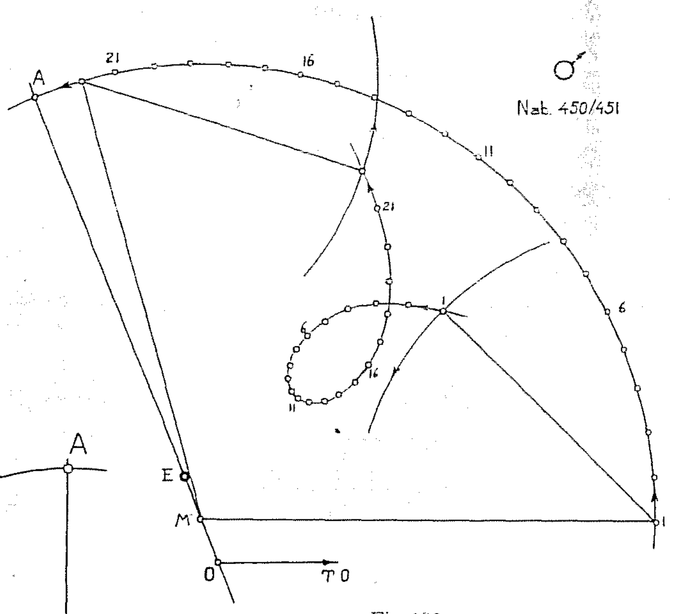

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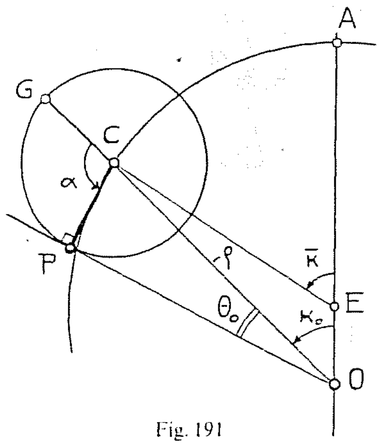

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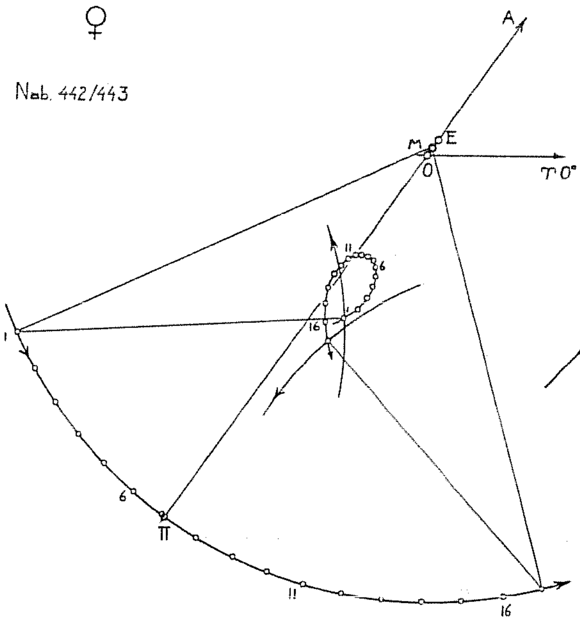

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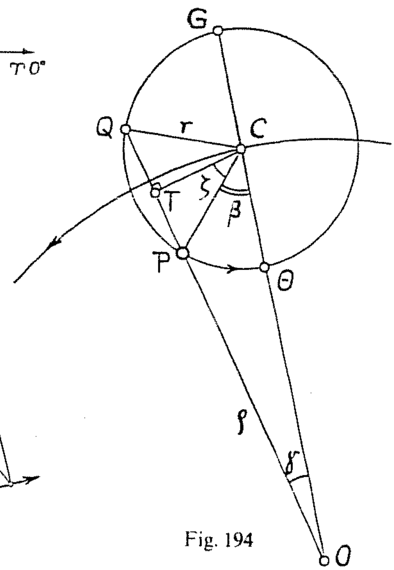

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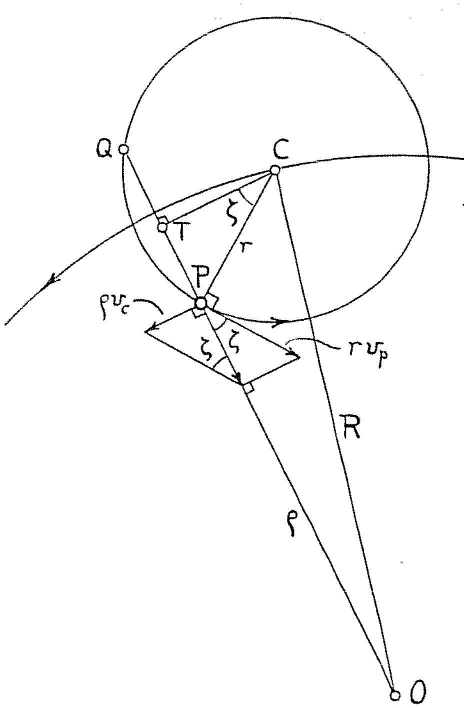


Fig. 195

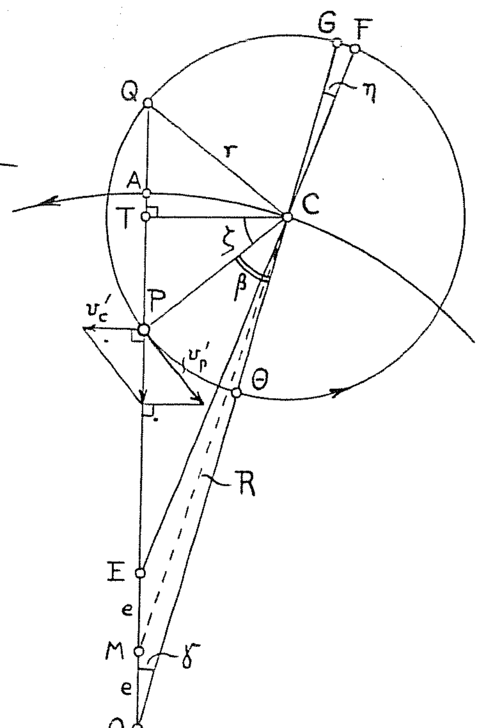
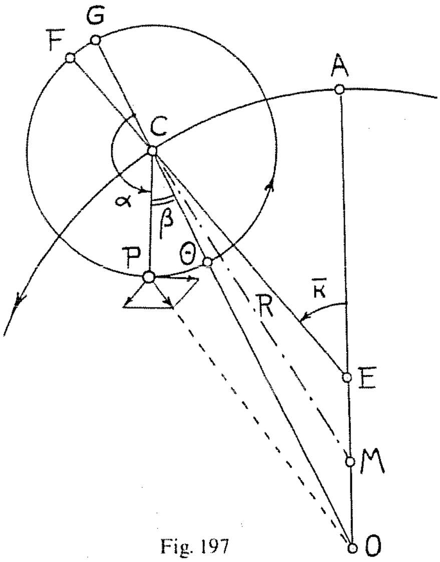


Fig. 196



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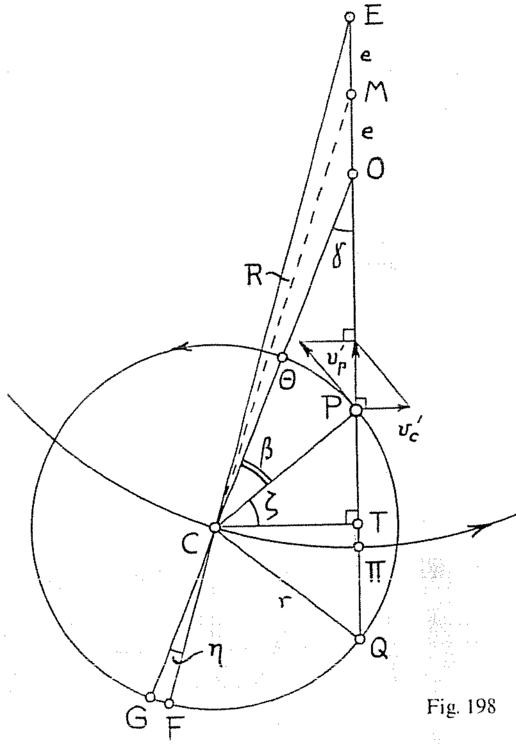

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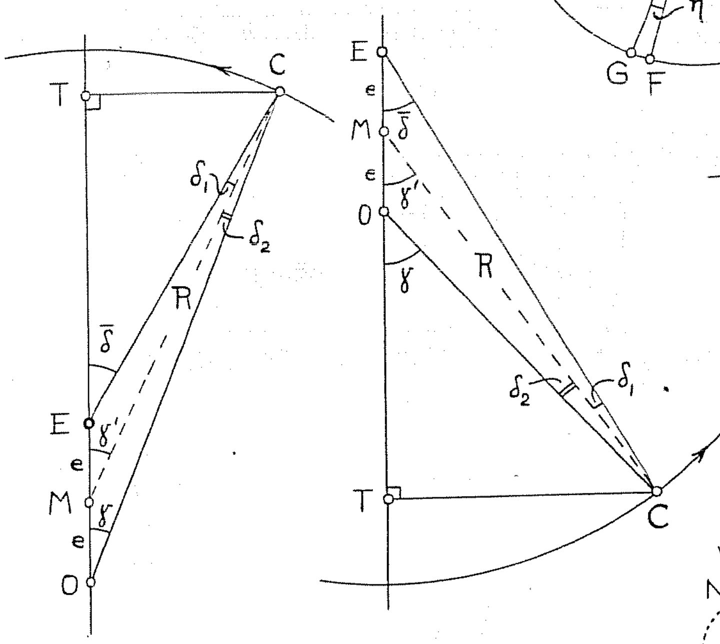

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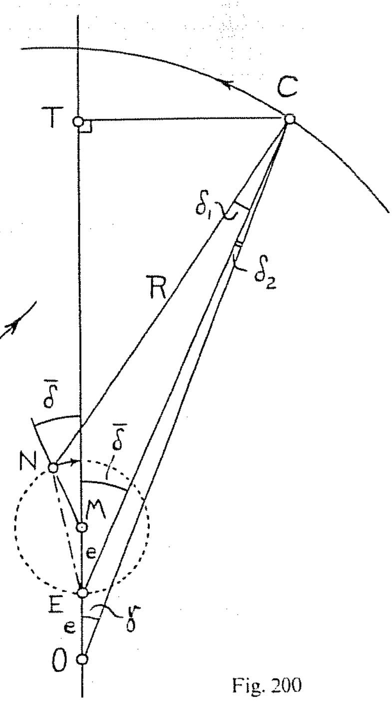


Fig. 200

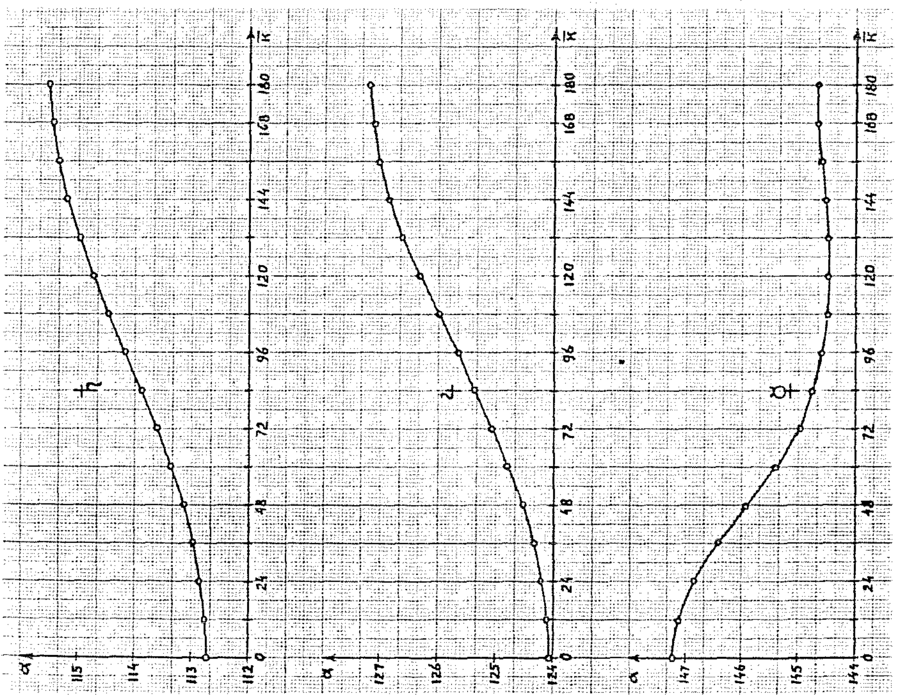


Fig. 201

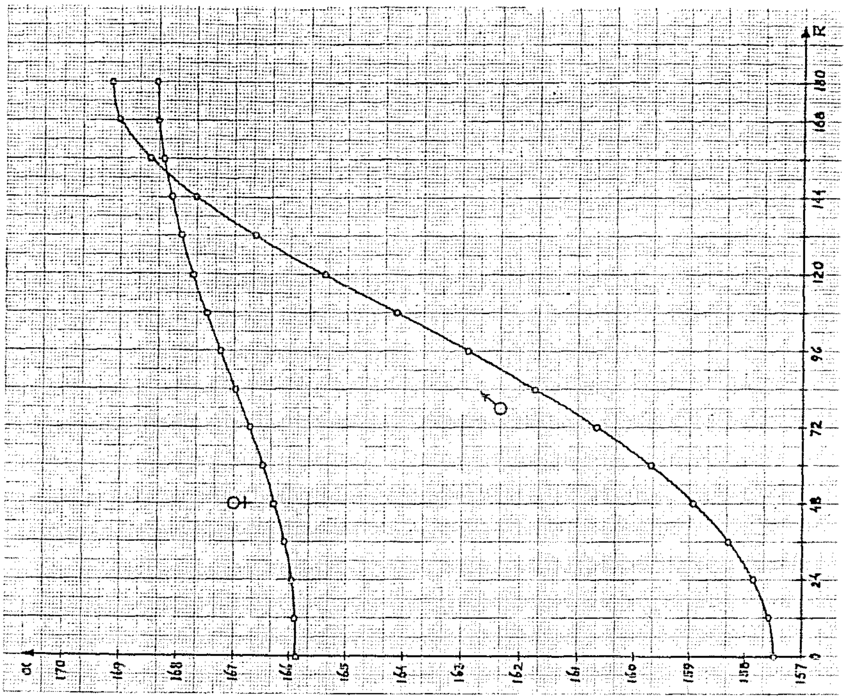


Fig. 202

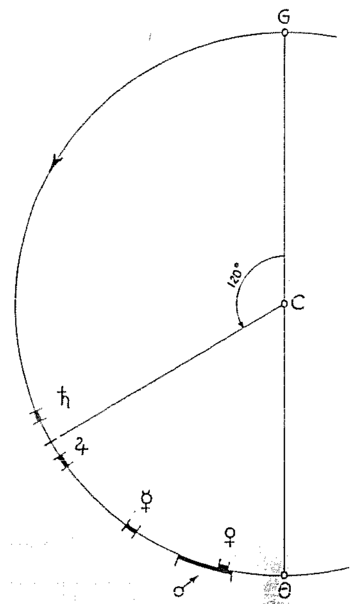
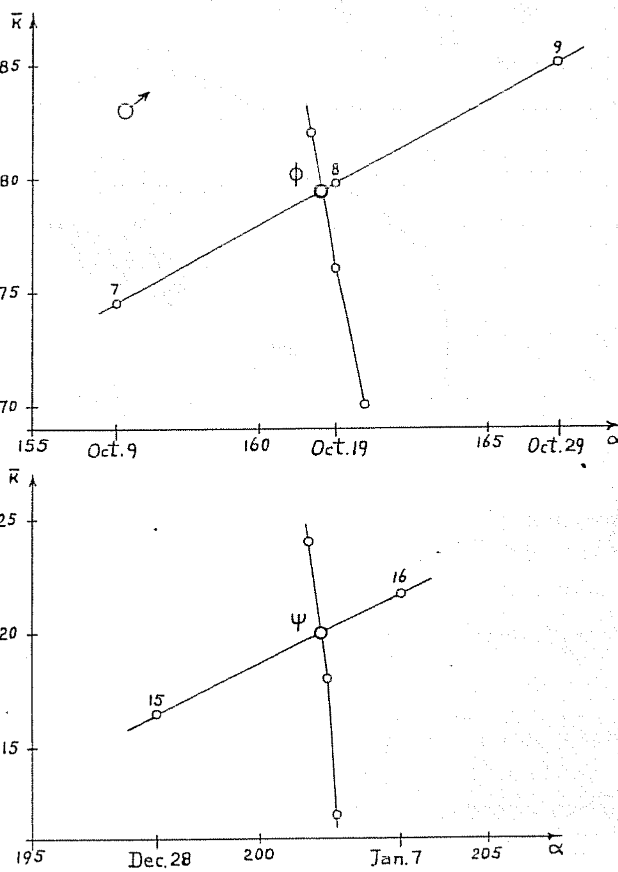


Fig. 203



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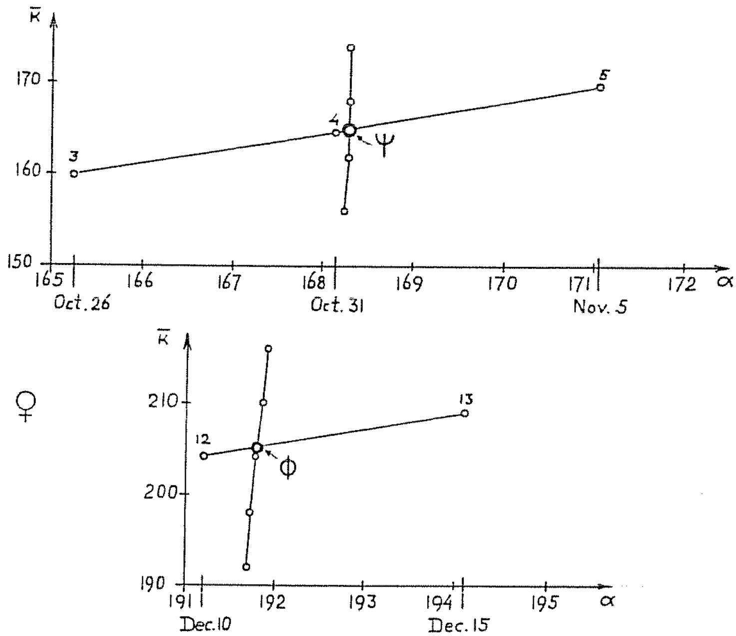

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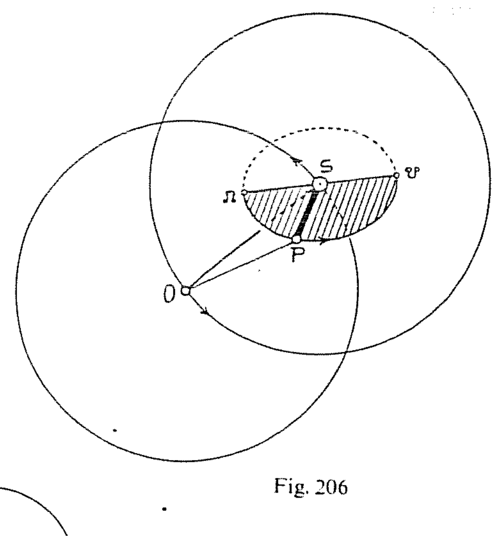

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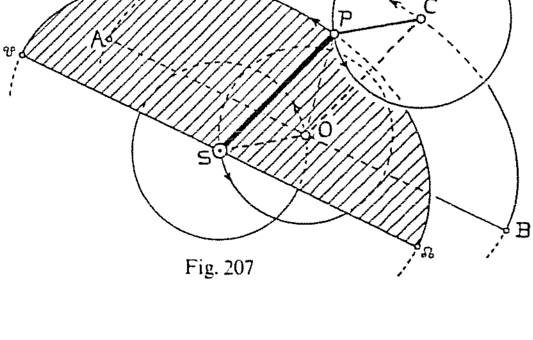

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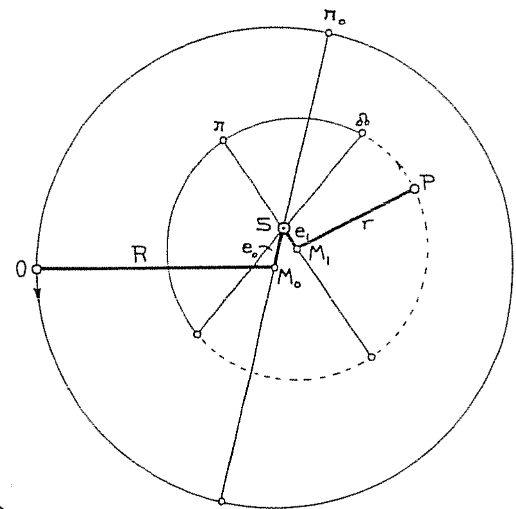


Fig. 208

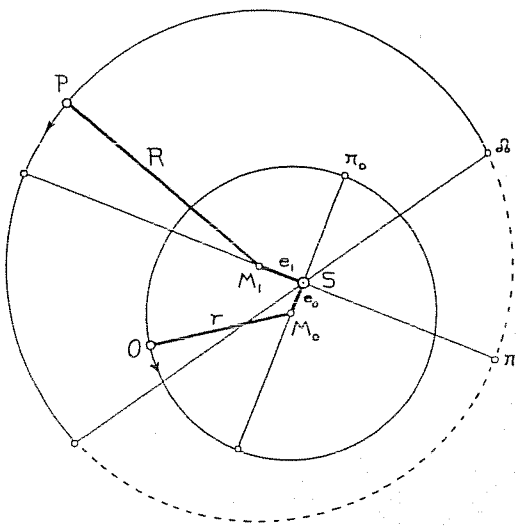
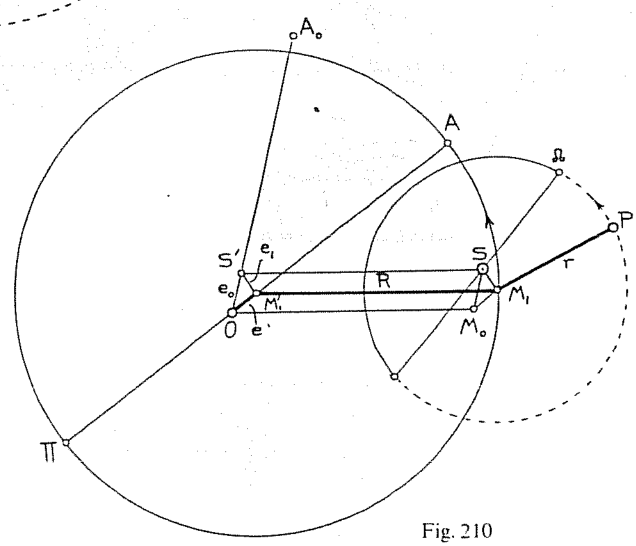


Fig. 209



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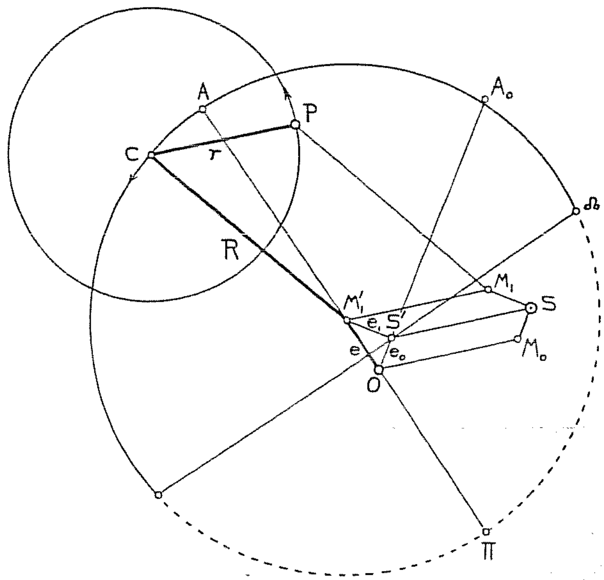

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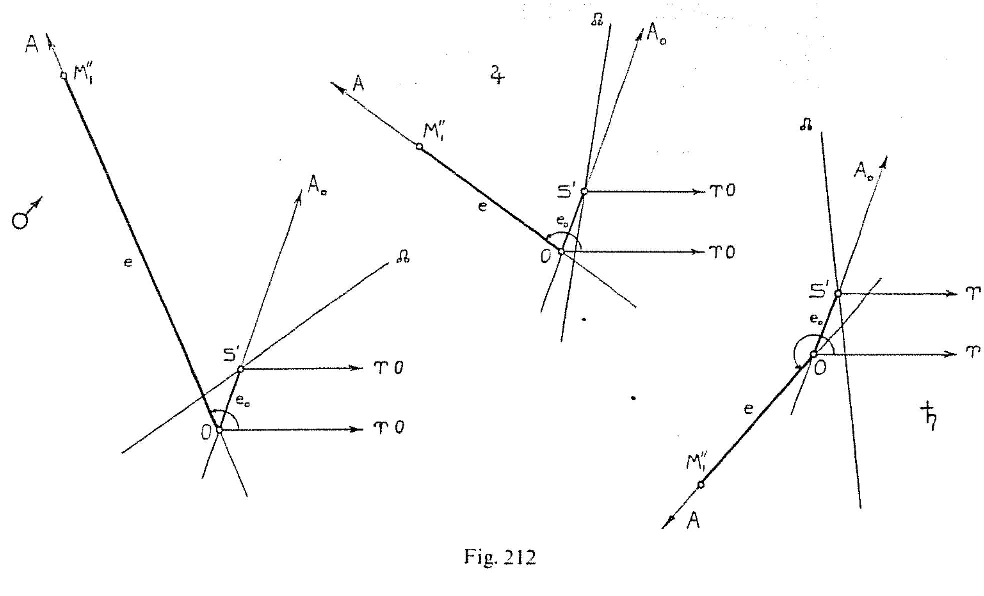

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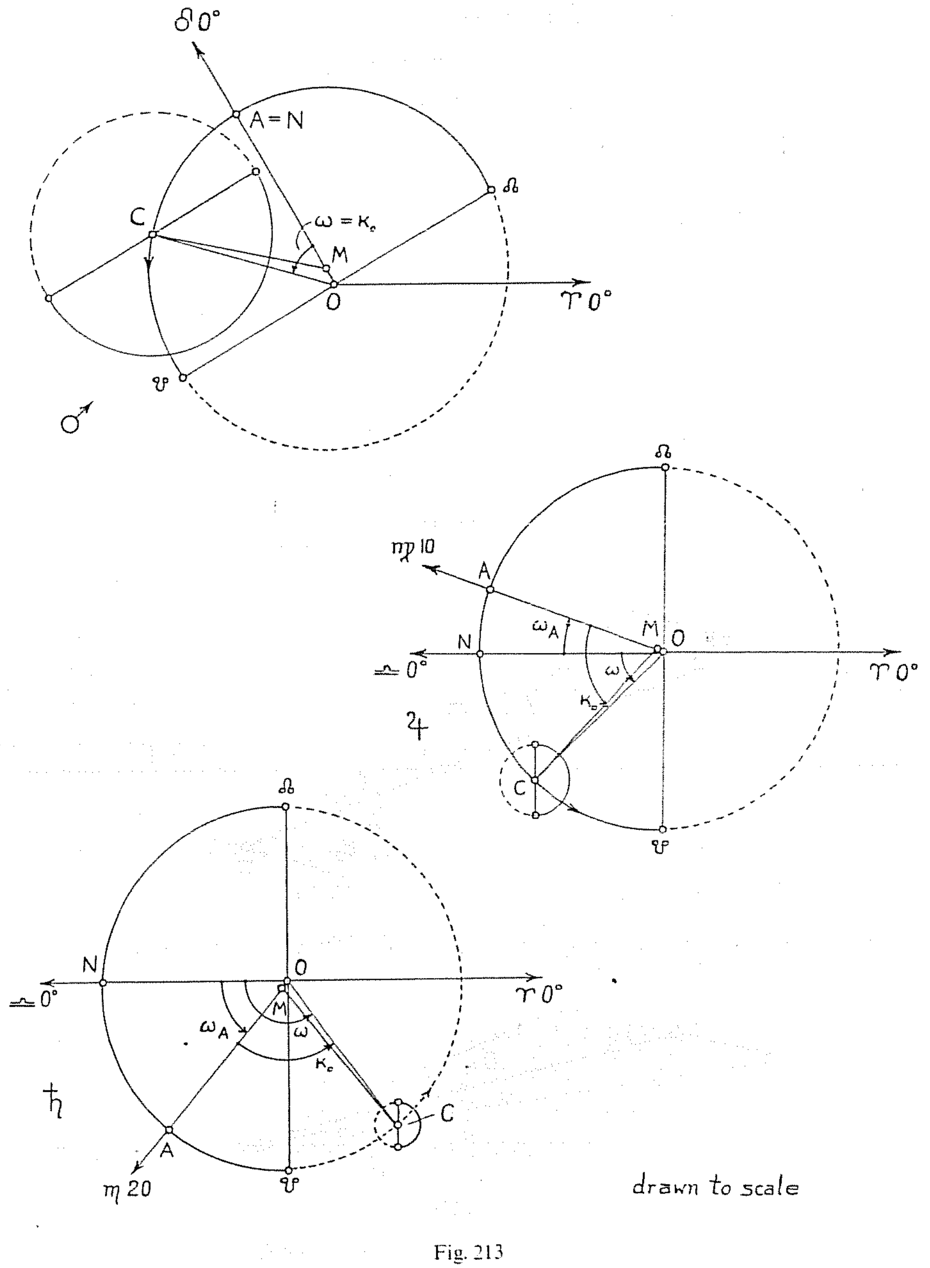

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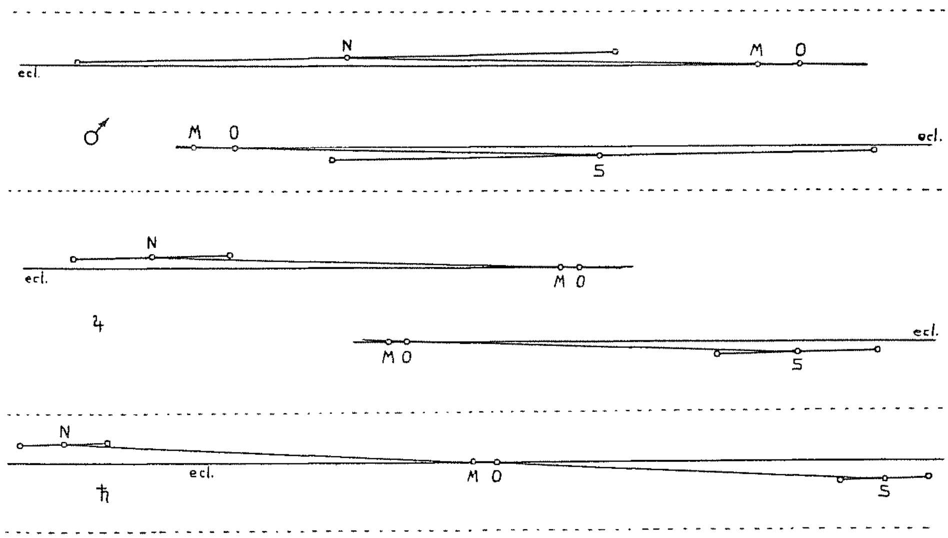
Fig. 213



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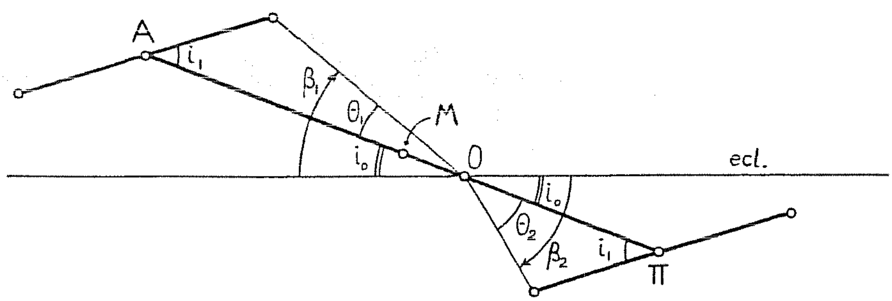

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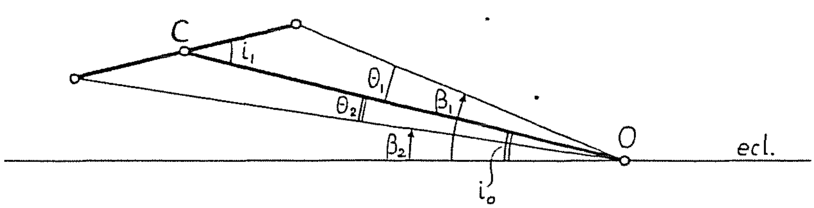

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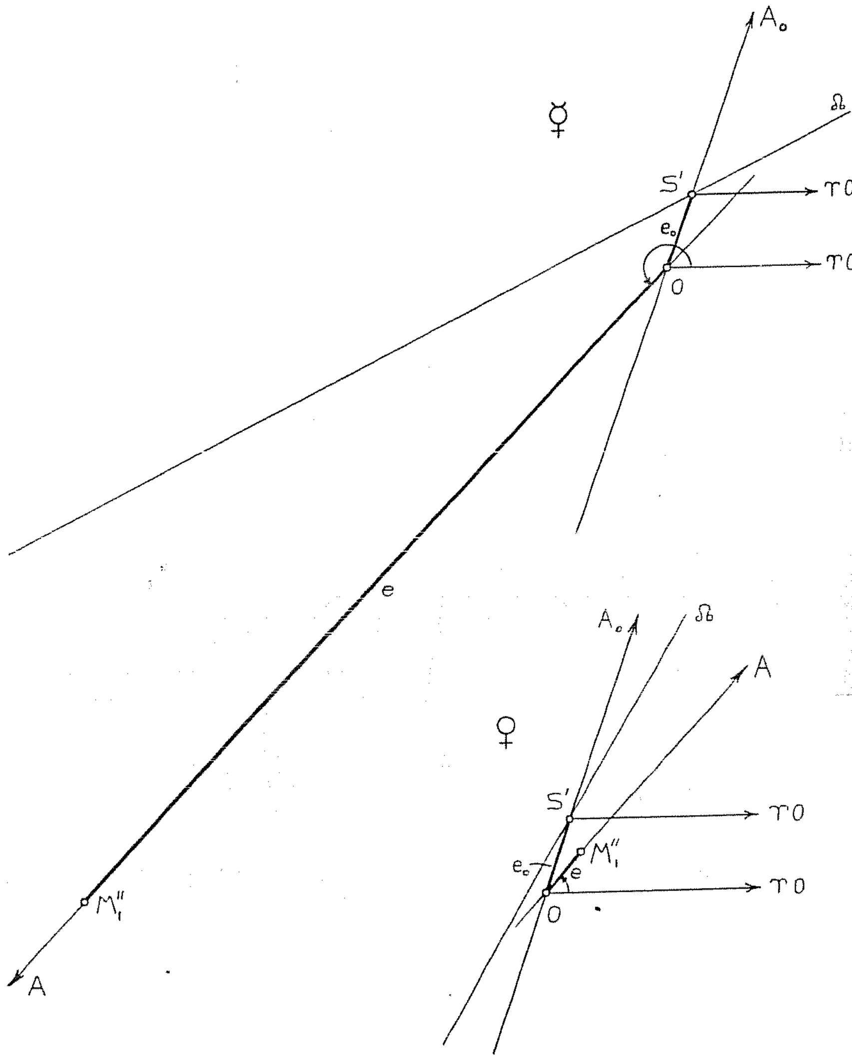

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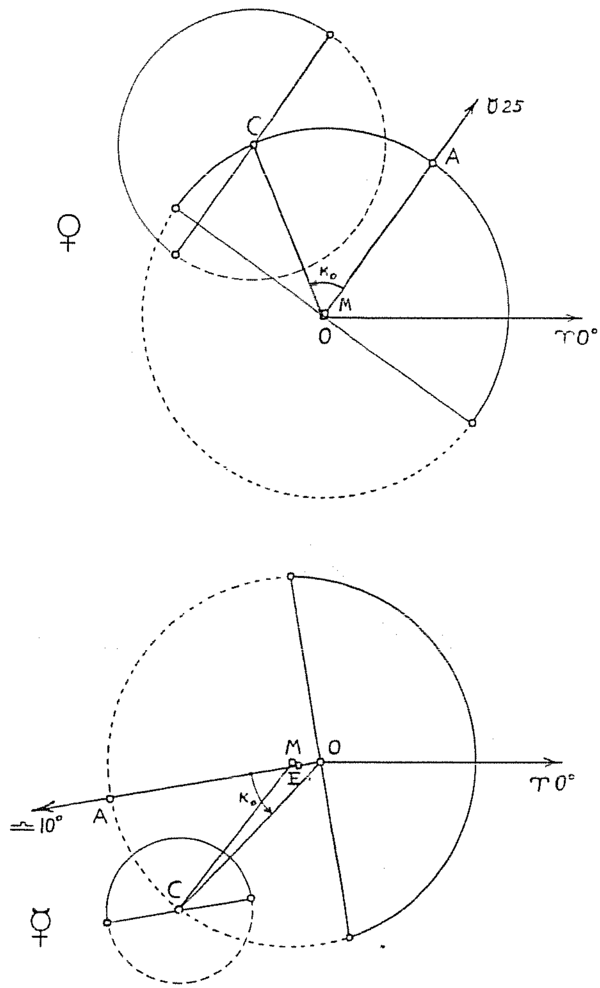

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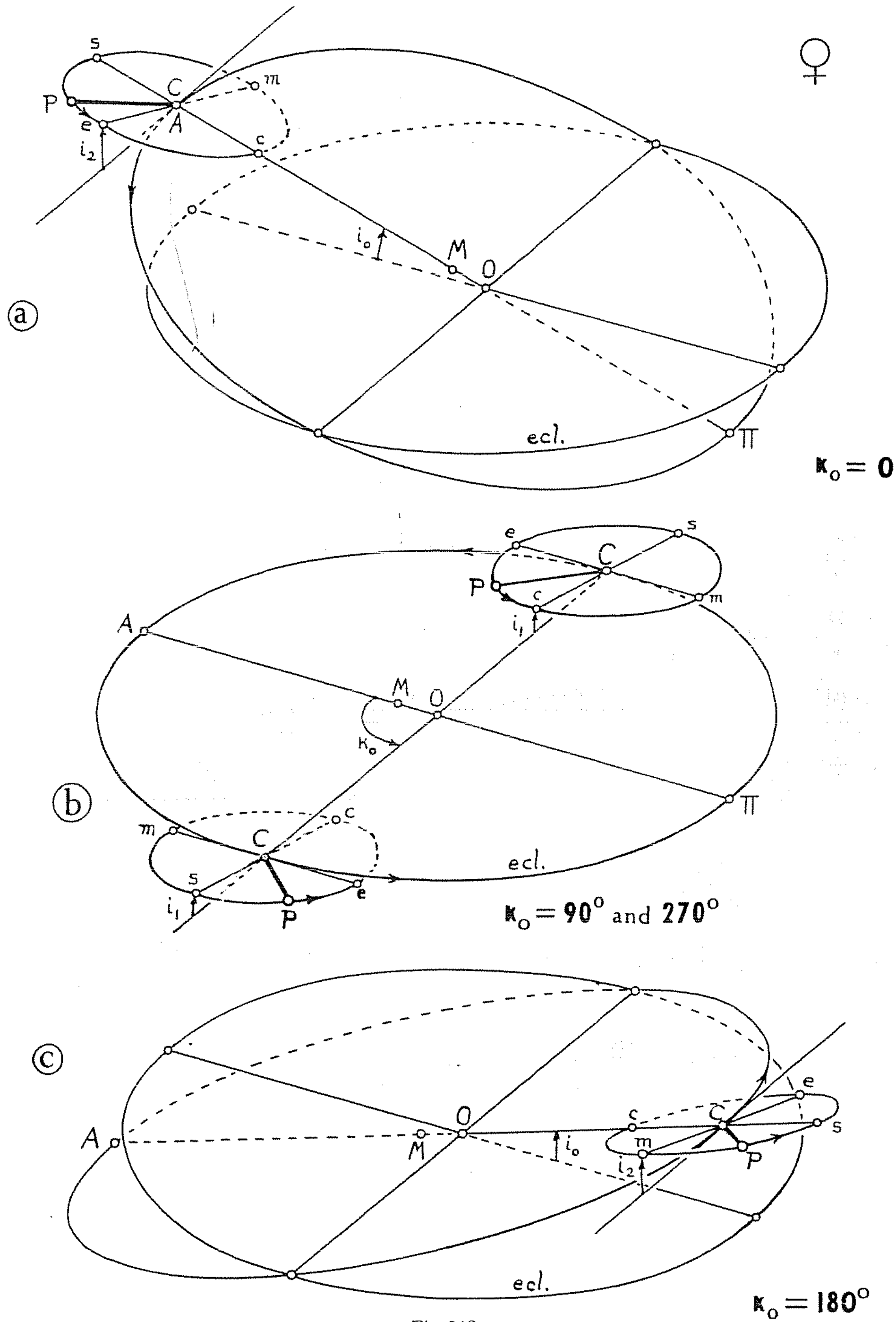
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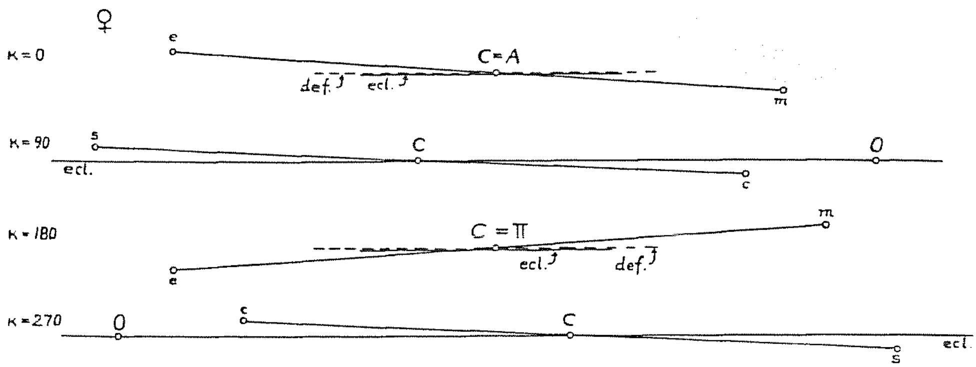
Fig. 219



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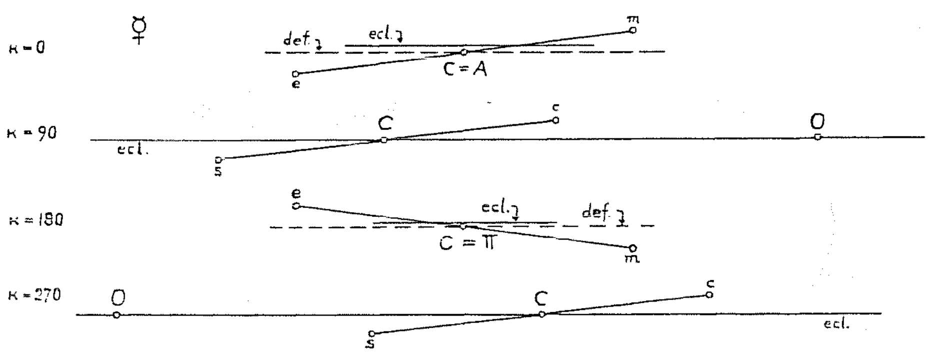

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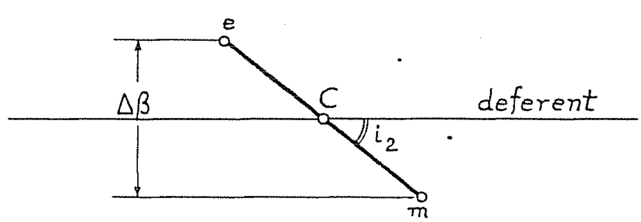


Fig. 222

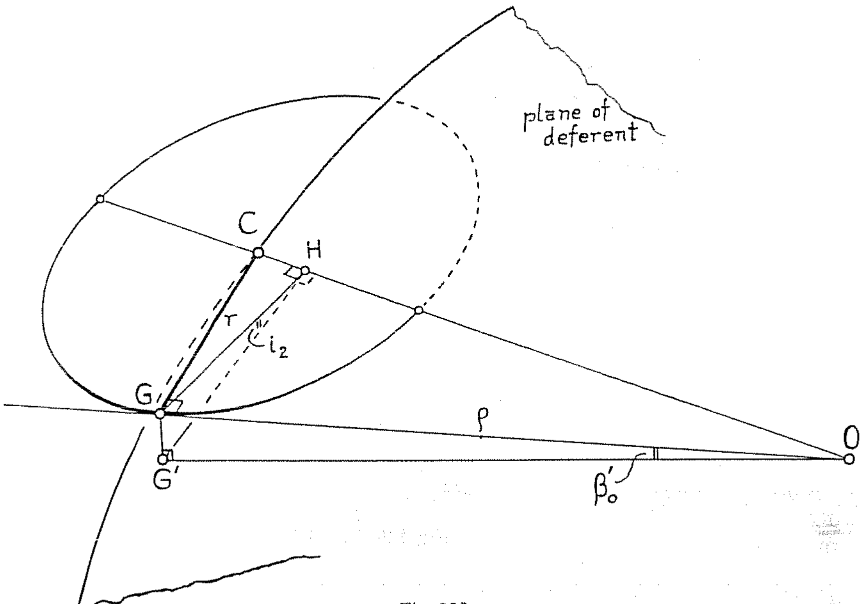
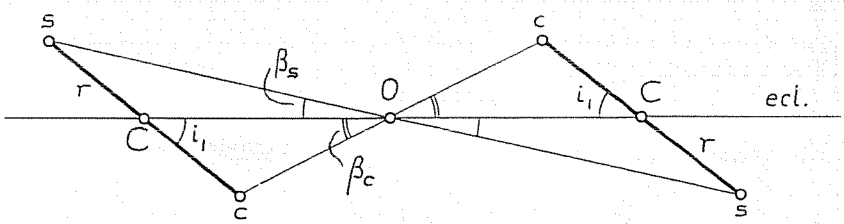


Fig. 223



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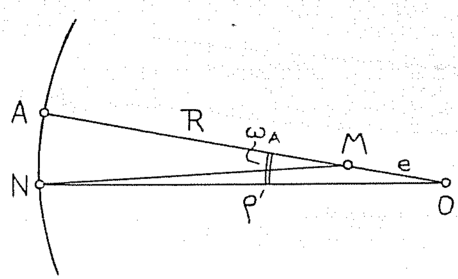

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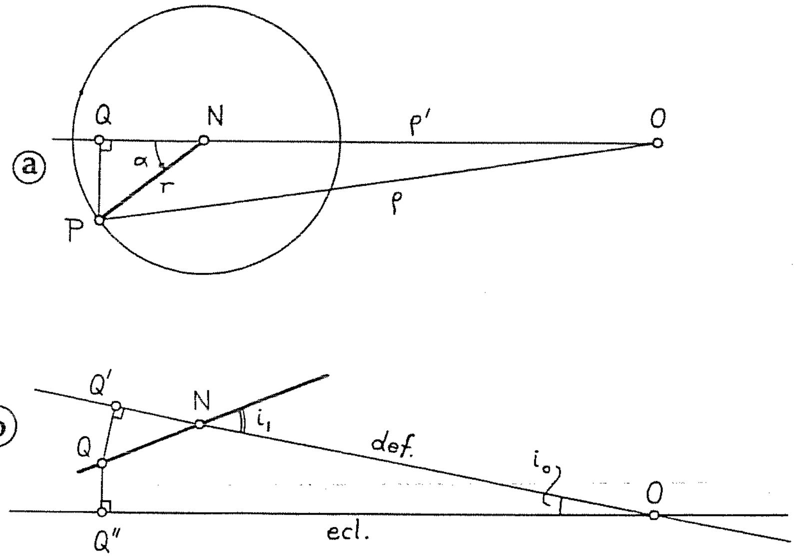

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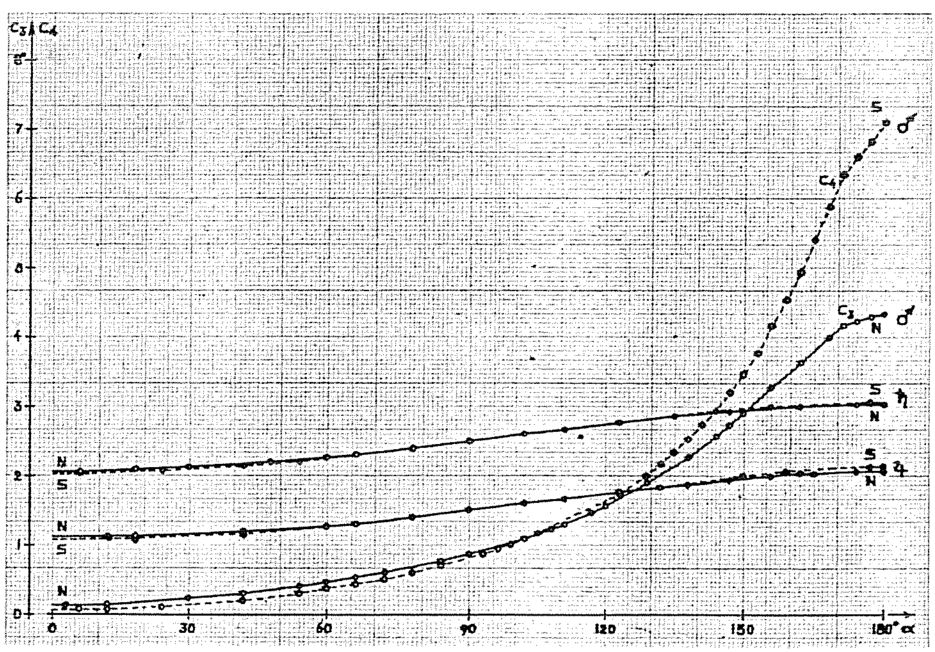
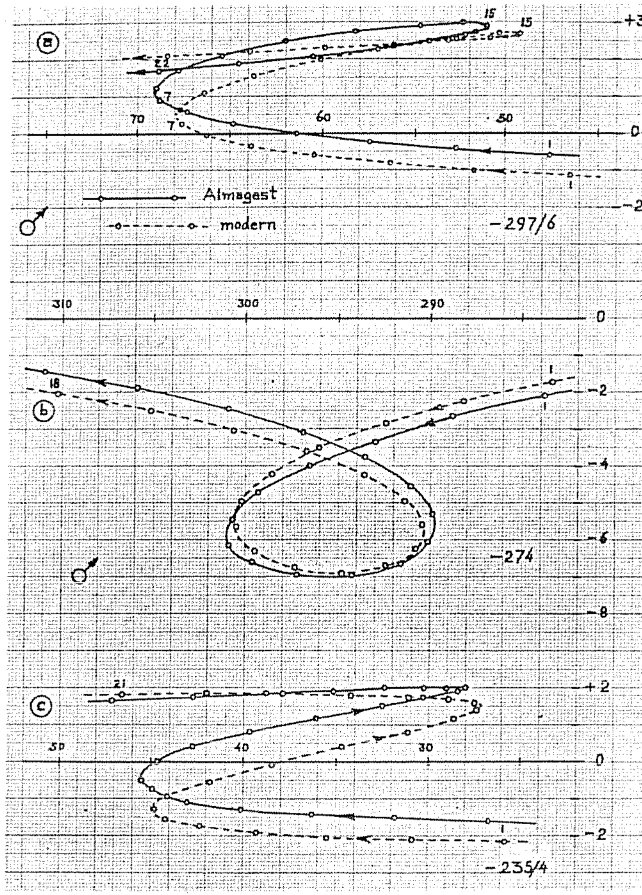


Fig. 227



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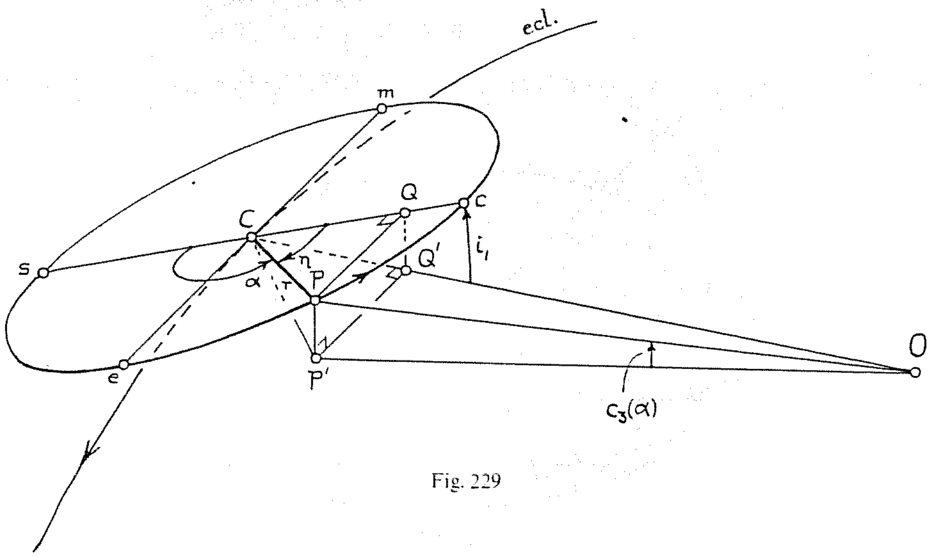

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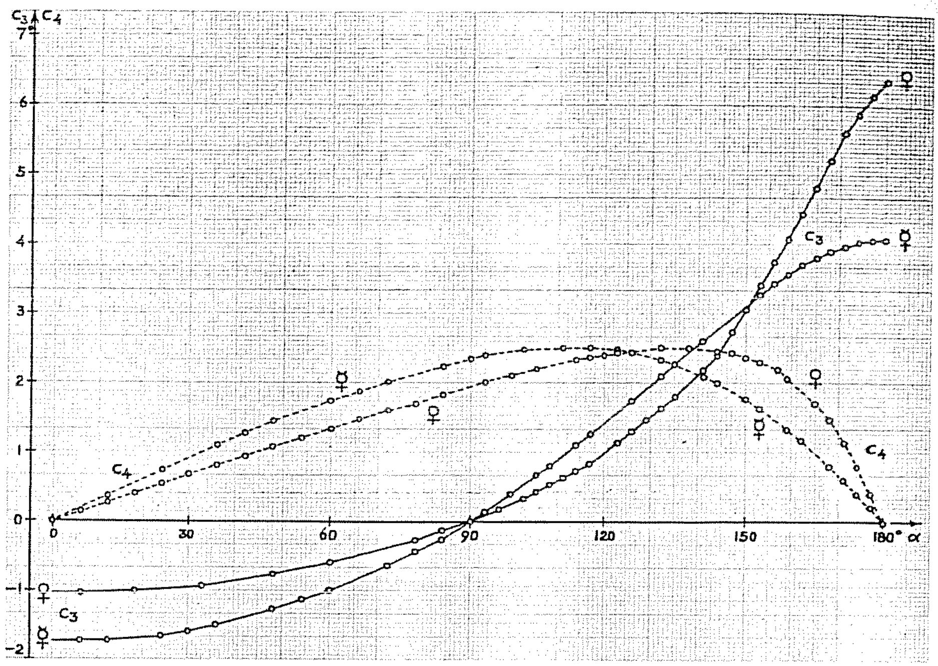

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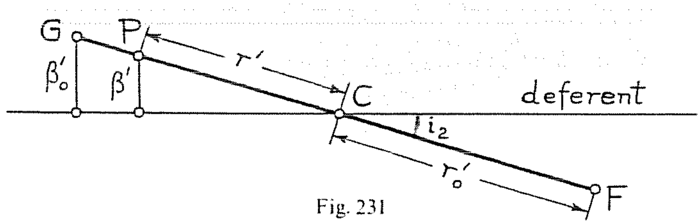

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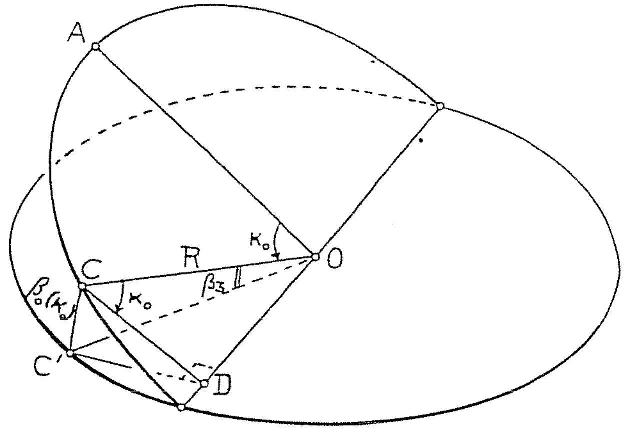

Fig. 232

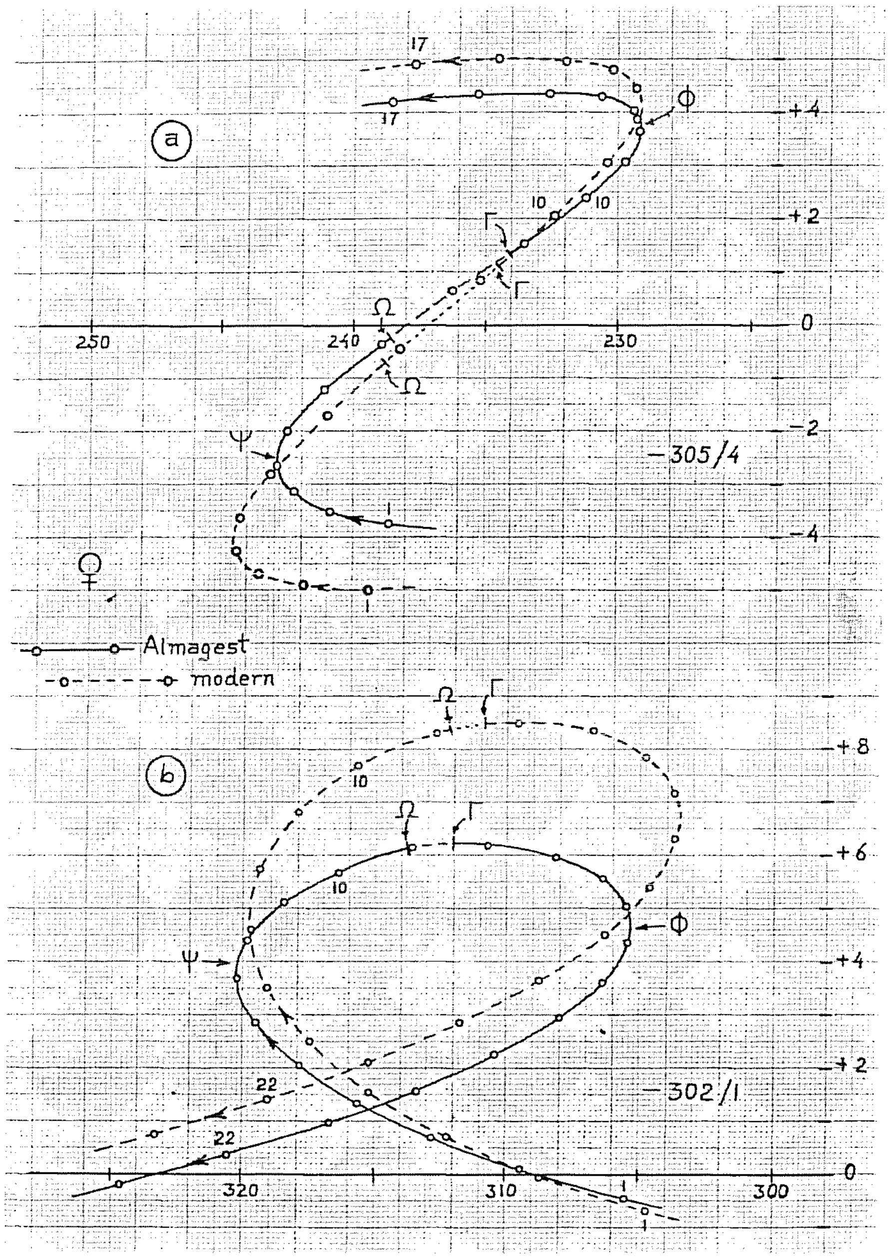
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Fig. 233

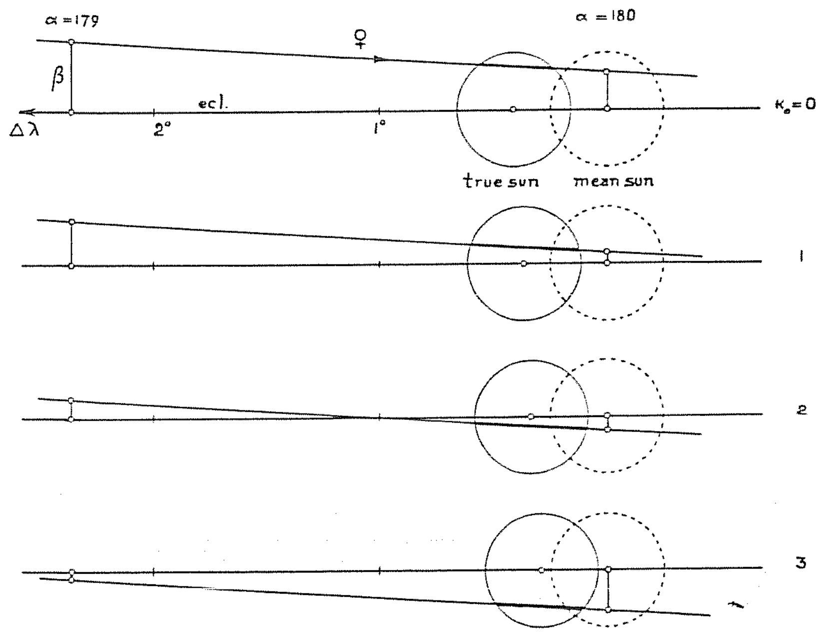


Fig. 234

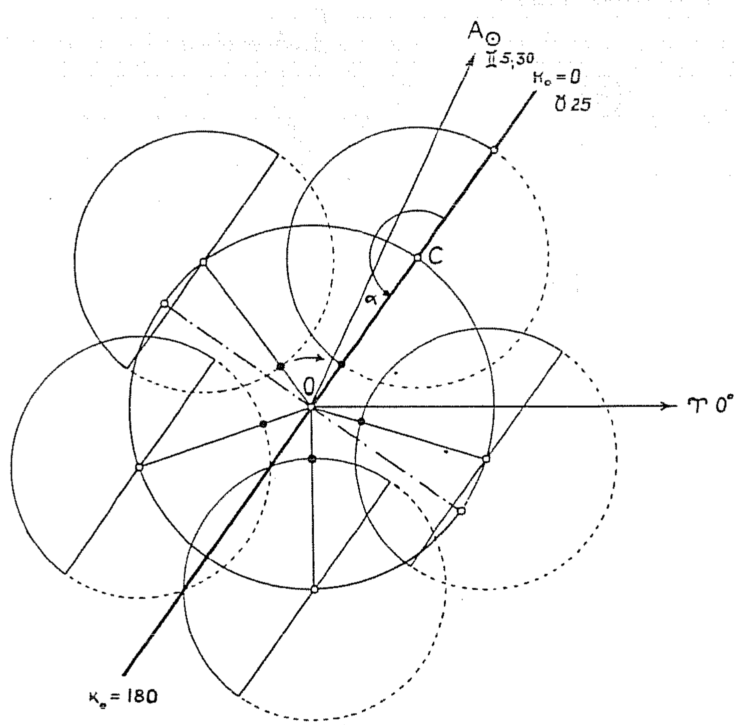
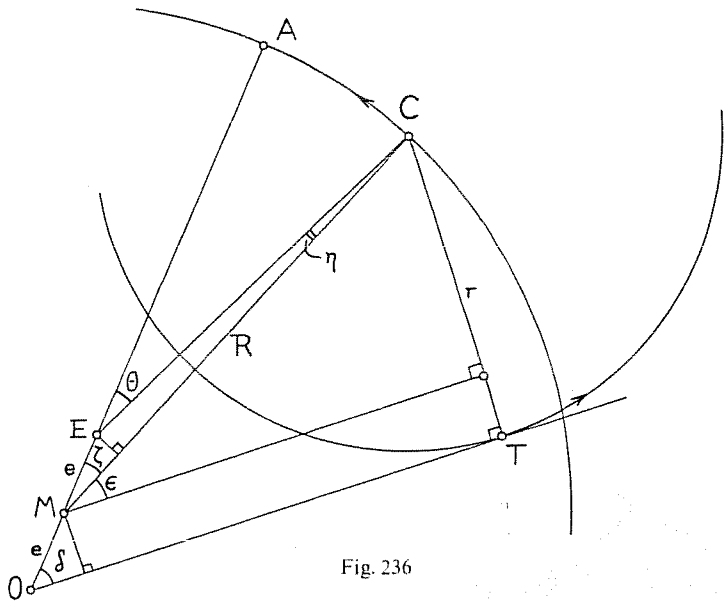


Fig. 235



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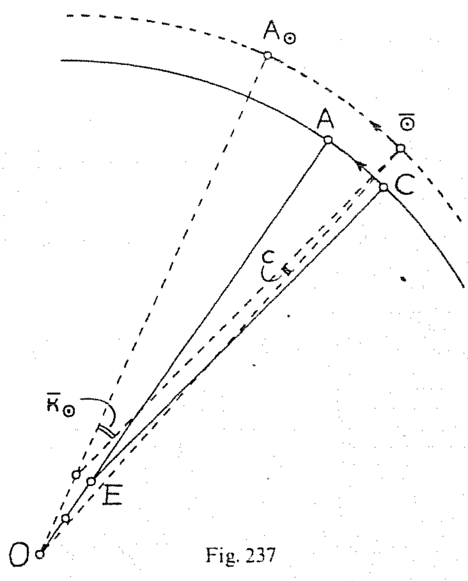

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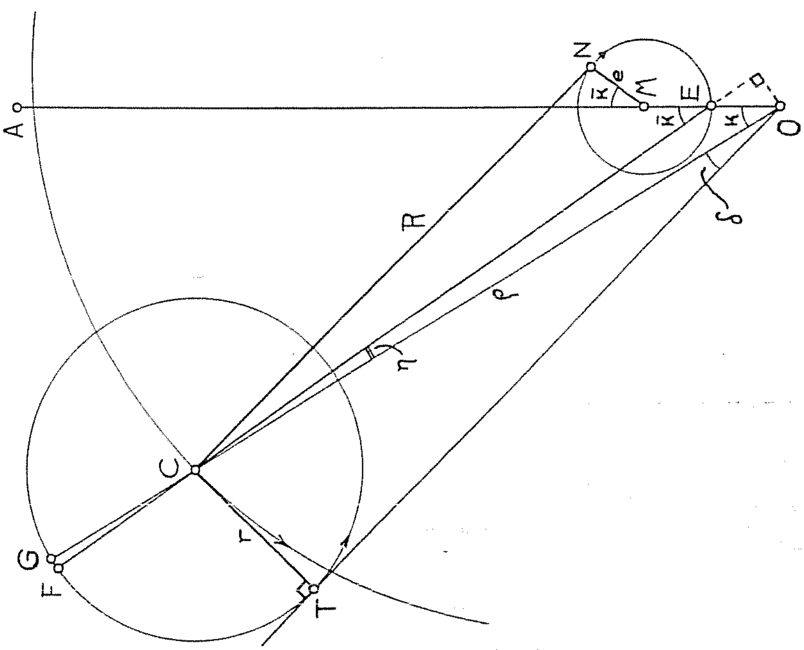

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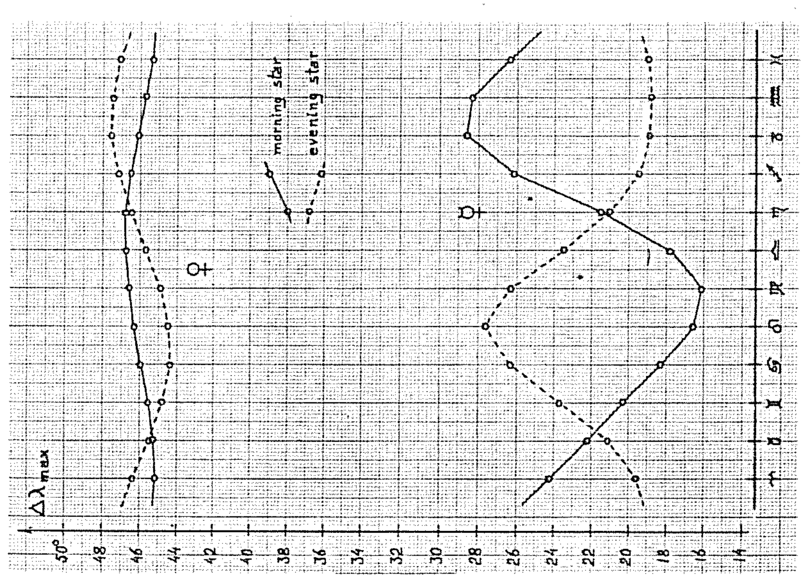

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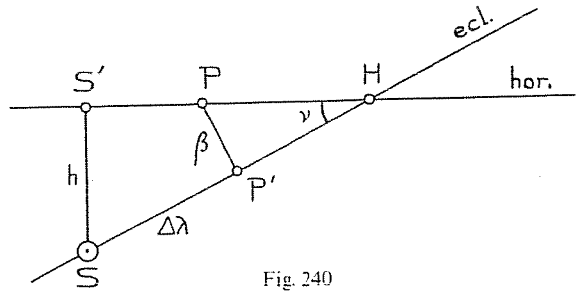

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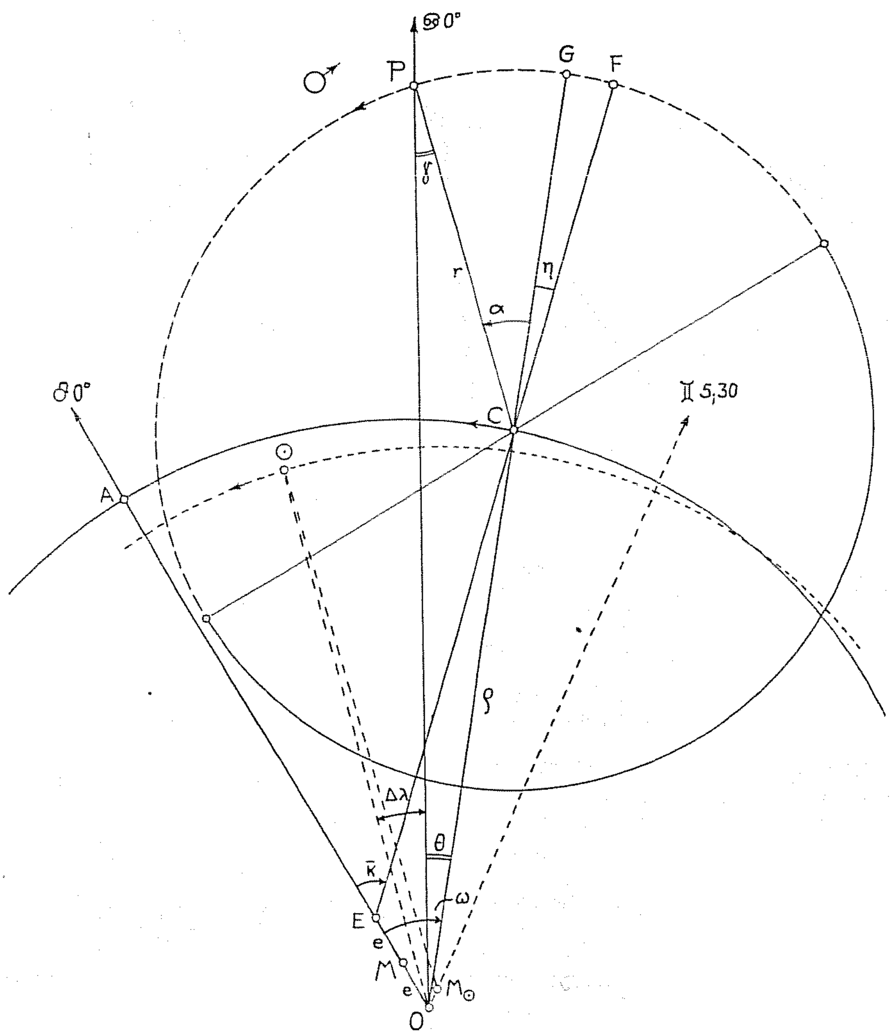

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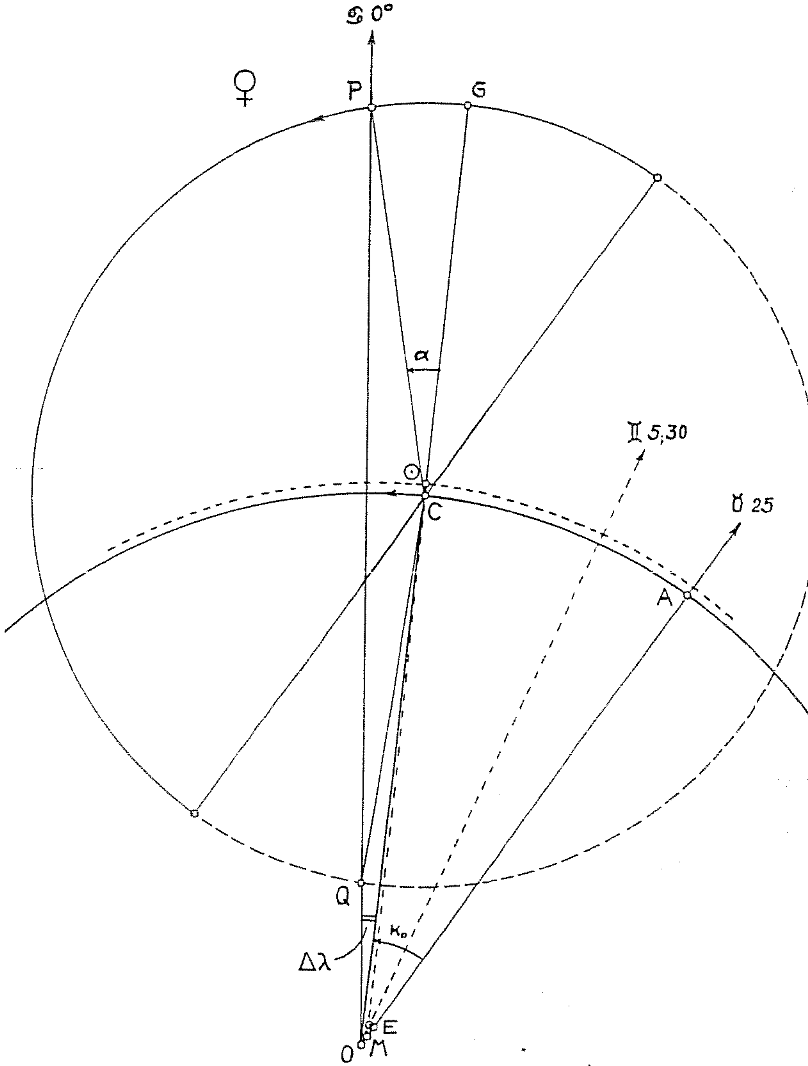

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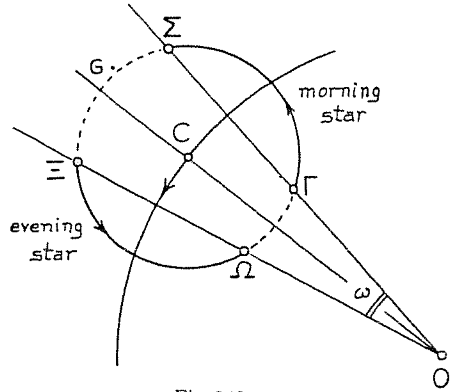

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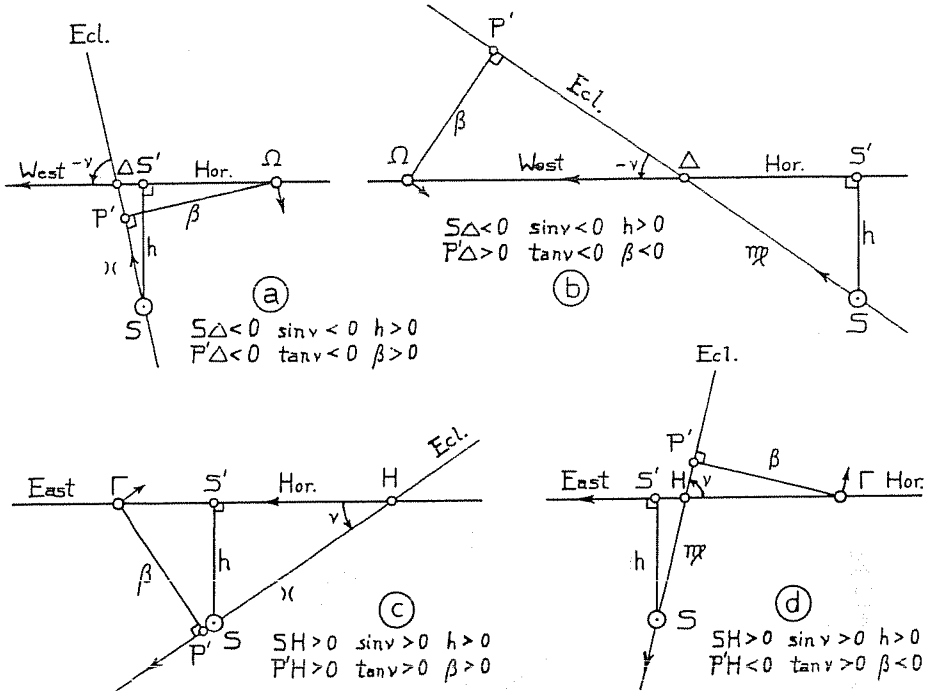

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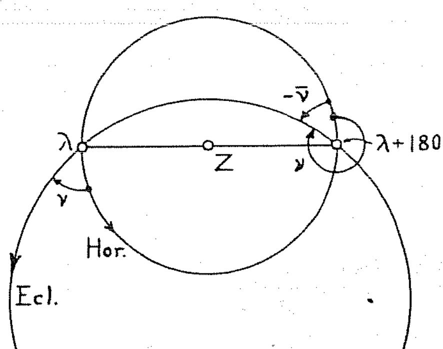

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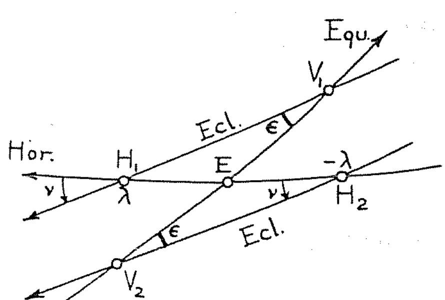

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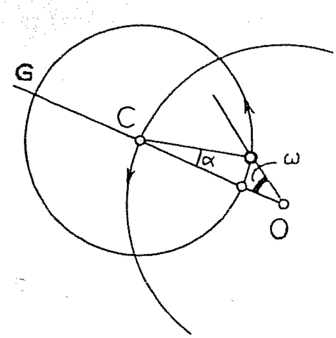

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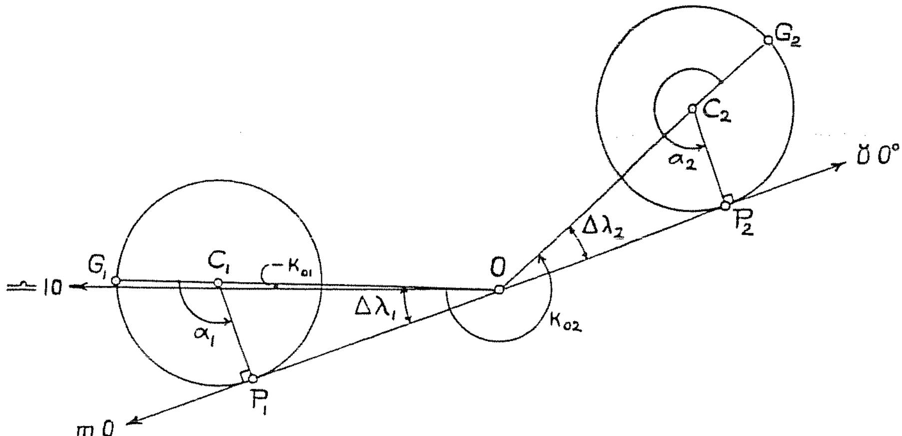

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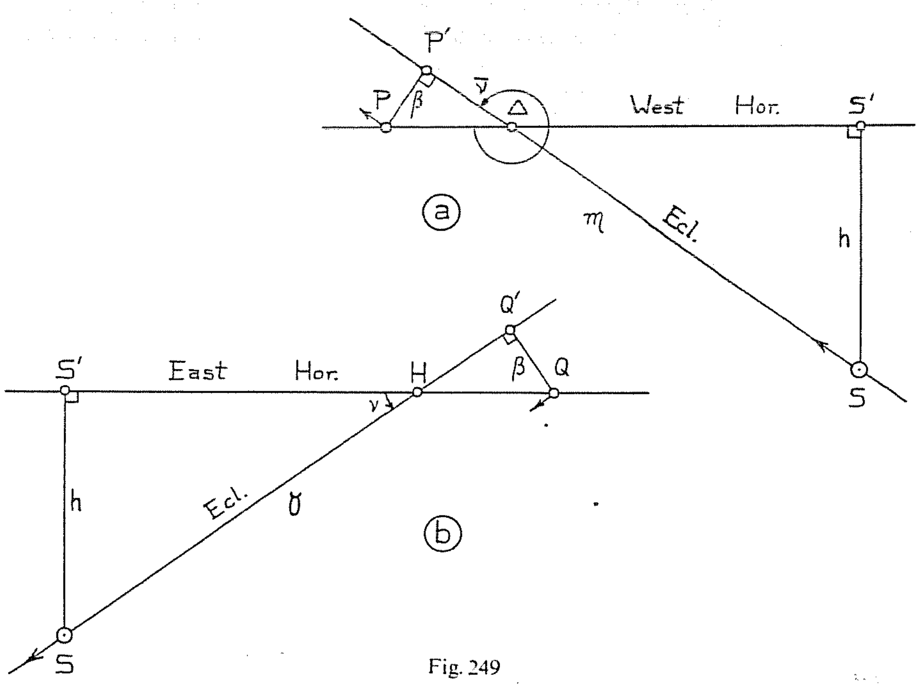

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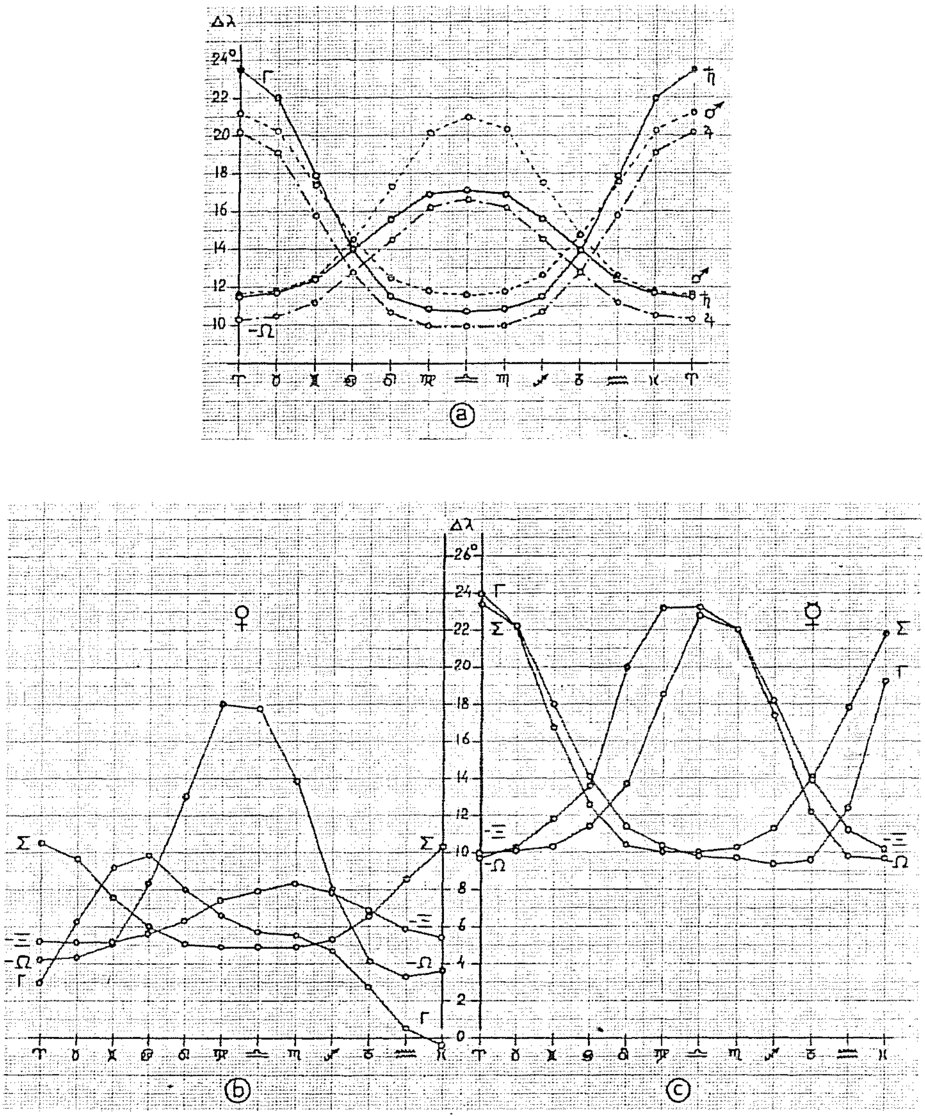
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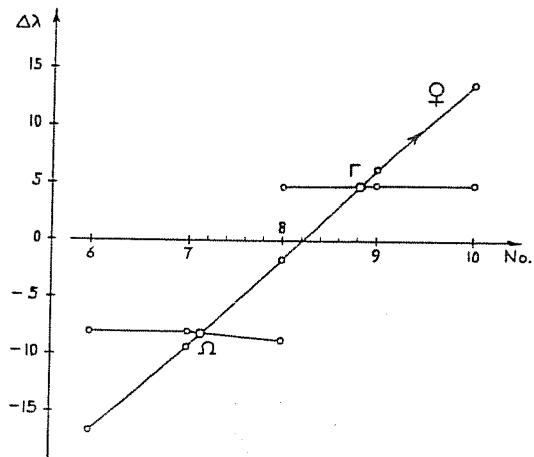

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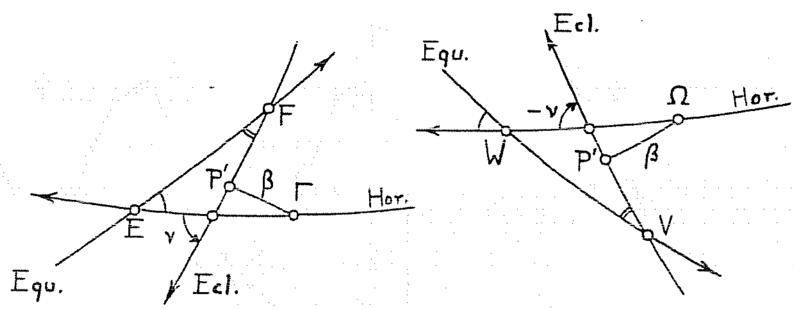

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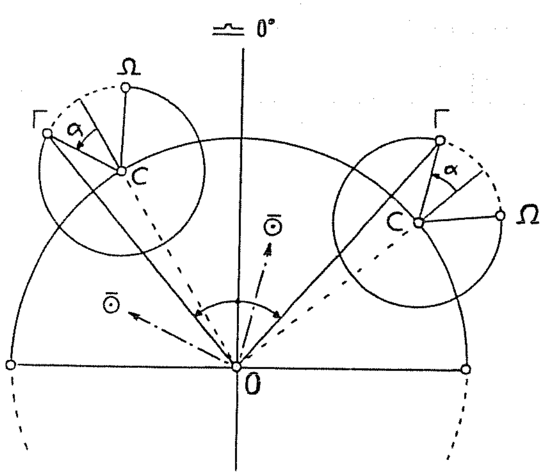

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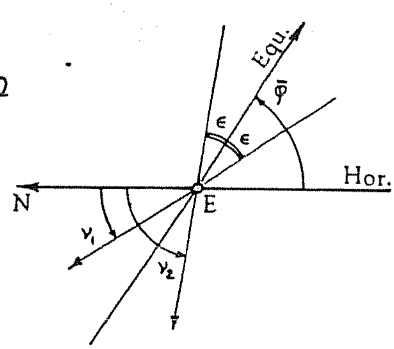

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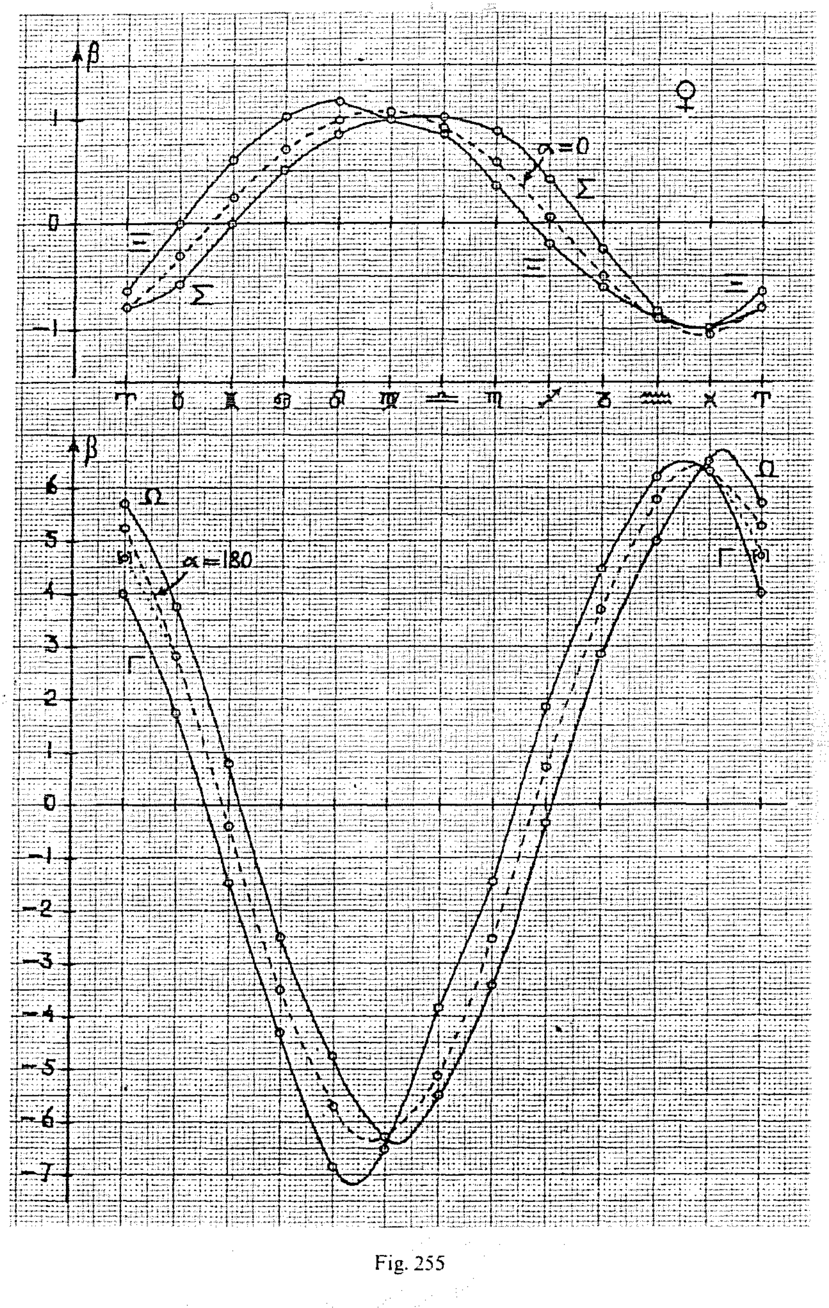

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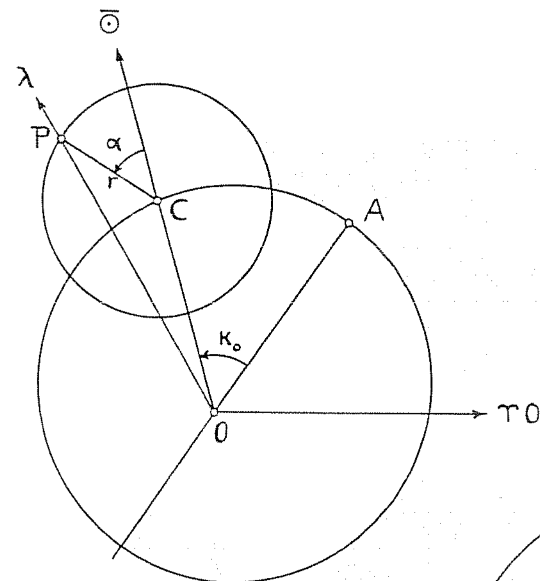

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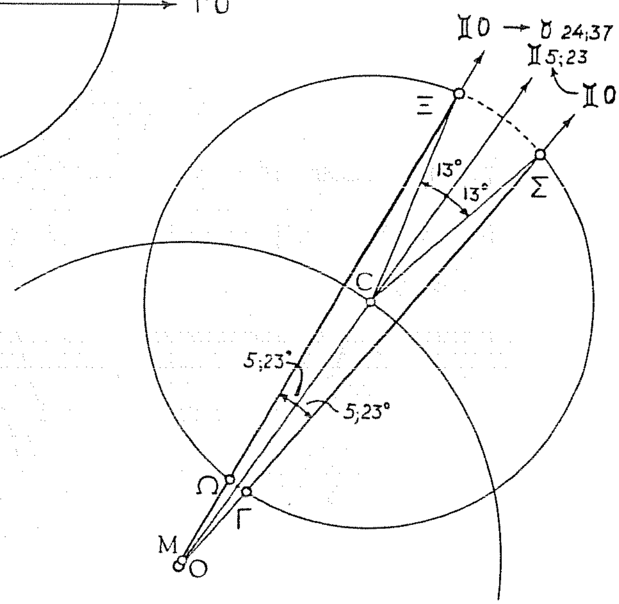

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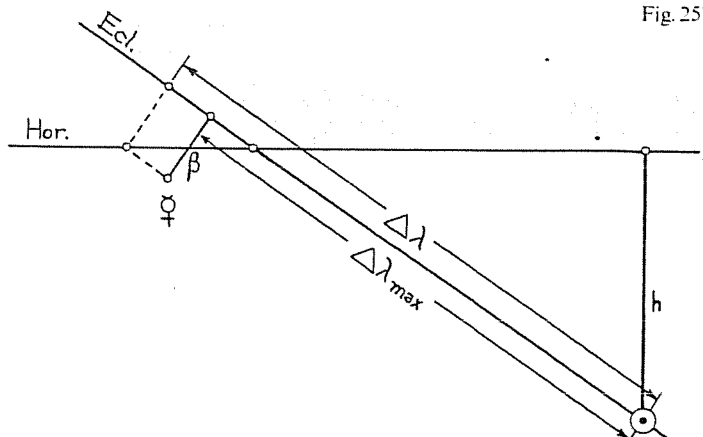

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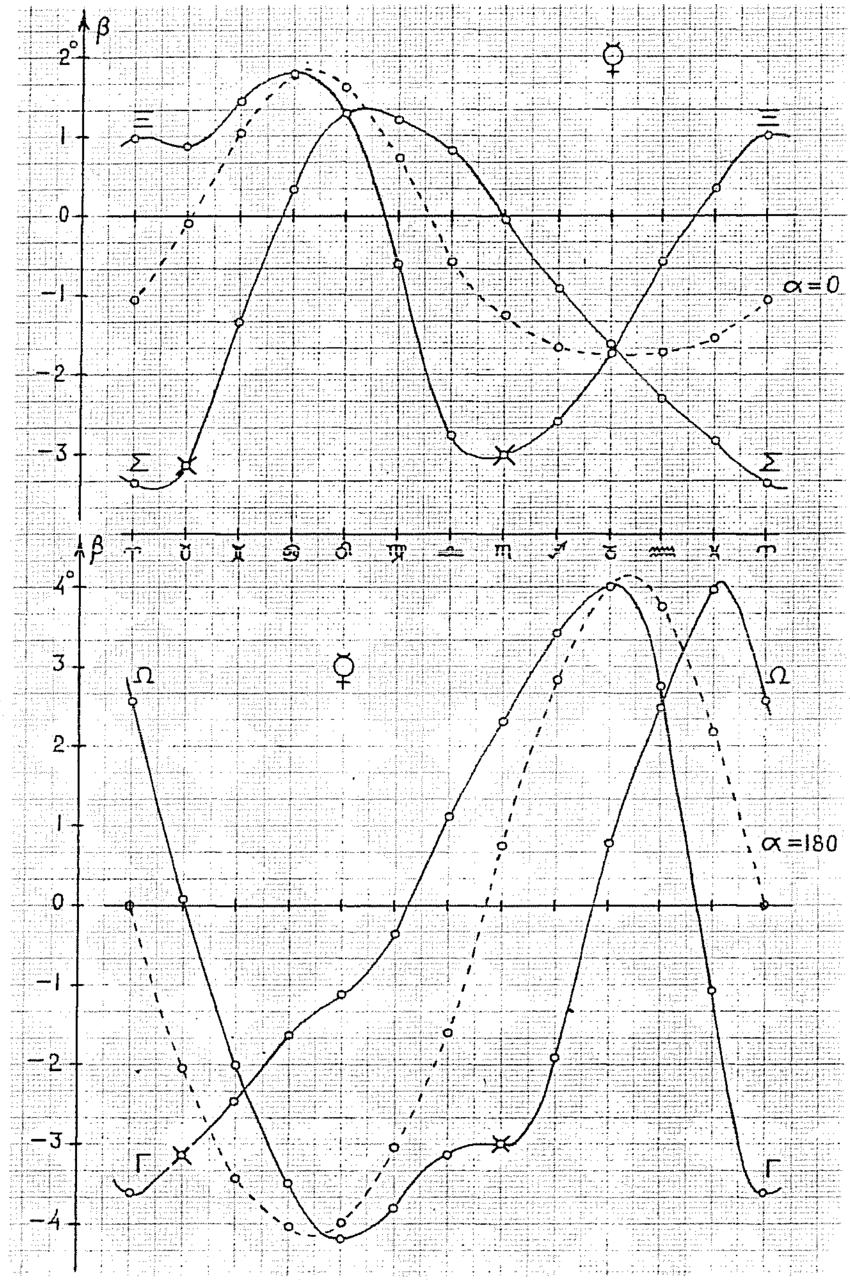


Fig. 259

Fig. 260 see page 1301

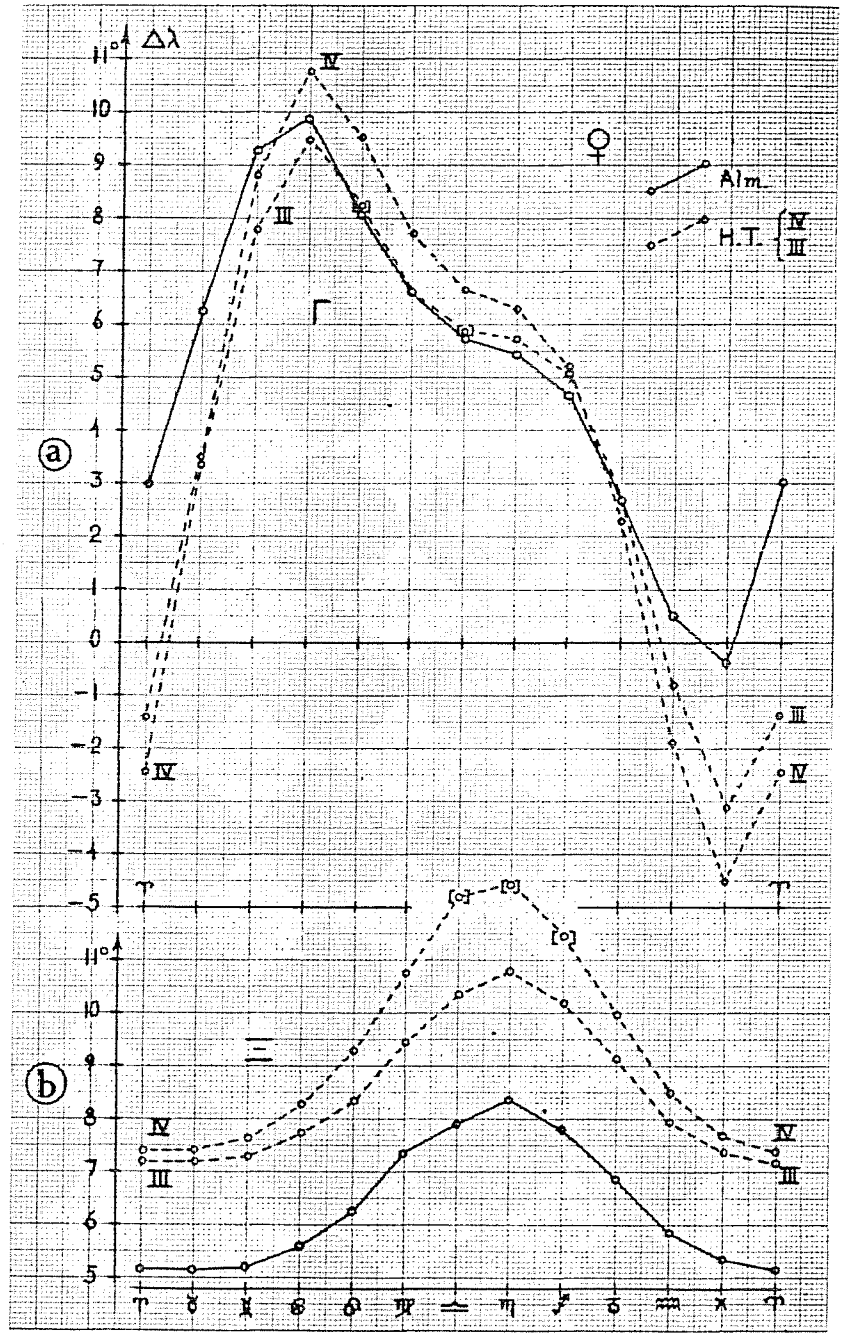


Fig. 261a and b

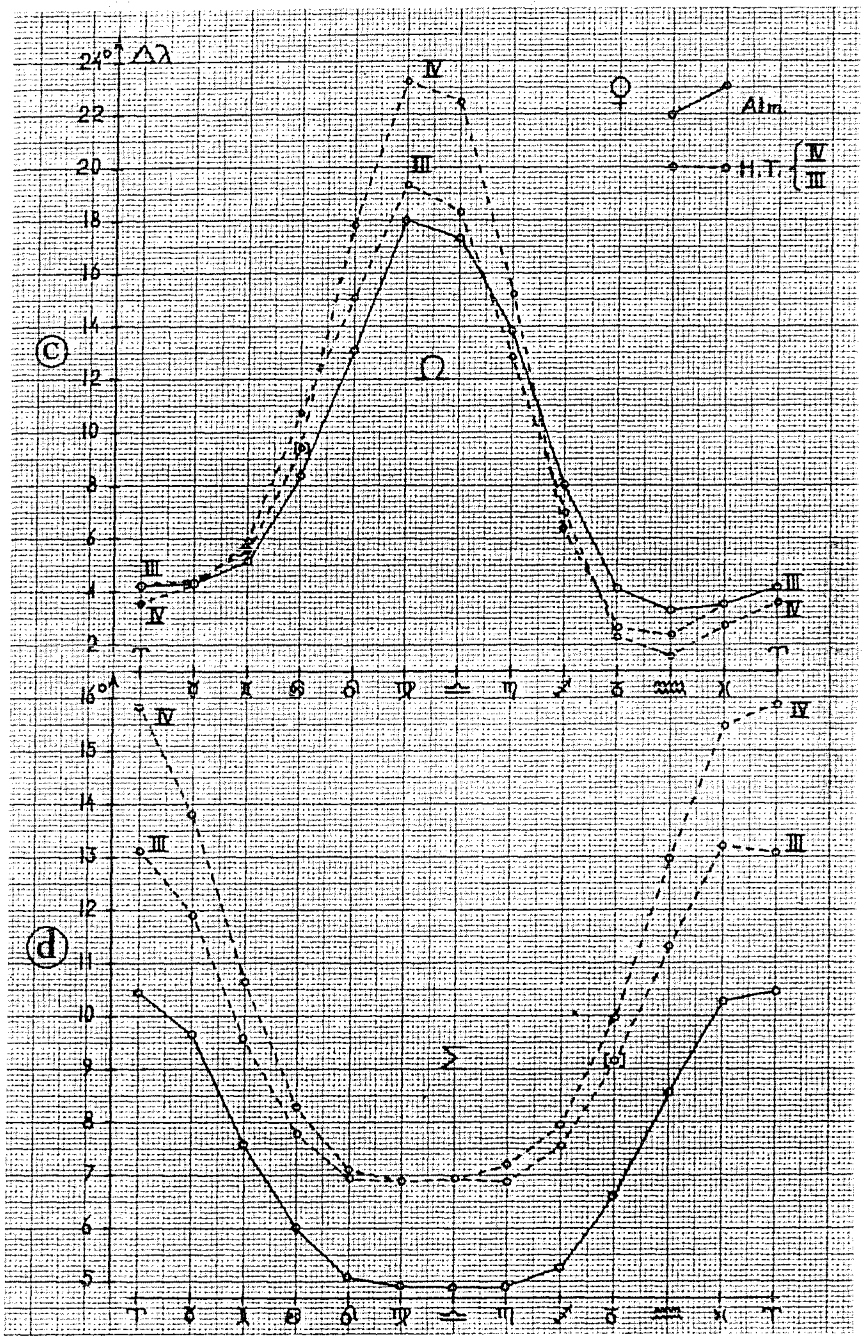


Fig. 261c and d

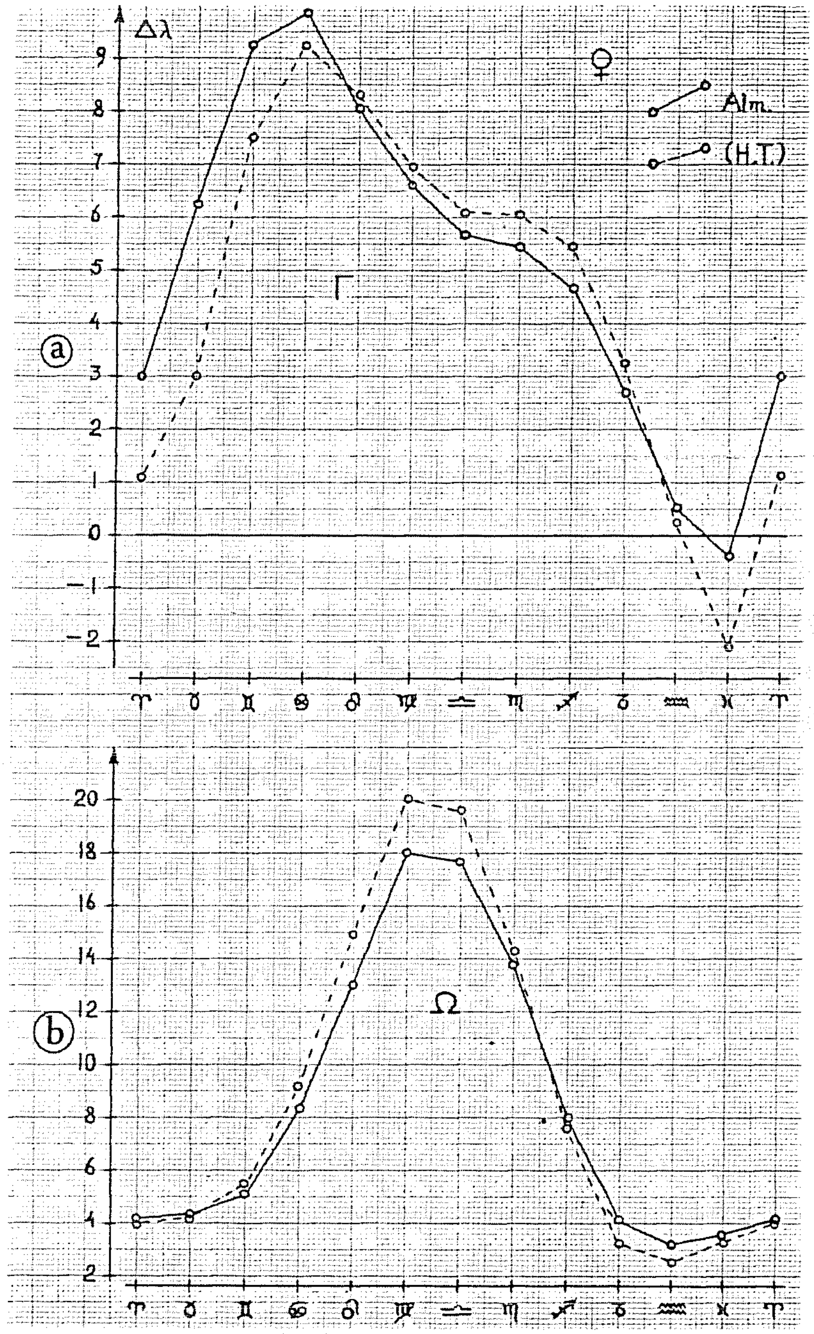


Fig. 262a and b

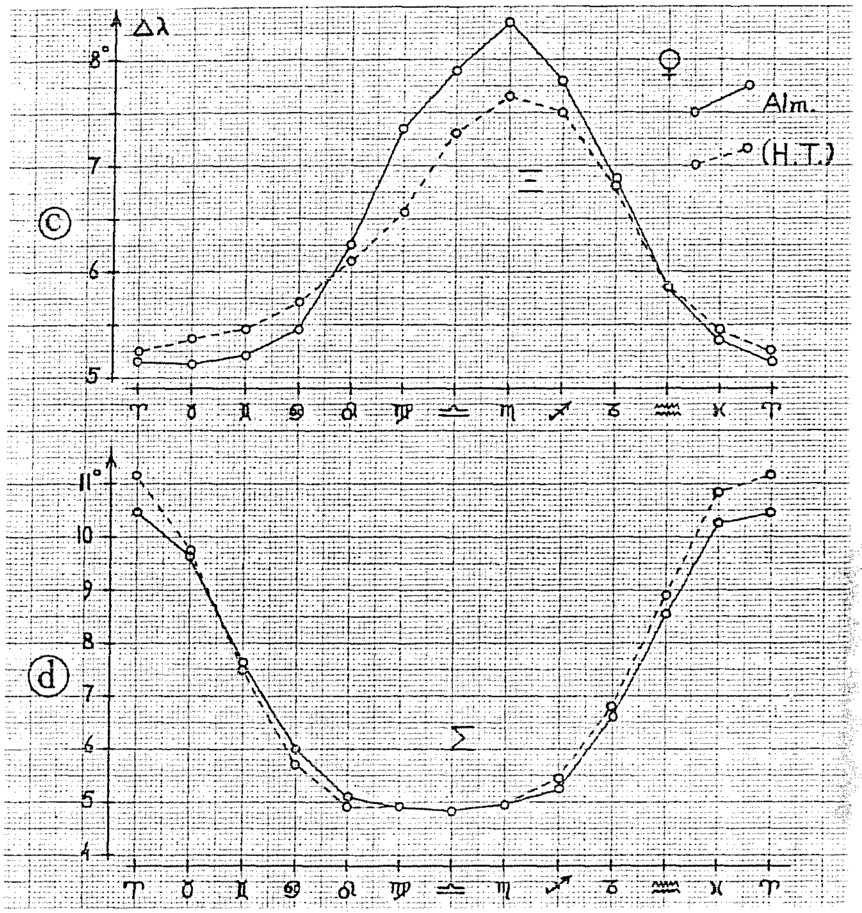
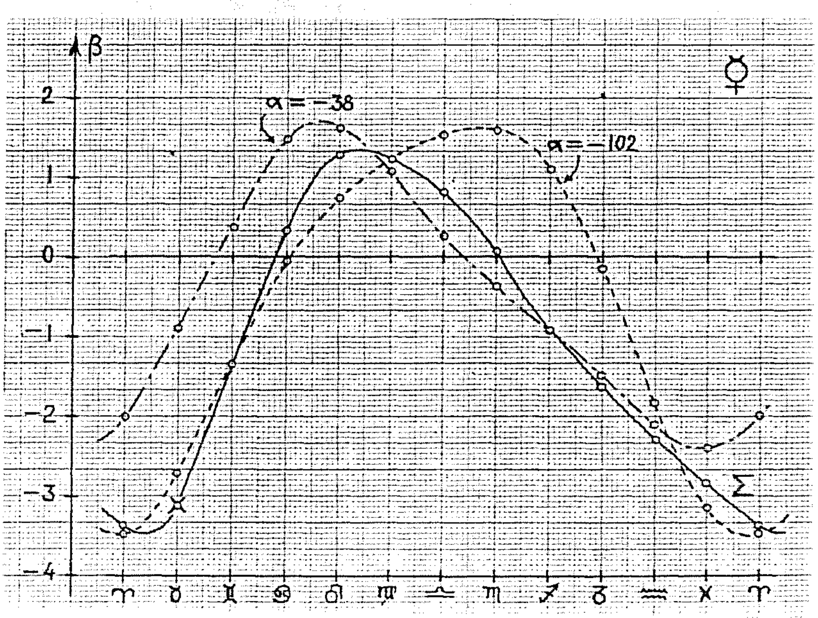


Fig. 262c and d



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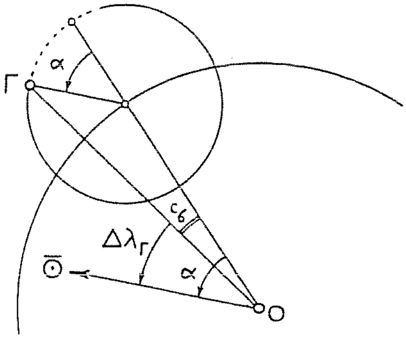

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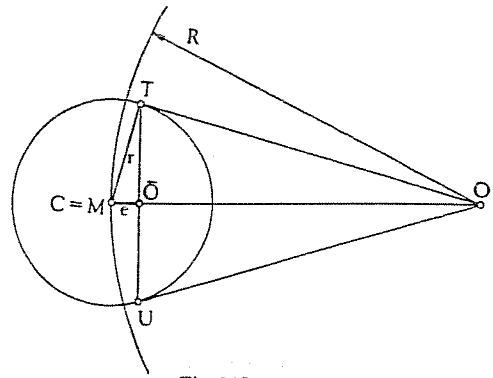

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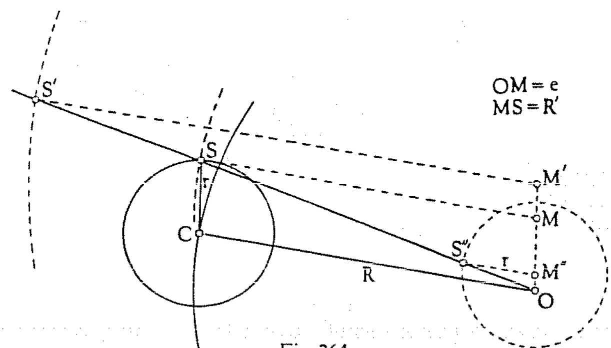

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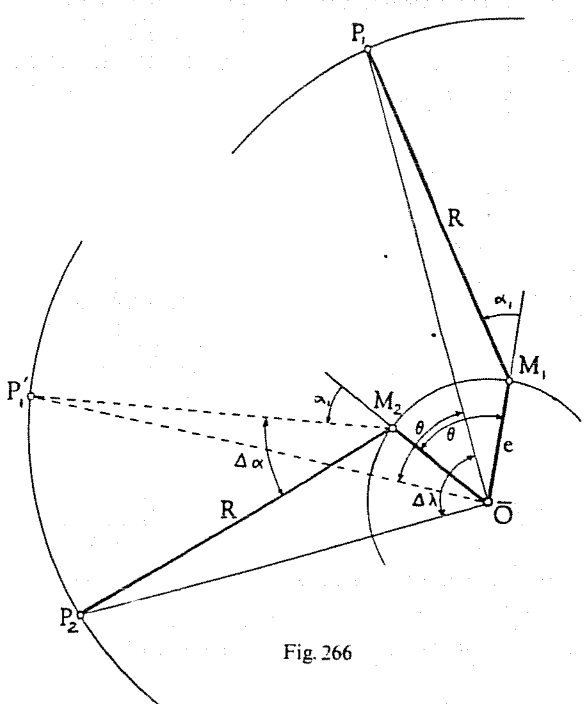

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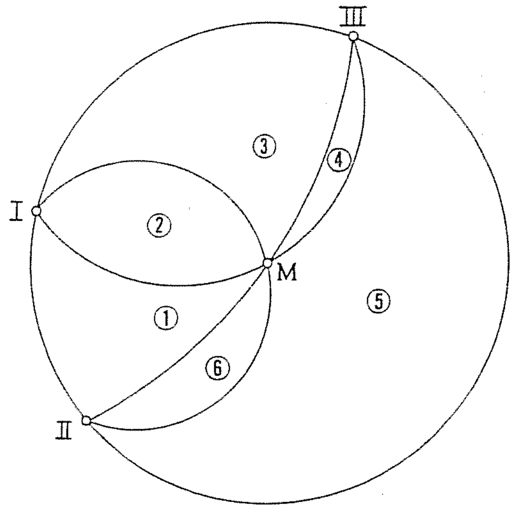

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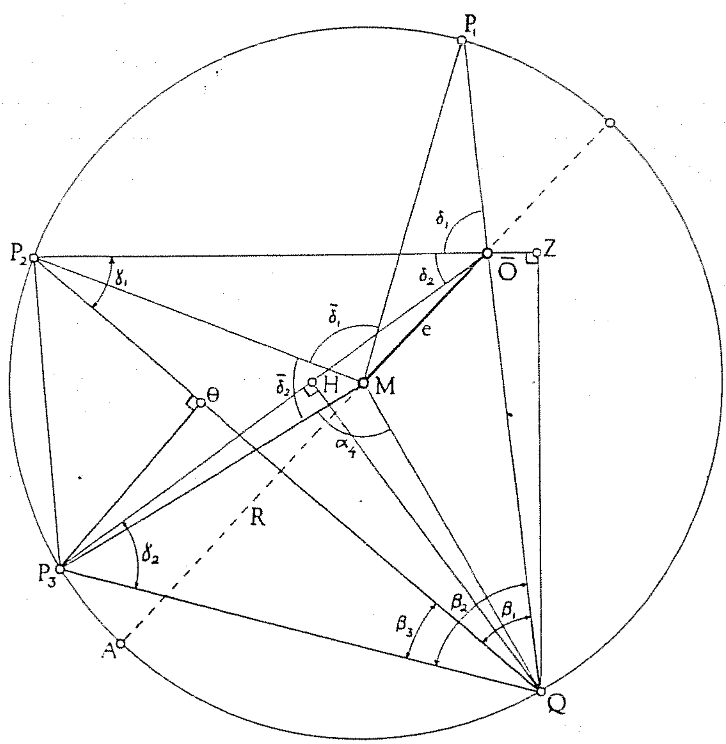

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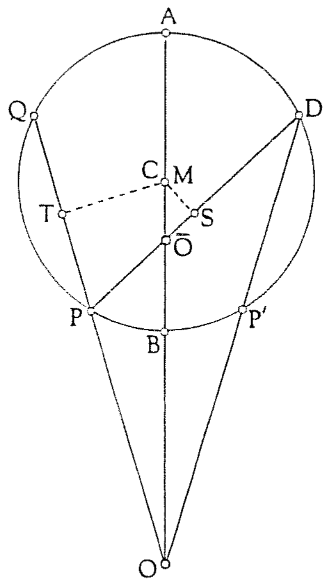

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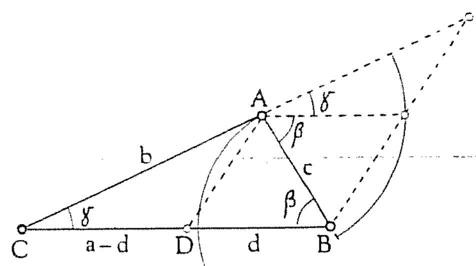

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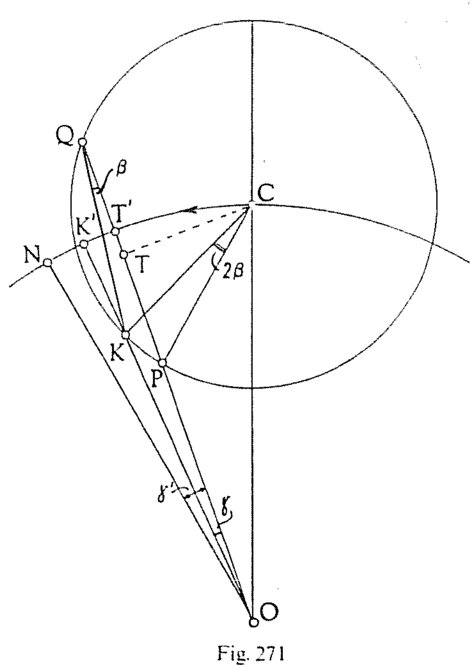

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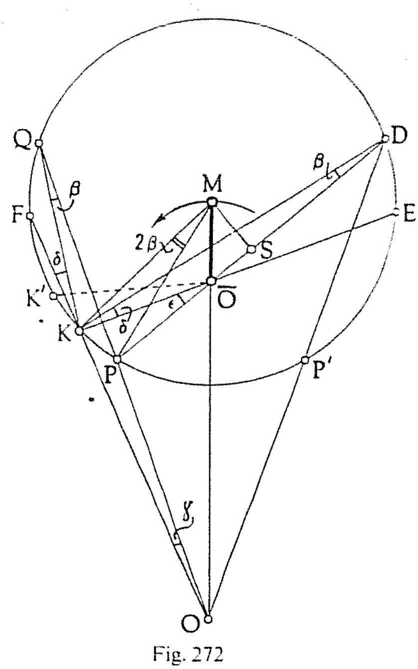

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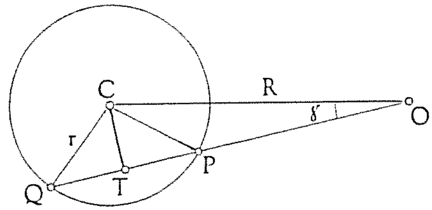

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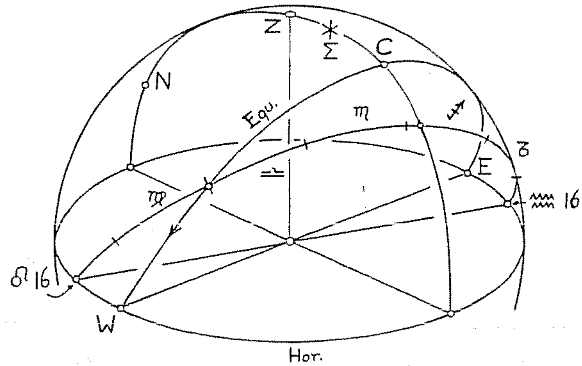

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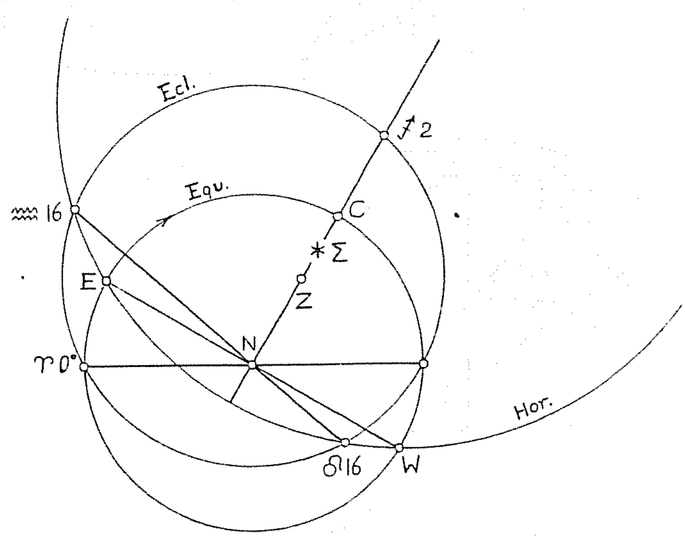

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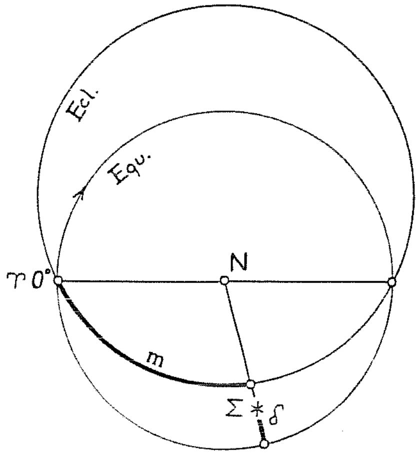

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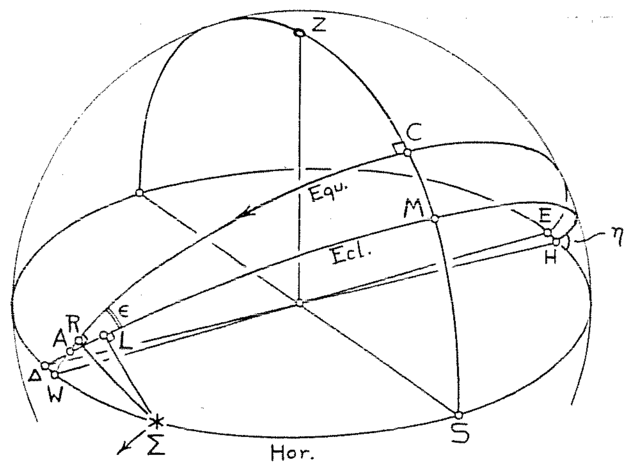

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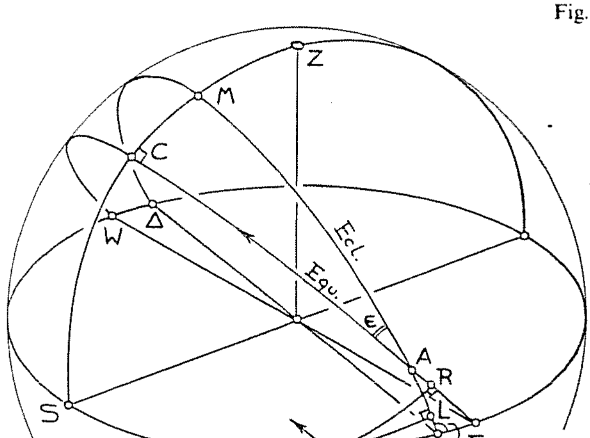

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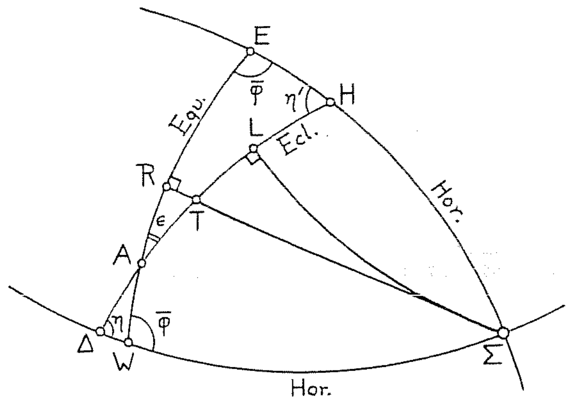

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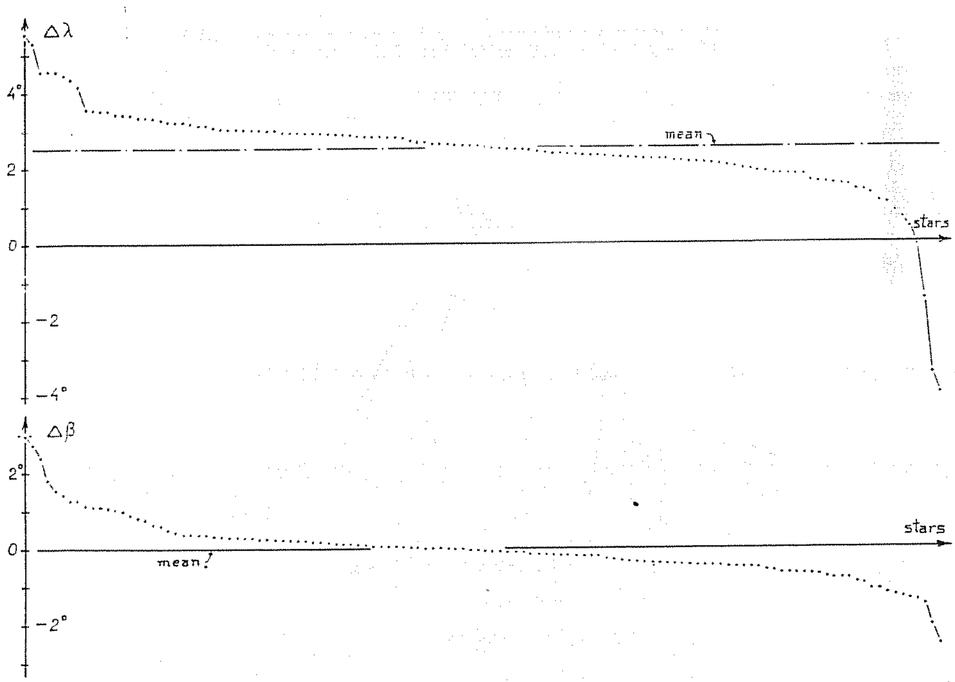

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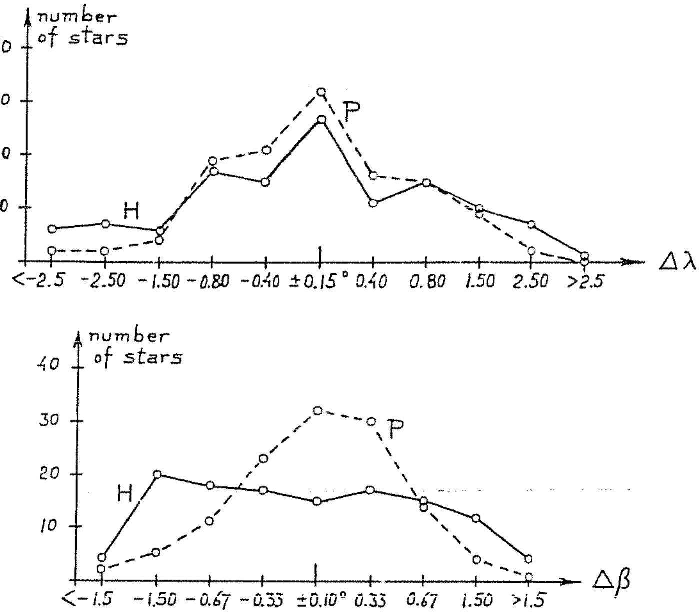

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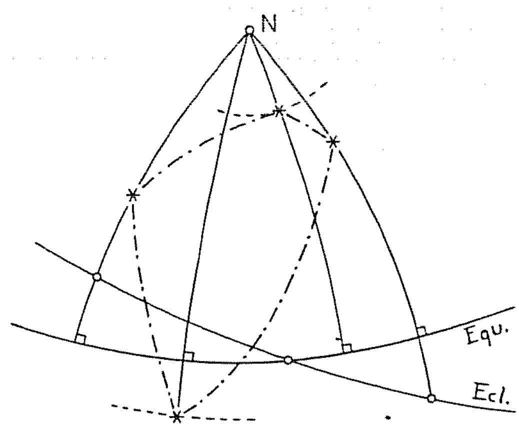

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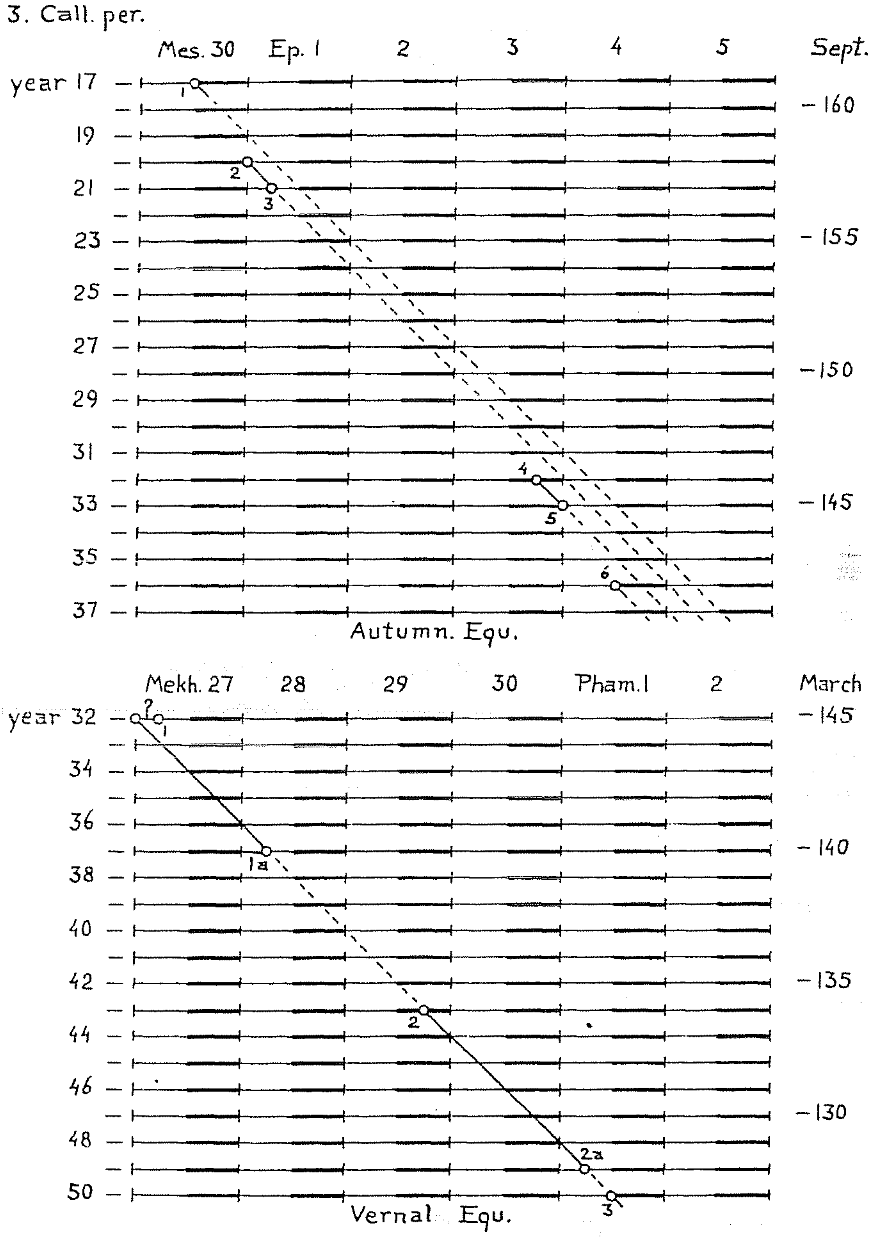
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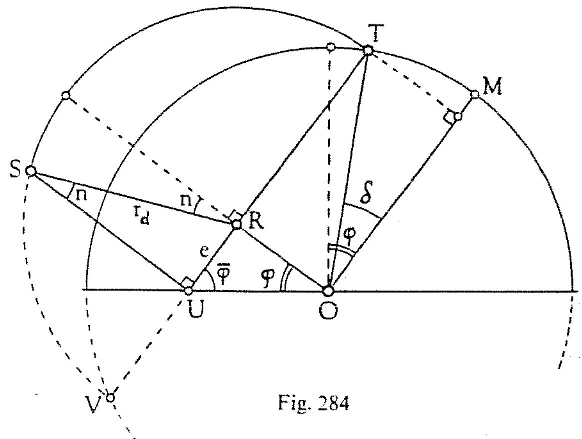
Fig. 283



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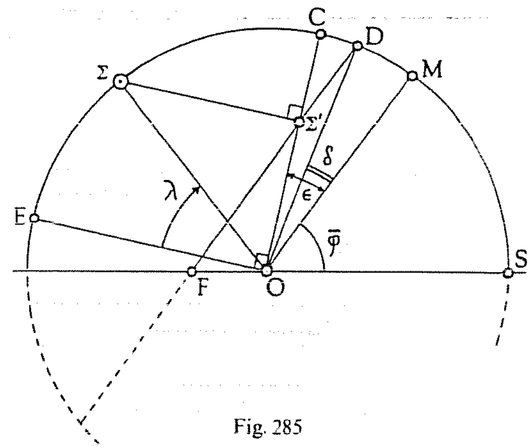

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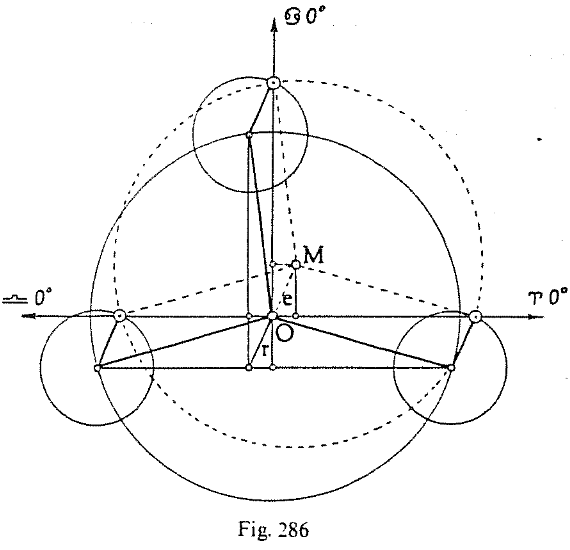

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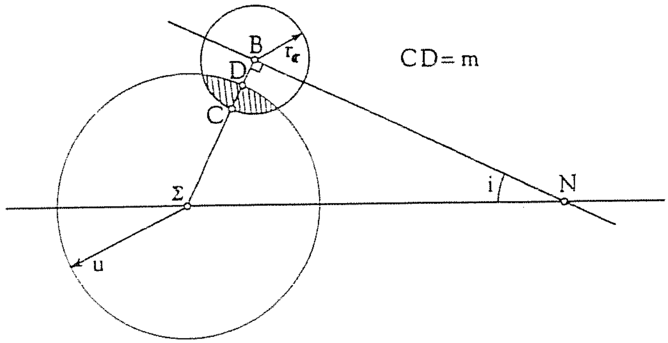

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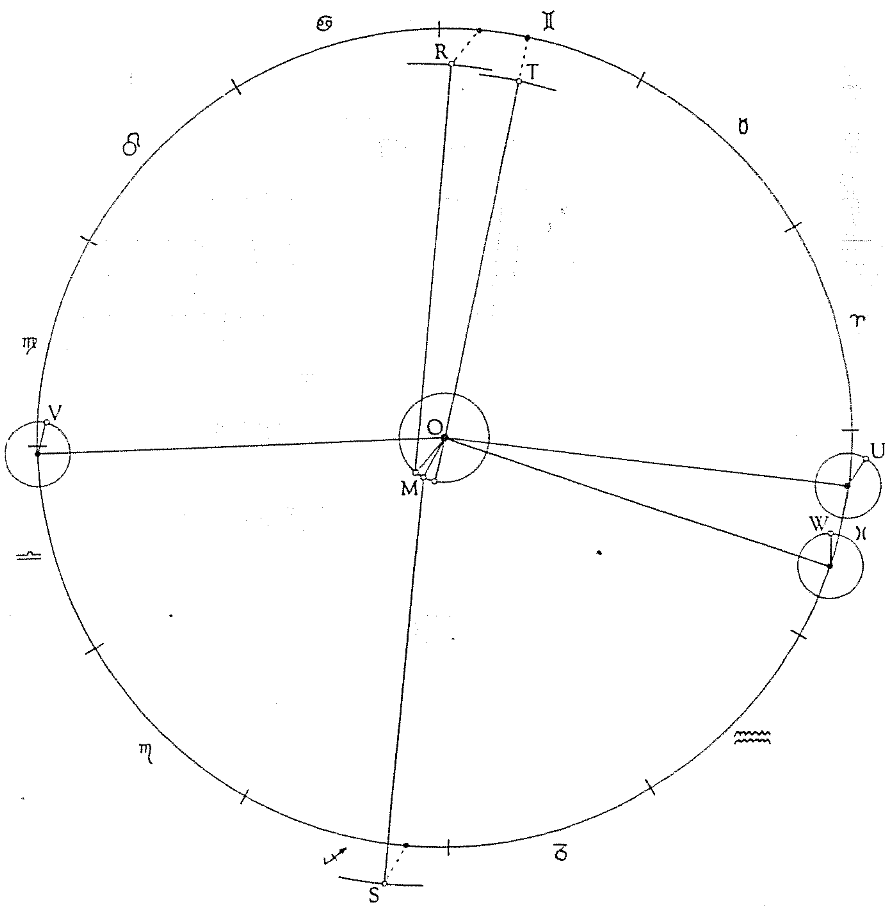

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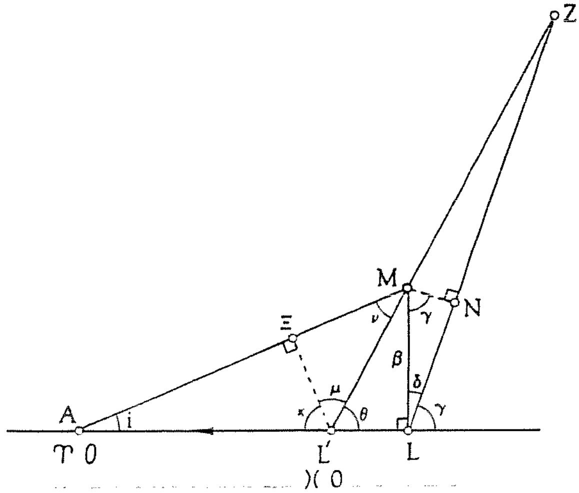

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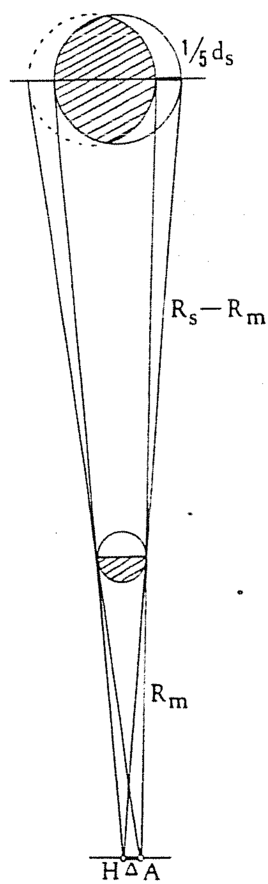

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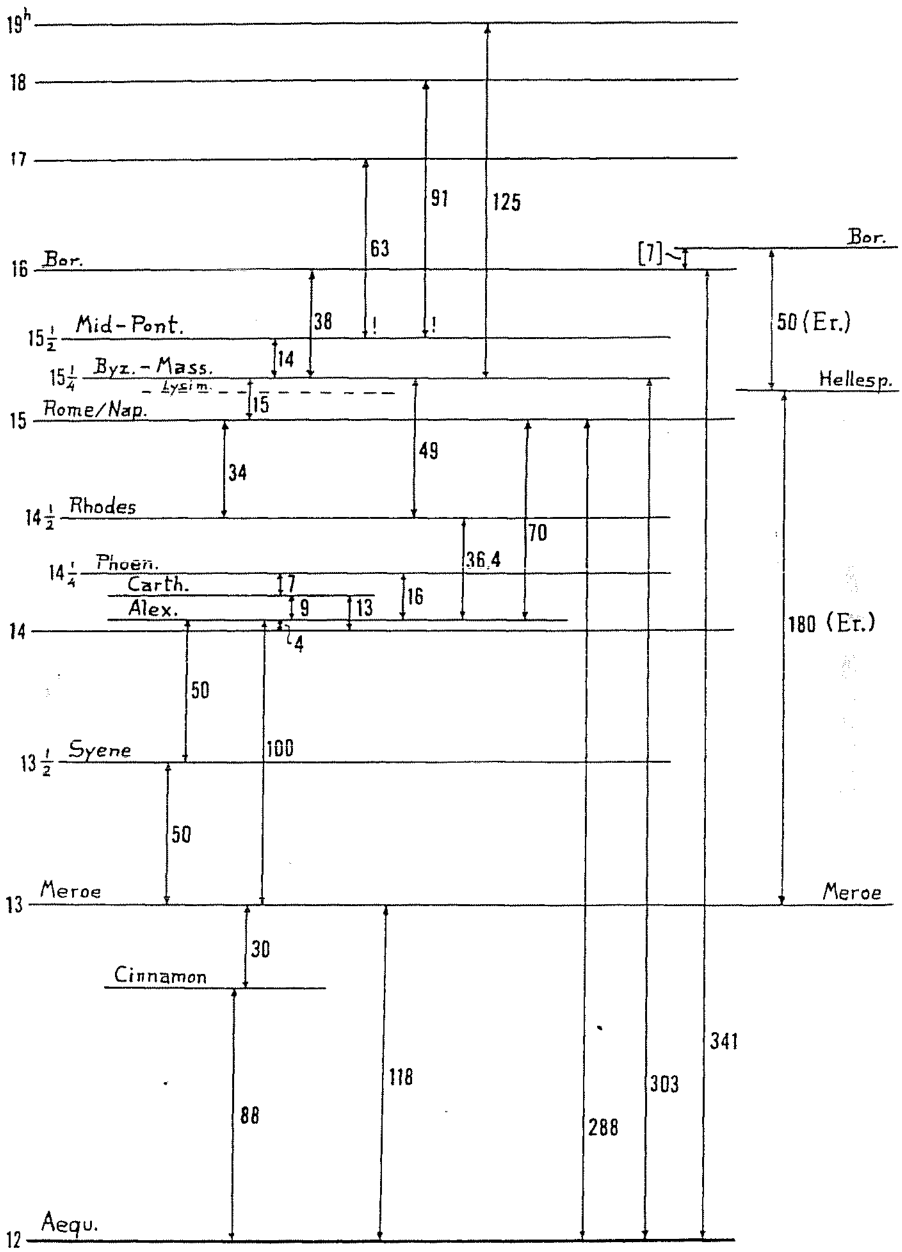
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Fig. 291

Figures to Book II

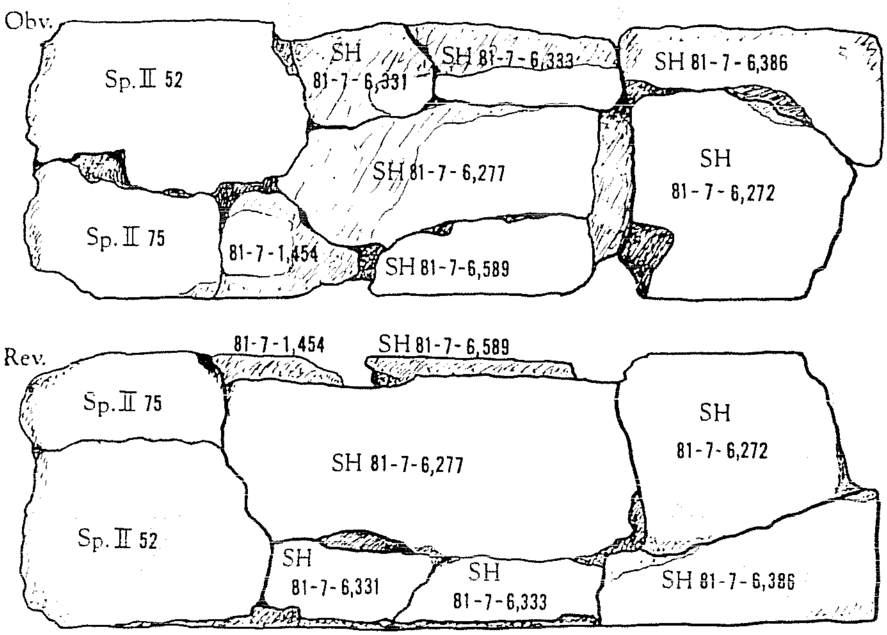
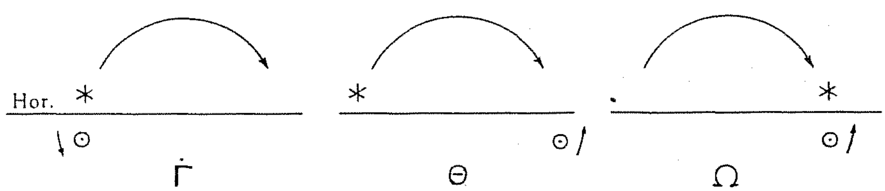


Fig. 1



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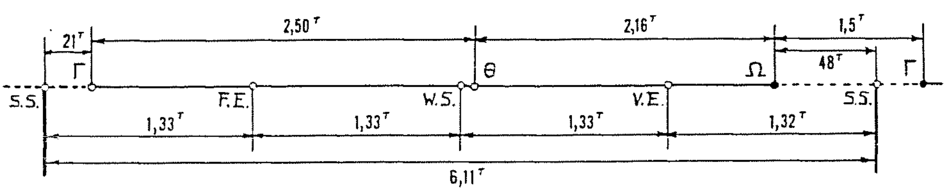
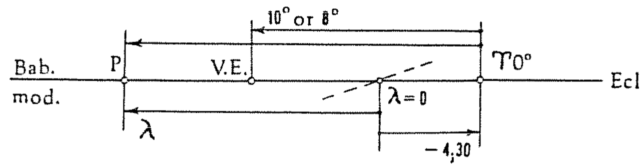


Fig. 3



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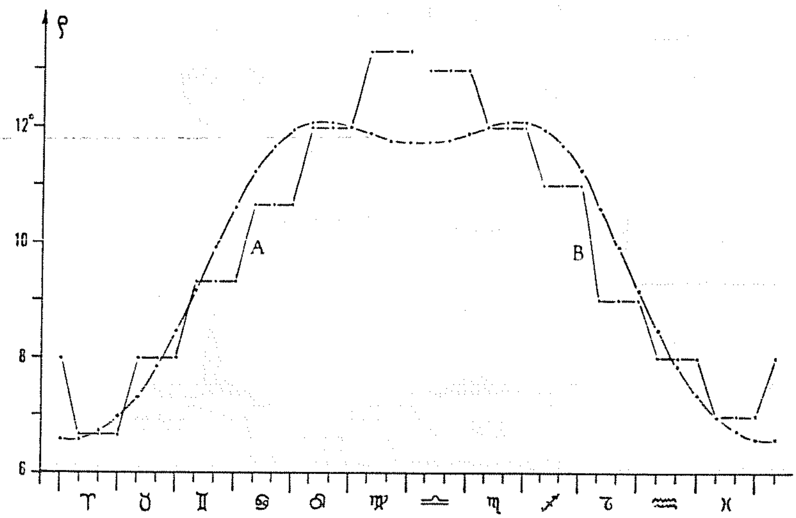

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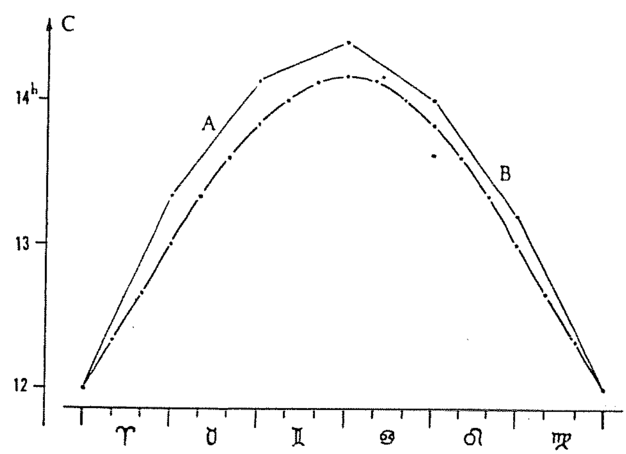

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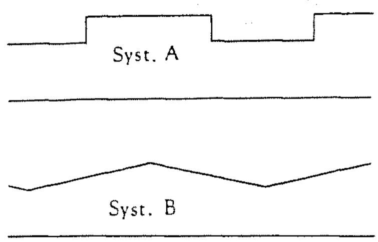

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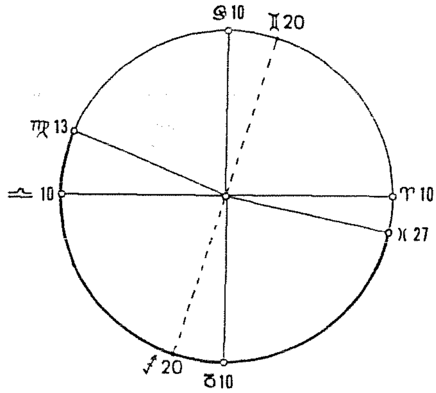

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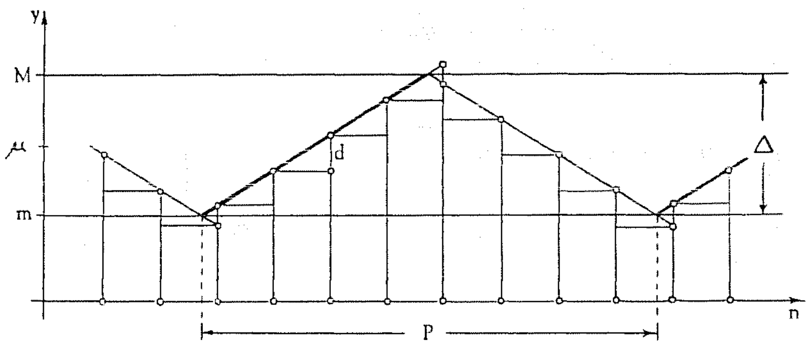

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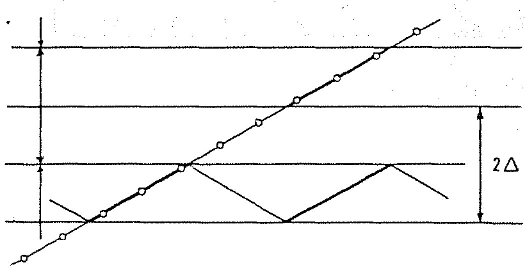

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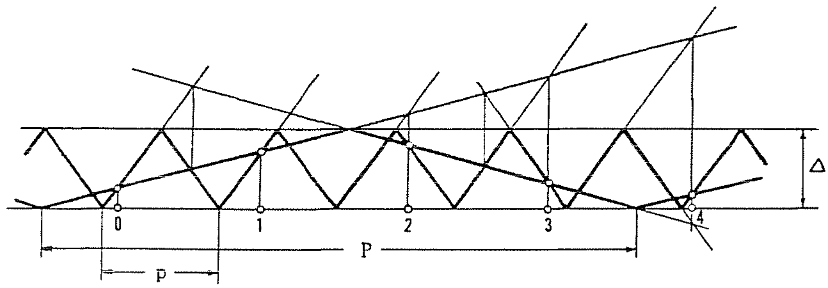


Fig. 11

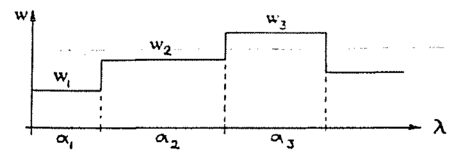


Fig. 12

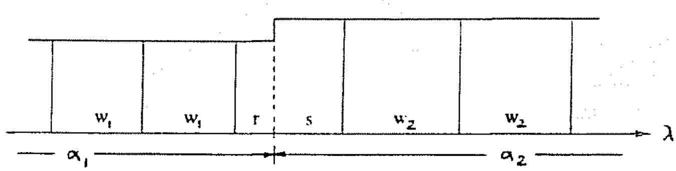


Fig. 13

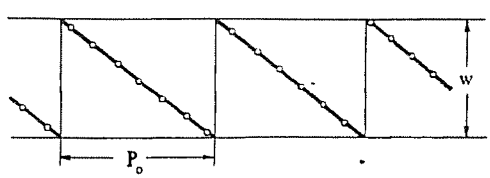


Fig. 14

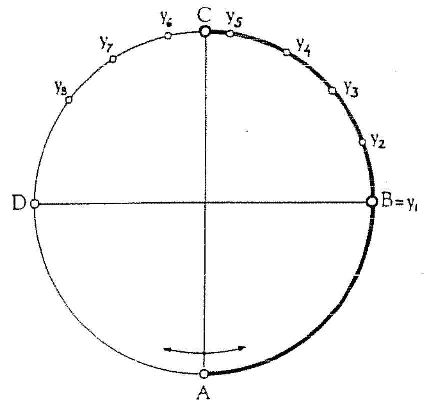


Fig. 15

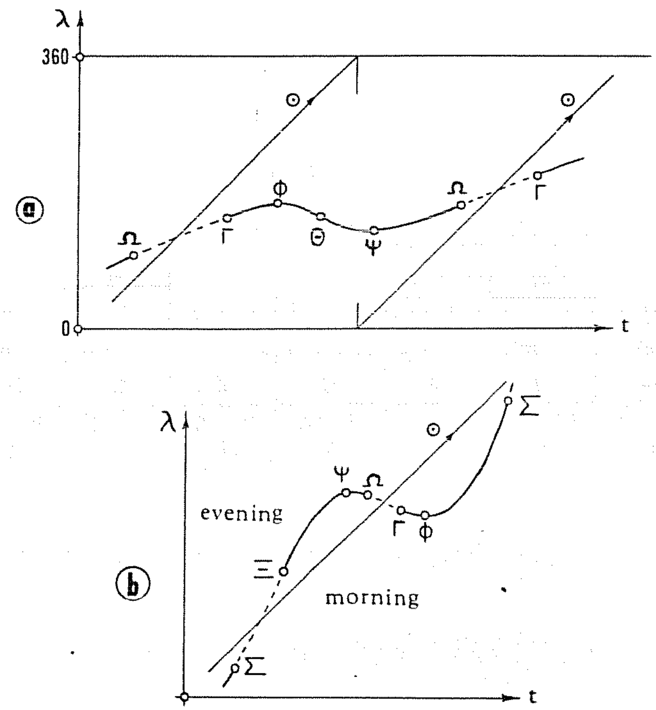


Fig. 16

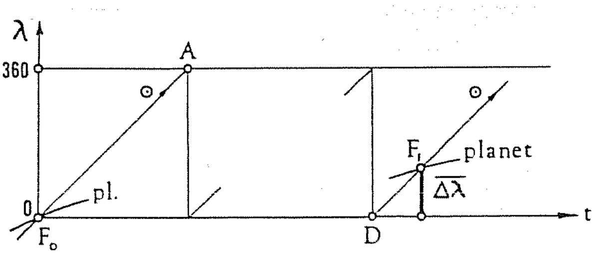


Fig. 17

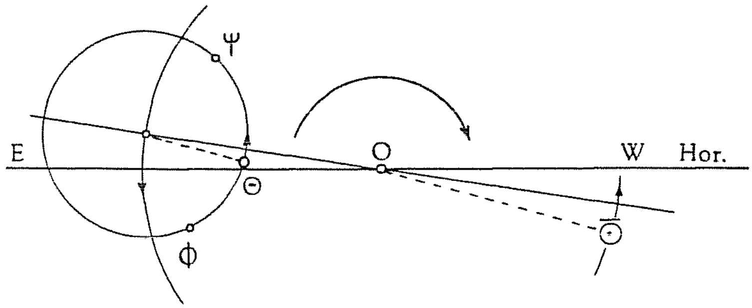


Fig. 18

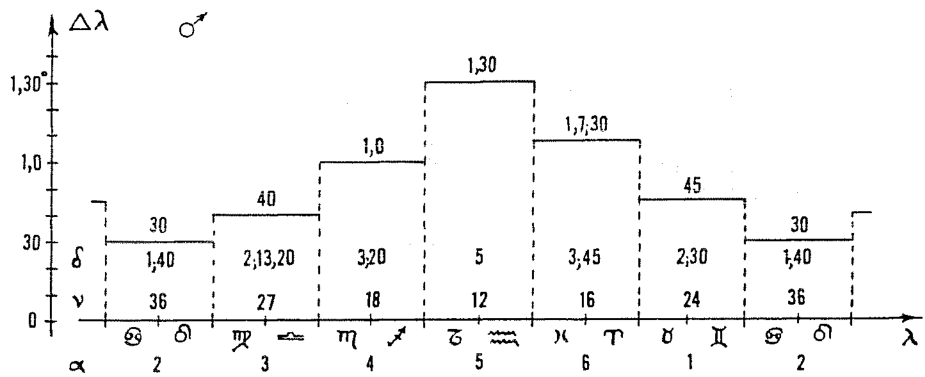


Fig. 19

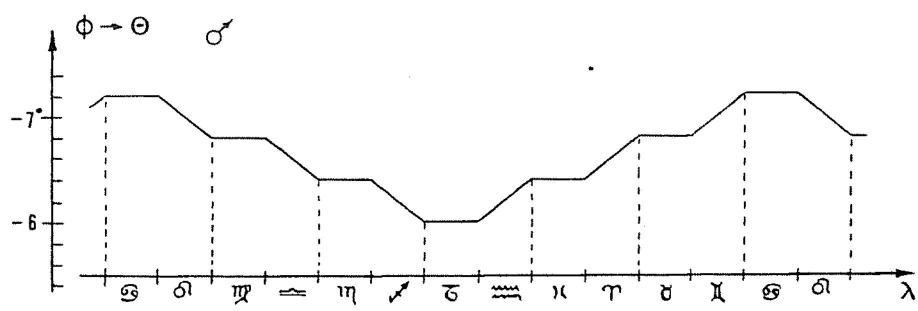


Fig. 20

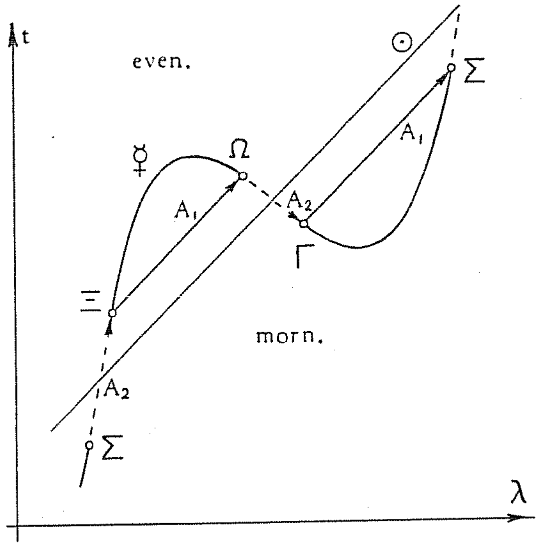


Fig. 21

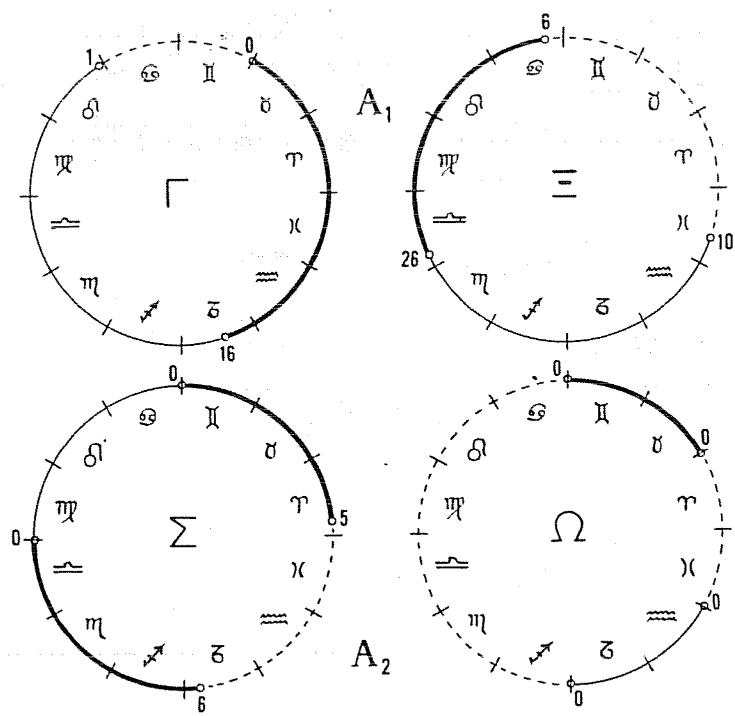
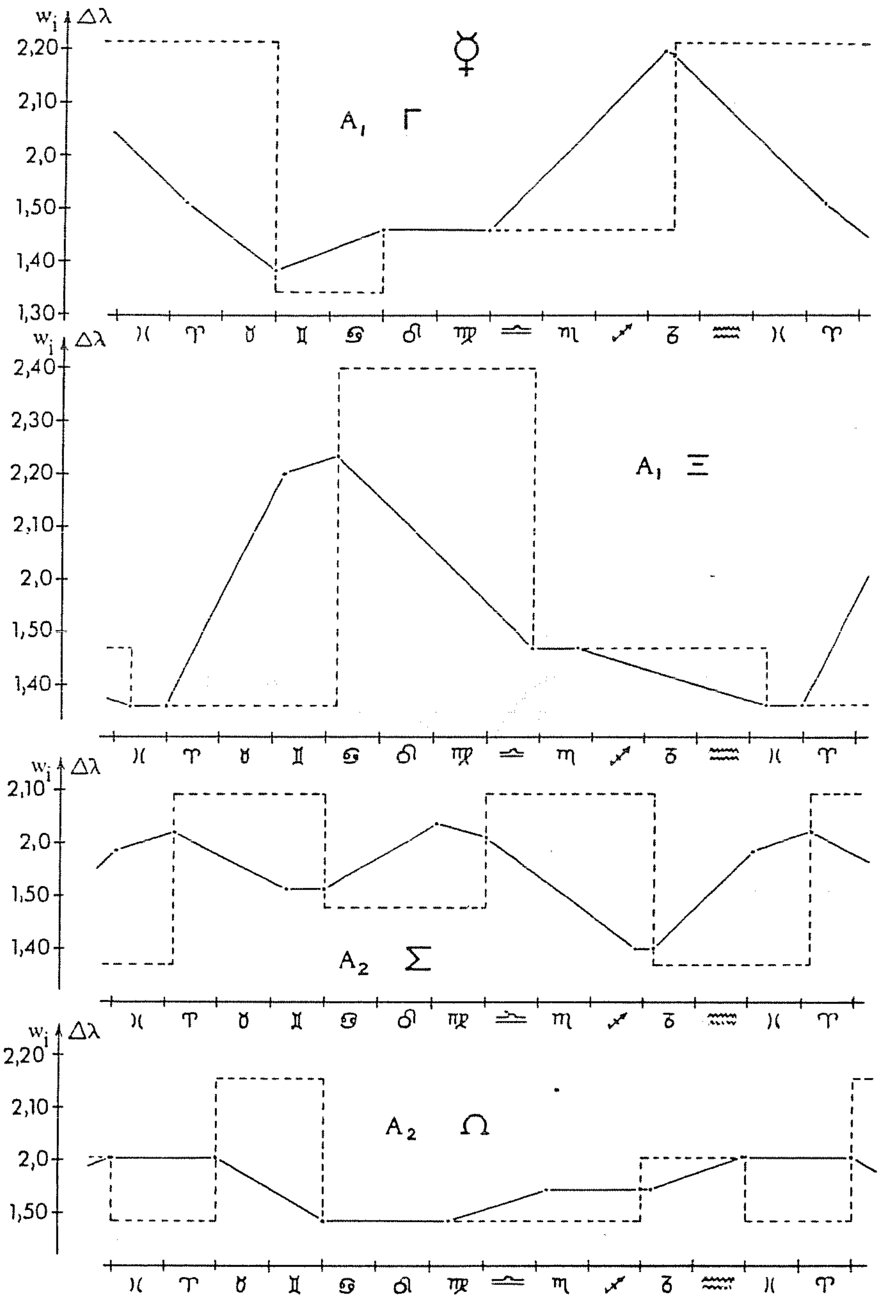


Fig. 22



w_i $\Delta\lambda$
2,20
2,10
2,0
1,50
1,40
1,30
A₁ Γ
☿
♓ ♈ ♉ ♊ ♋ ♌ ♍ ♎ ♏ ♐ ♑ ♒ ♓ ♈
w_i $\Delta\lambda$
2,40
2,30
2,20
2,10
2,0
1,50
1,40
A₁ Ξ
♓ ♈ ♉ ♊ ♋ ♌ ♍ ♎ ♏ ♐ ♑ ♒ ♓ ♈
w_i $\Delta\lambda$
2,10
2,0
1,50
1,40
A₂ Σ
♓ ♈ ♉ ♊ ♋ ♌ ♍ ♎ ♏ ♐ ♑ ♒ ♓ ♈
w_i $\Delta\lambda$
2,20
2,10
2,0
1,50
A₂ Ω
♓ ♈ ♉ ♊ ♋ ♌ ♍ ♎ ♏ ♐ ♑ ♒ ♓ ♈

Fig. 23

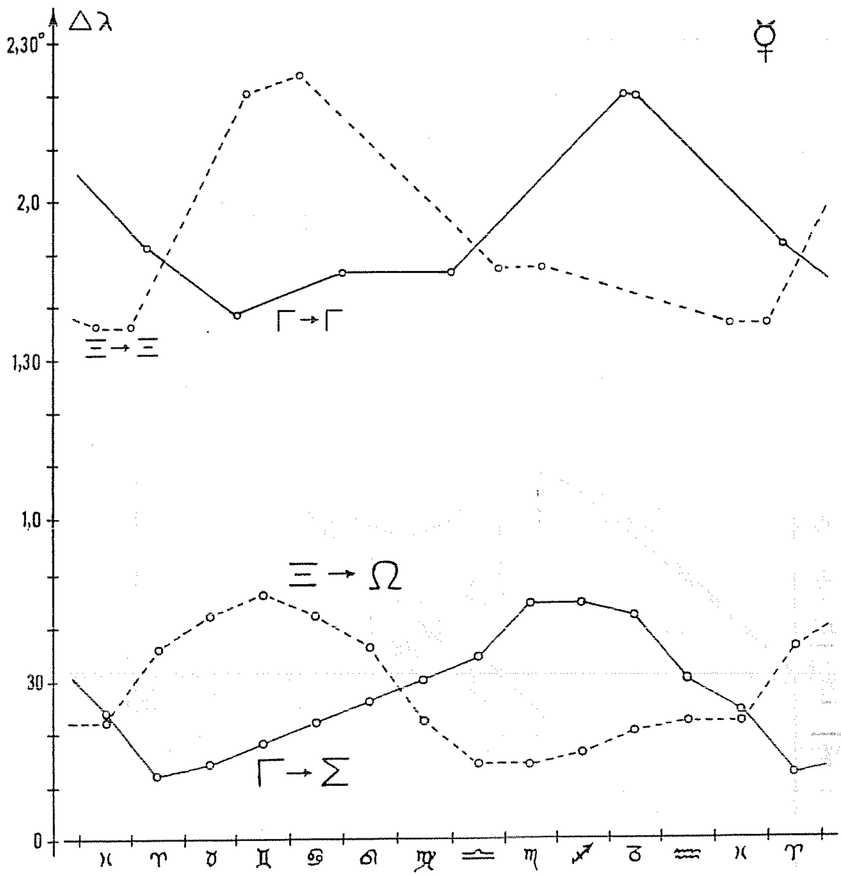


Fig. 24

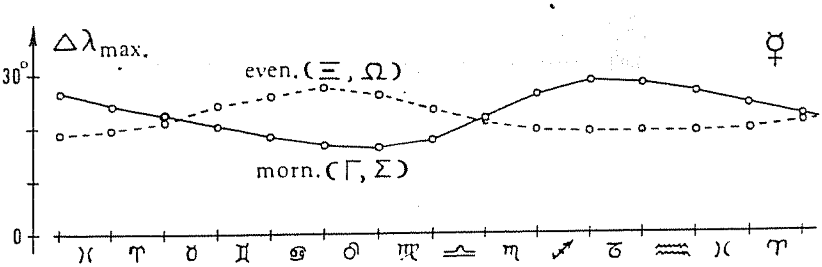


Fig. 25

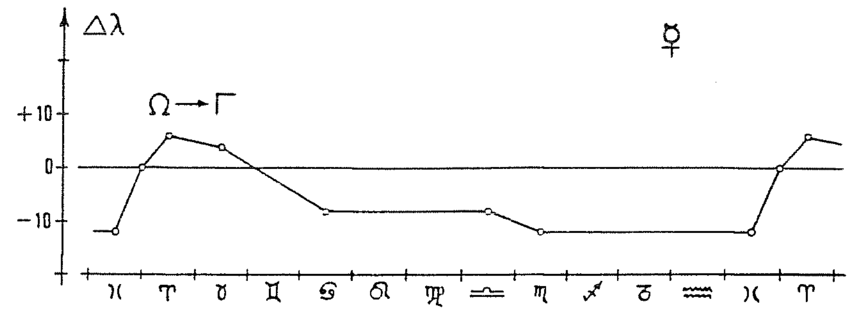


Fig. 26

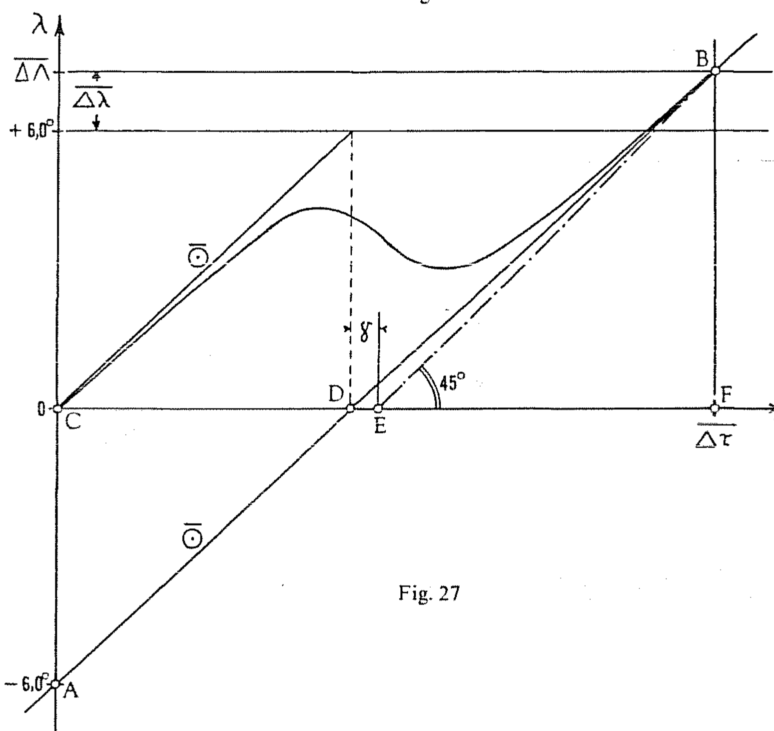


Fig. 27

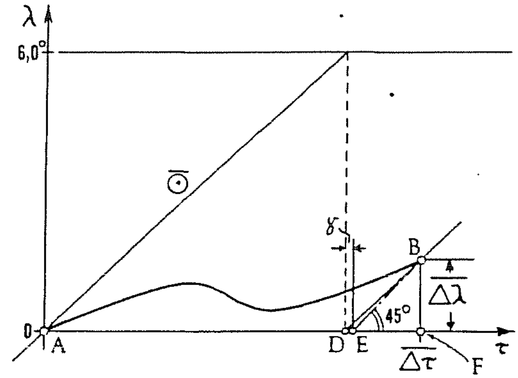
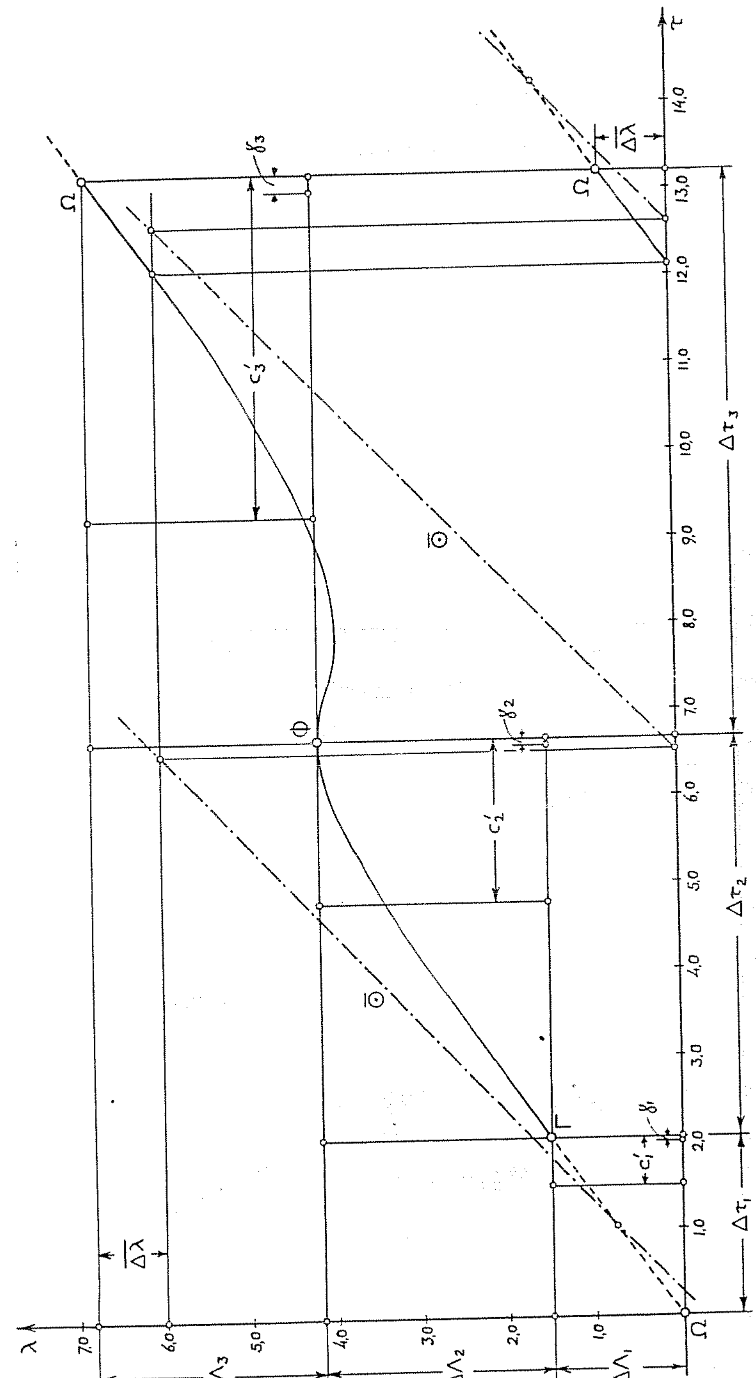


Fig. 28



τ
λ
1,0
2,0
3,0
4,0
5,0
6,0
7,0
8,0
9,0
10,0
11,0
12,0
13,0
14,0
Ω
Φ
Γ
c'_1
c'_2
c'_3
γ_1
γ_2
γ_3
$\Delta\tau_1$
$\Delta\tau_2$
$\Delta\tau_3$
$\Delta\Lambda_1$
$\Delta\Lambda_2$
$\Delta\Lambda_3$
$\overline{\Delta\lambda}$
$\bar{\odot}$

Fig. 29

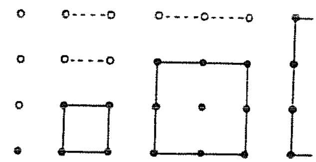


Fig. 30

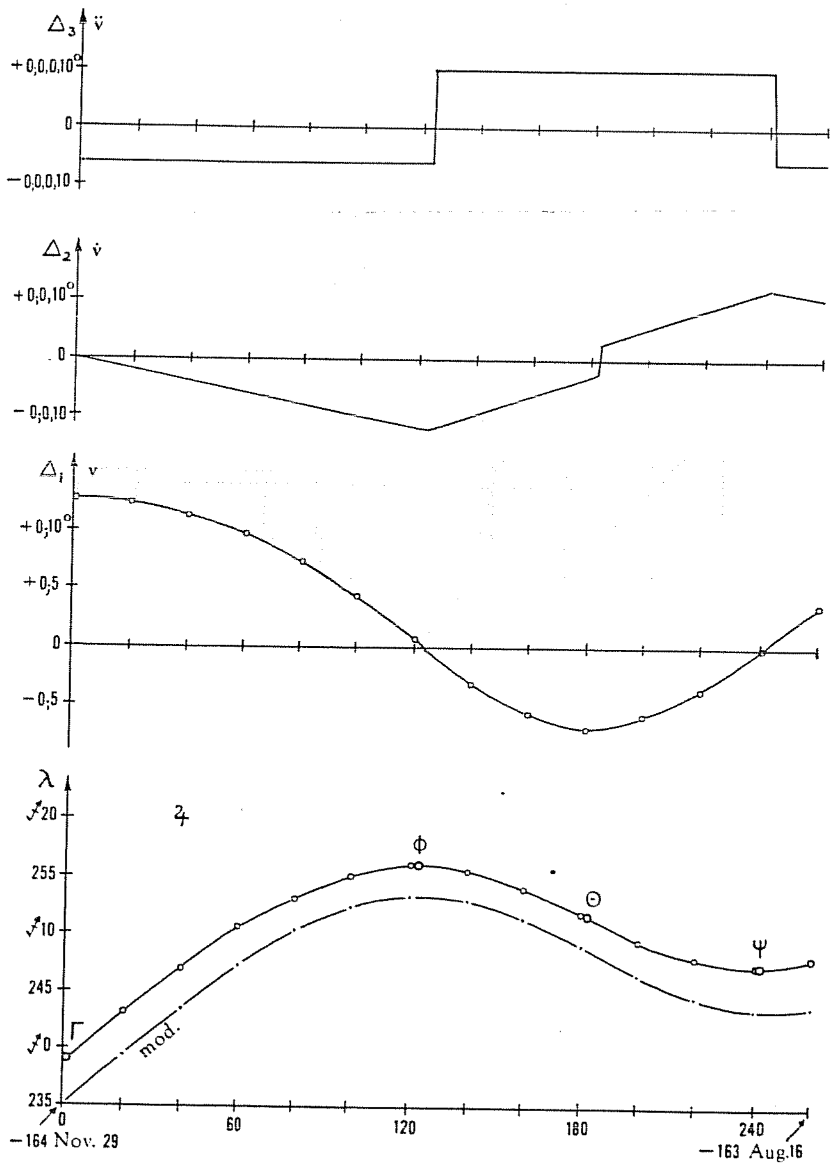

Δ_3
$\ddot{v}$
+ 0;0,0,10°
0
− 0;0,0,10
Δ_2
$\dot{v}$
+ 0;0,10°
0
− 0;0,10
Δ_1
v
+ 0;10°
+ 0;5
0
− 0;5
λ
♐20
♃
Φ
255
Θ
♐10
Ψ
245
Γ
♐0
mod.
235
0
60
120
180
240
−164 Nov. 29
−163 Aug.16

Fig. 31

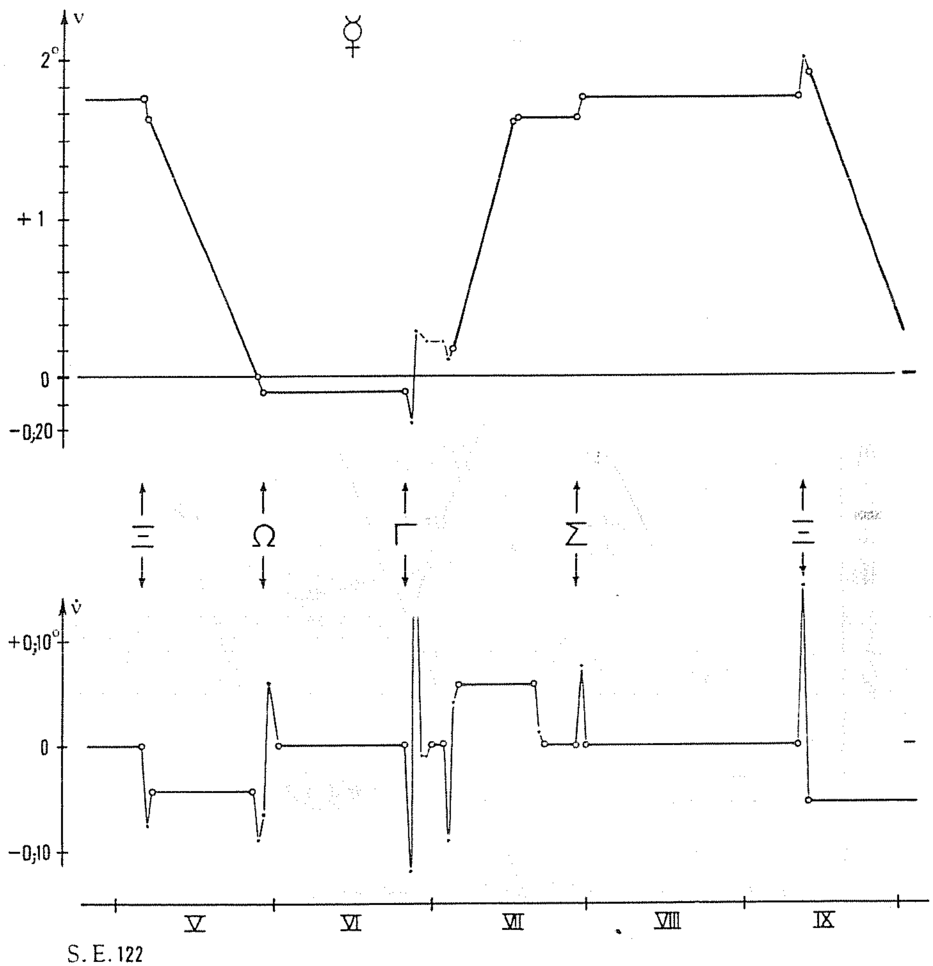


Fig. 32

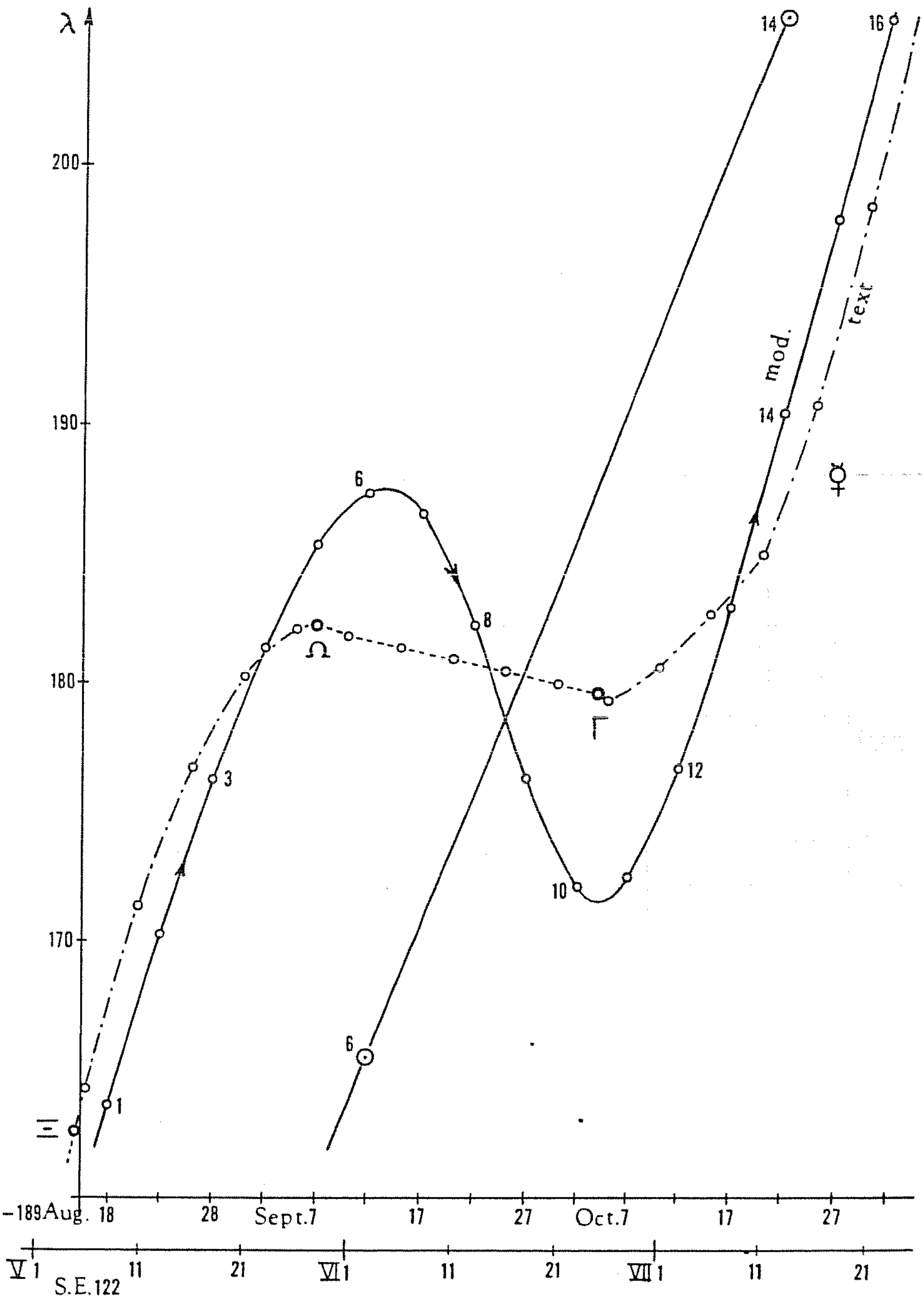
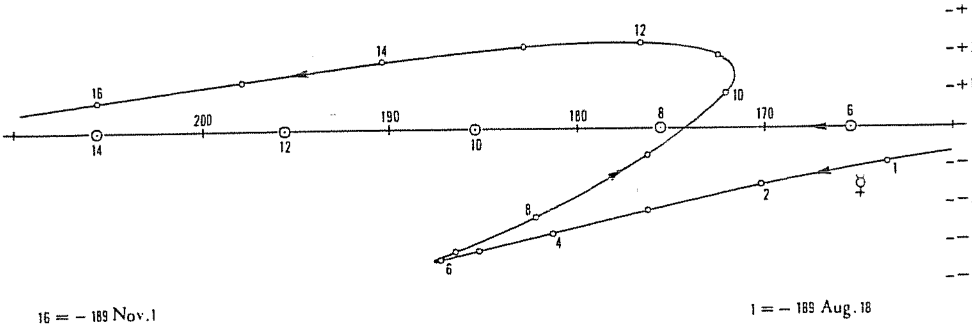
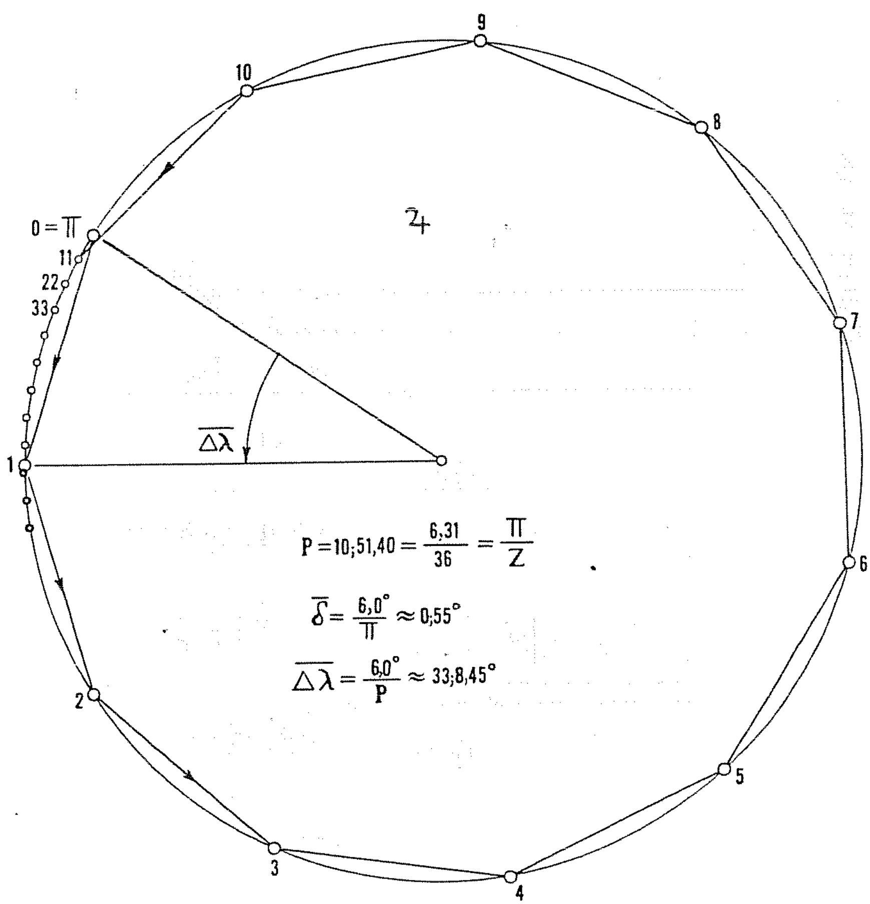

λ
200
190
180
170
14
16
6
8
3
1
10
12
14
6
mod.
text
☿
Ω
Γ
Ξ
-189 Aug. 18
28
Sept. 7
17
27
Oct. 7
17
27
V 1
S.E. 122
11
21
VI 1
11
21
VII 1
11
21

Fig. 33



12
14
16
10
200
190
180
8
170
6
14
12
10
+3
+2
+1
0
−1
−2
−3
−4
1
2
8
4
6
16 = − 189 Nov. 1
1 = − 189 Aug. 18

Fig. 34



9
10
8
0 = Π
11
22
33
7
$\overline{\Delta\lambda}$
1
$P = 10;51,40 = \frac{6,31}{36} = \frac{\Pi}{Z}$
$\overline{\delta} = \frac{6,0^\circ}{\Pi} \approx 0;55^\circ$
$\overline{\Delta\lambda} = \frac{6,0^\circ}{P} \approx 33;8,45^\circ$
6
2
5
3
4

Fig. 35

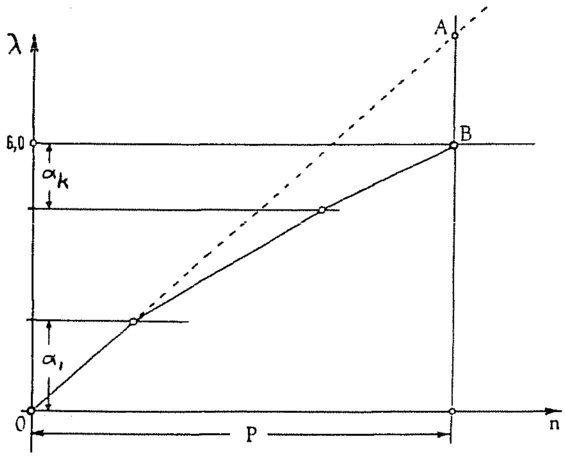


Fig. 36

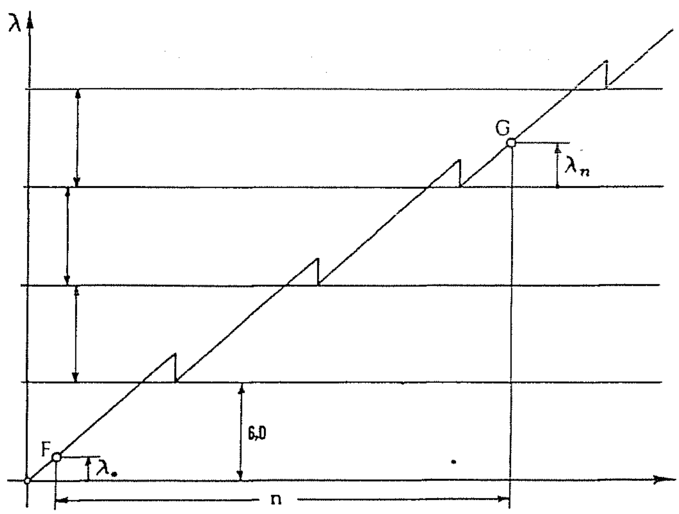


Fig. 37

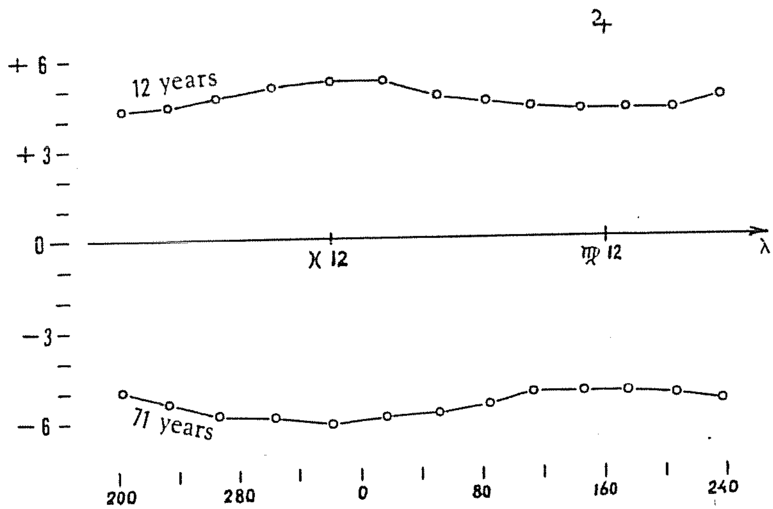

♃
+ 6
12 years
+ 3
0
♓ 12
♍ 12
λ
− 3
− 6
71 years
200
280
0
80
160
240

Fig. 38

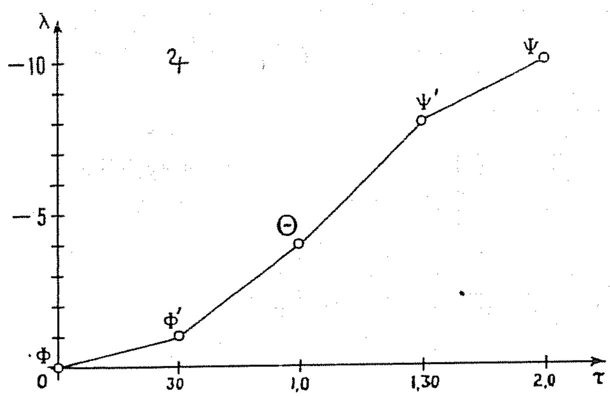

λ
−10
♃
Ψ
Ψ′
− 5
Θ
Φ′
Φ
0
30
1,0
1,30
2,0
τ

Fig. 39

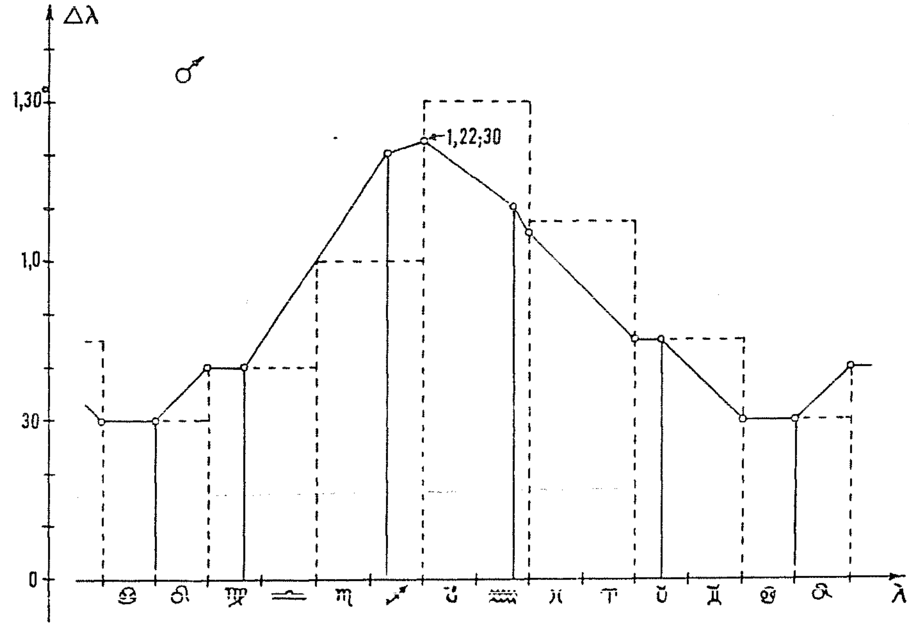


Fig. 40

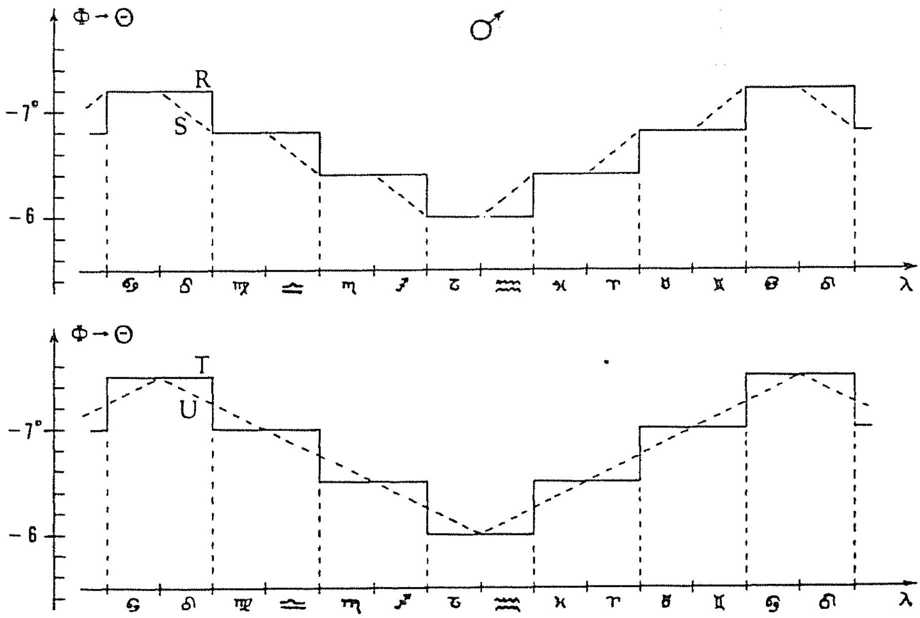


Fig. 41

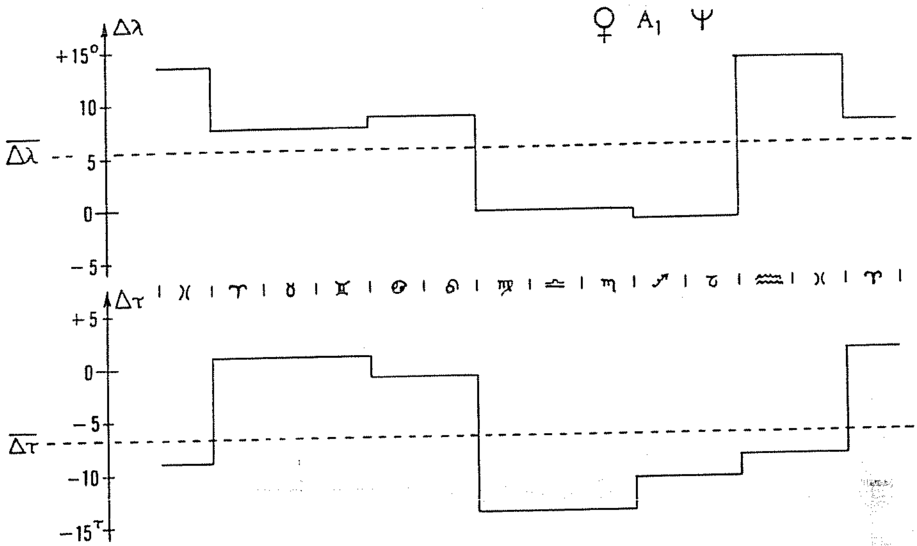

♀ A_1 Ψ
$\Delta\lambda$
+15°
10
5
0
−5
$\overline{\Delta\lambda}$
$\Delta\tau$
+5
0
−5
−10
-15^τ
$\overline{\Delta\tau}$
♓ ♈ ♉ ♊ ♋ ♌ ♍ ♎ ♏ ♐ ♑ ♒ ♓ ♈

Fig. 42

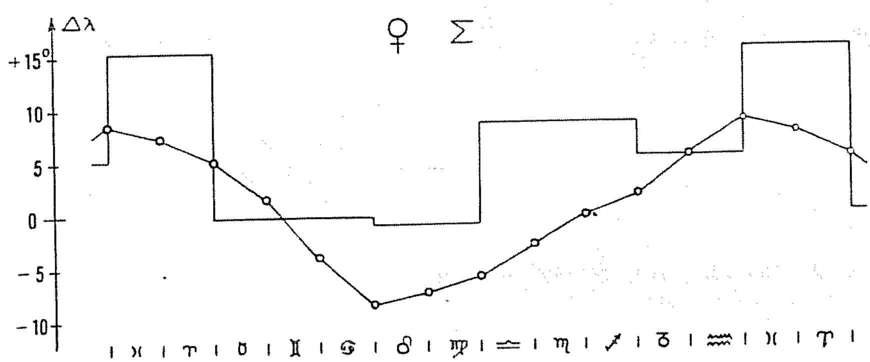

♀ Σ
$\Delta\lambda$
+15°
10
5
0
−5
−10
♓ ♈ ♉ ♊ ♋ ♌ ♍ ♎ ♏ ♐ ♑ ♒ ♓ ♈

Fig. 43

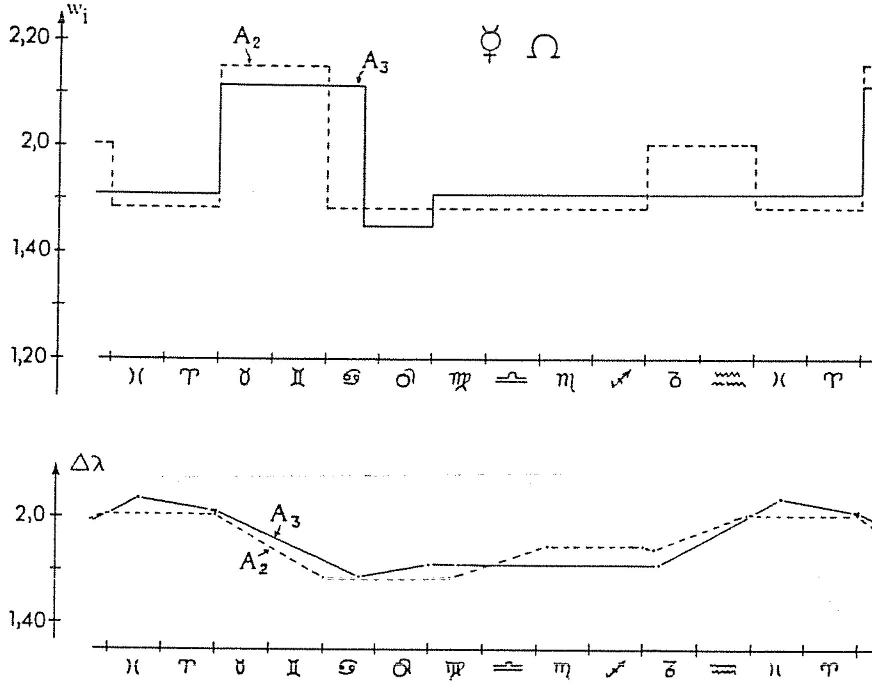


Fig. 44

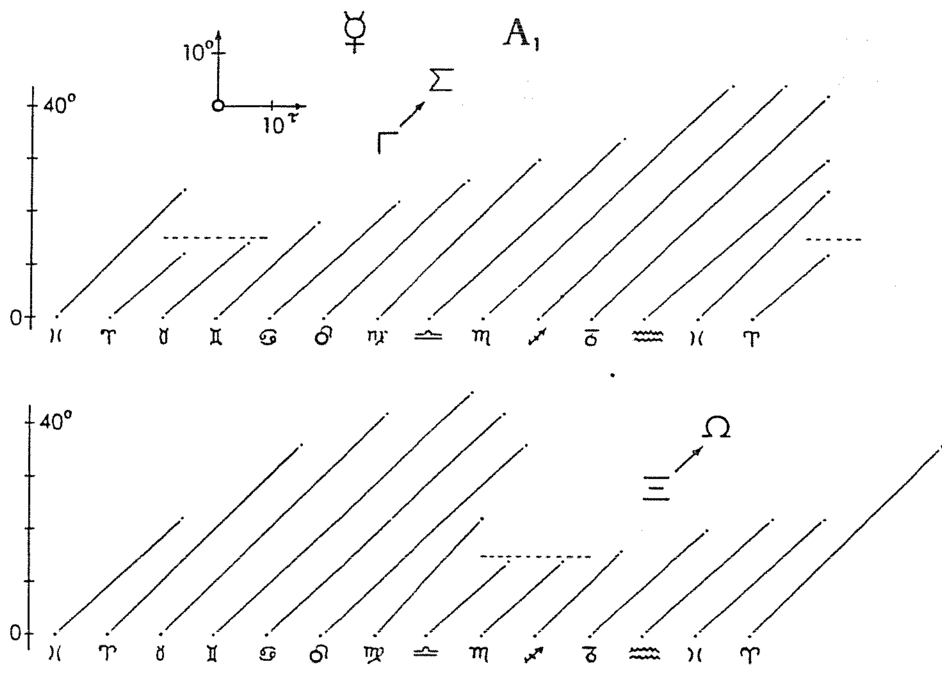


Fig. 45

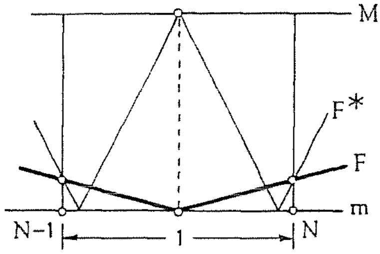


Fig. 46

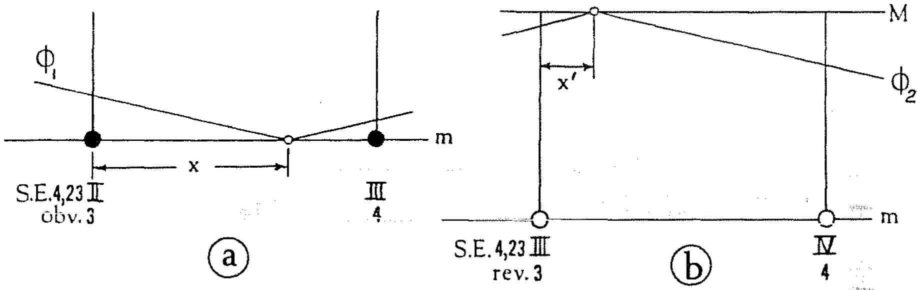


Fig. 47

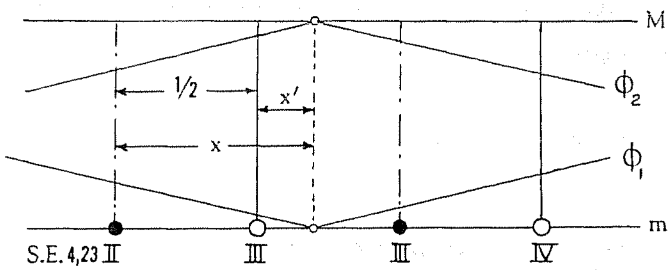


Fig. 48

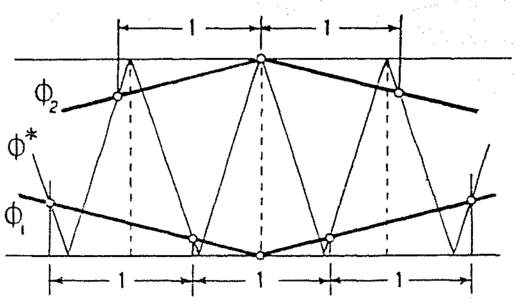


Fig. 49

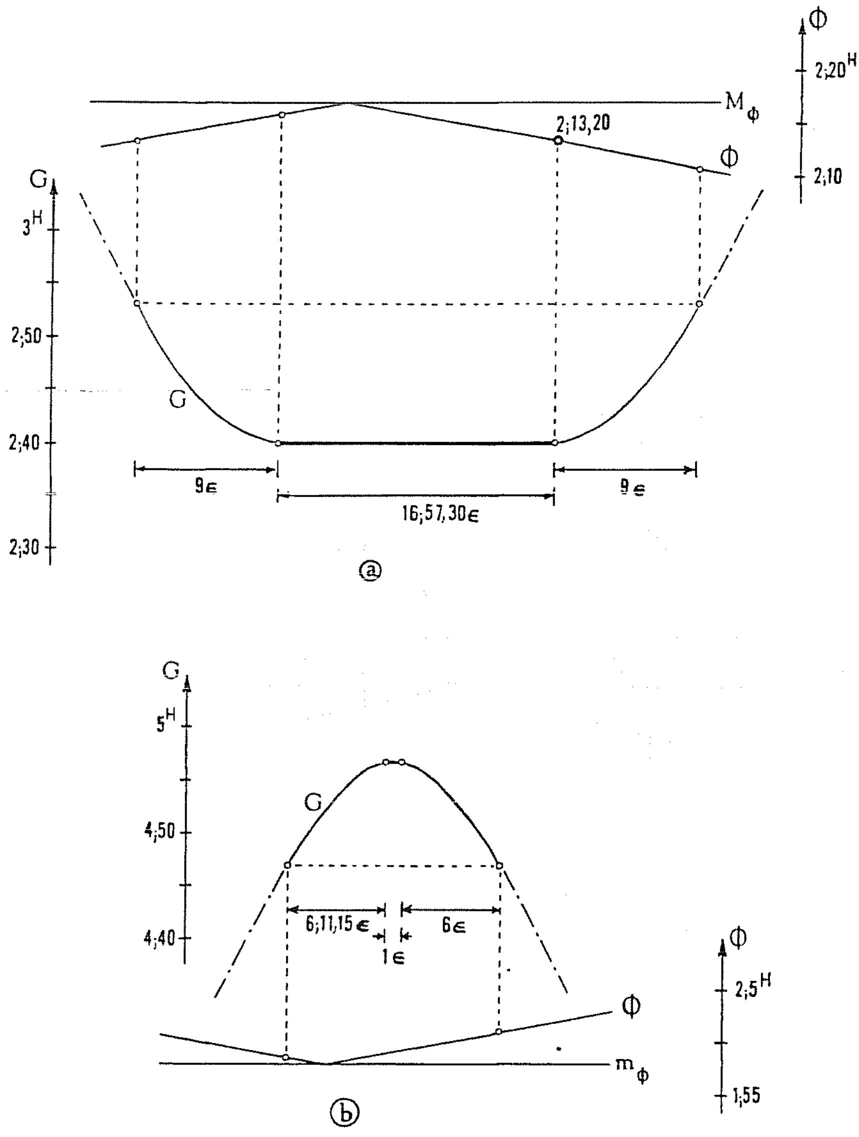
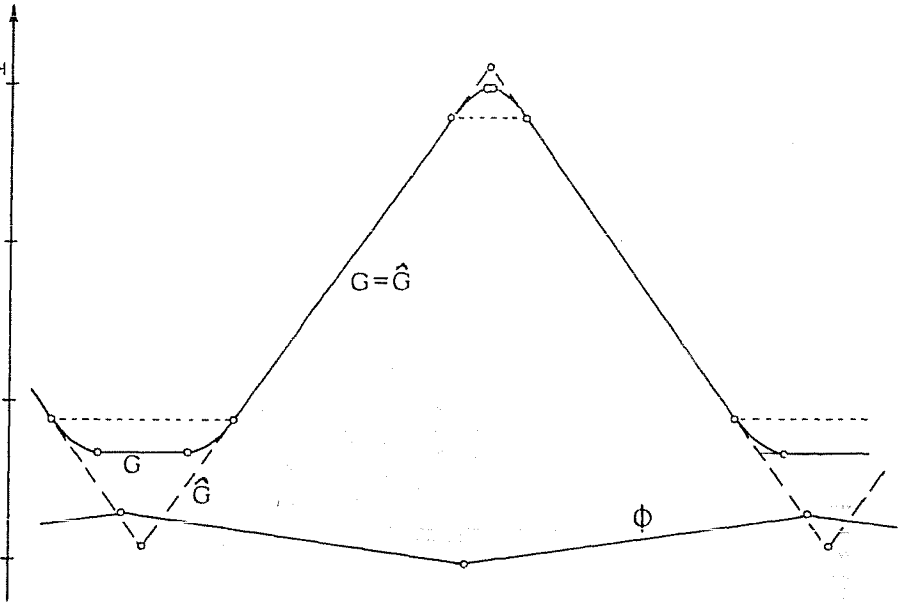
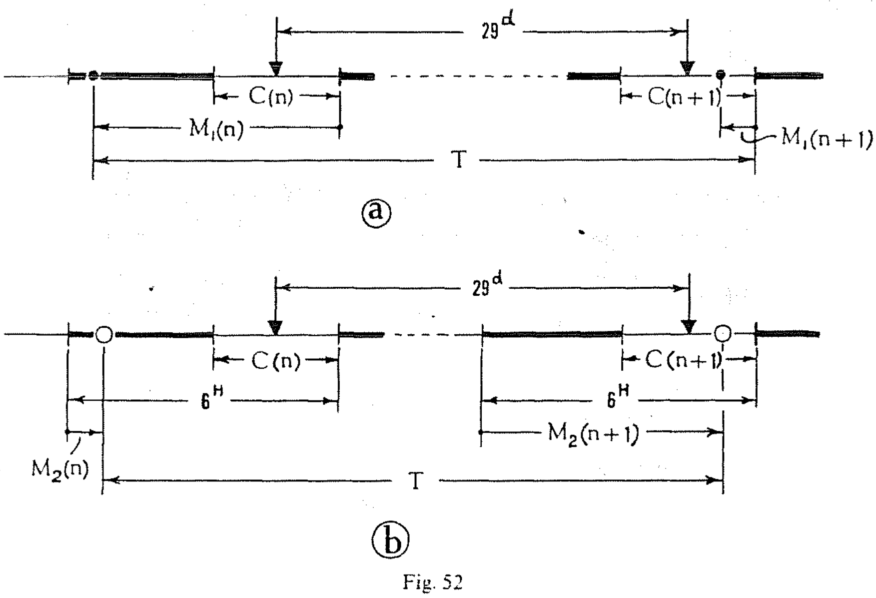

Φ
2;20^H
M_Φ
2;13,20
Φ
2;10
G
3^H
2;50
G
2;40
2;30
9ε
16;57,30ε
9ε
ⓐ
G
5^H
G
4;50
4;40
6;11,15ε
6ε
1ε
Φ
2;5^H
Φ
m_Φ
1;55
ⓑ

Fig. 50



5^H
4
3
2
$G=\hat{G}$
G
$\hat{G}$
ϕ

Fig. 51



29^d
C(n)
$M_1(n)$
C(n+1)
$M_1(n+1)$
T
ⓐ
29^d
C(n)
6^H
$M_2(n)$
C(n+1)
6^H
$M_2(n+1)$
T
ⓑ

Fig. 52

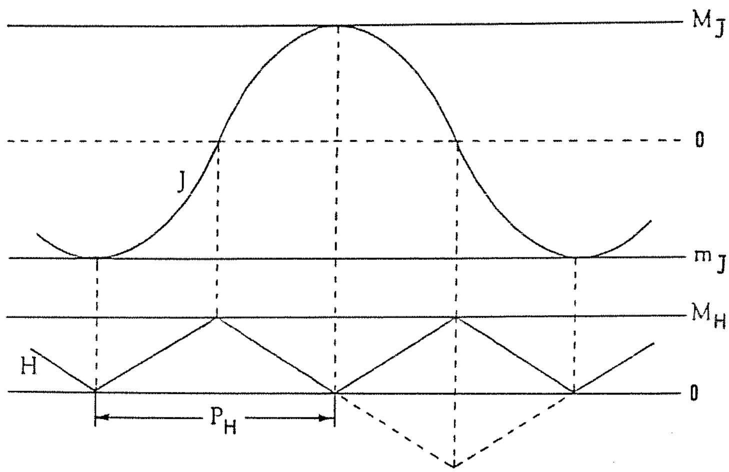


Fig. 53

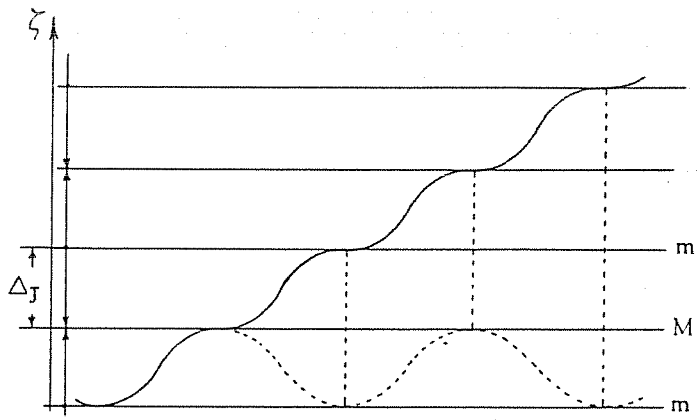


Fig. 54

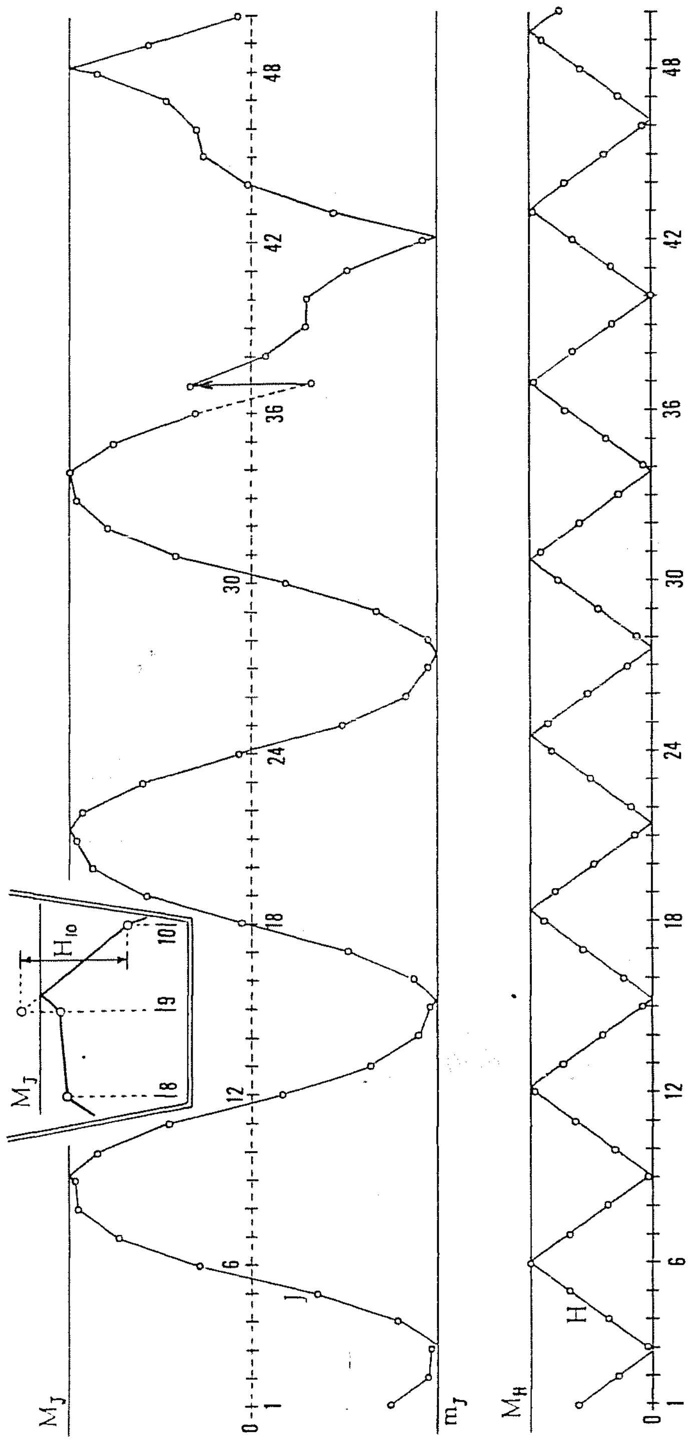
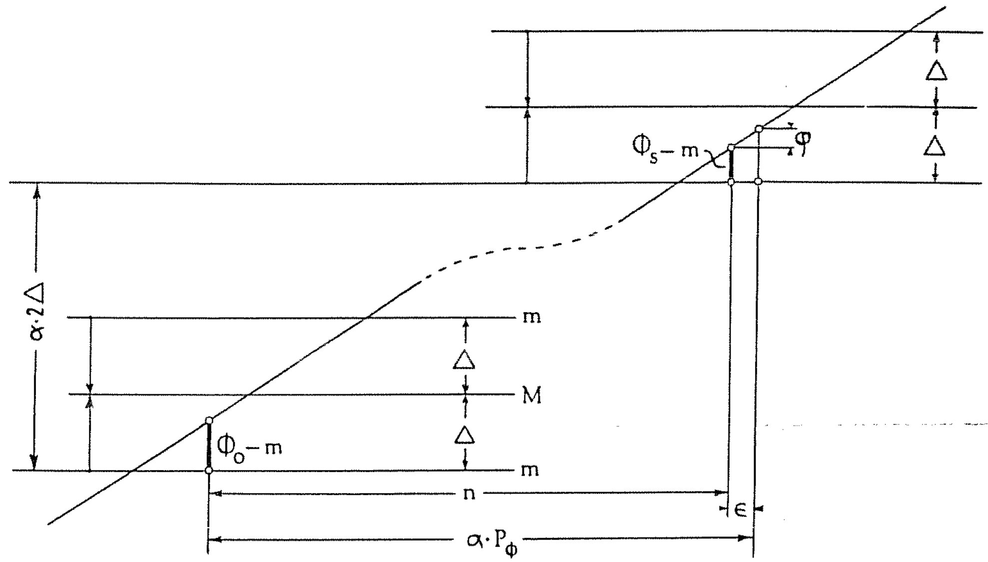
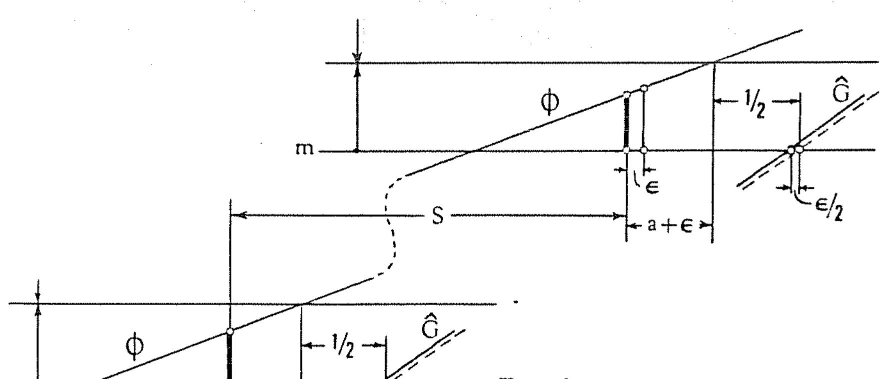


Fig. 55



Δ
Δ
$\Phi_s - m$
φ
$\alpha \cdot 2\Delta$
m
Δ
M
Δ
$\Phi_o - m$
m
n
ε
$\alpha \cdot P_\phi$

Fig. 56



ϕ
m
1/2
Ĝ
ε
ε/2
S
a + ε
ϕ
1/2
Ĝ
m
a
ε/2

Fig. 57

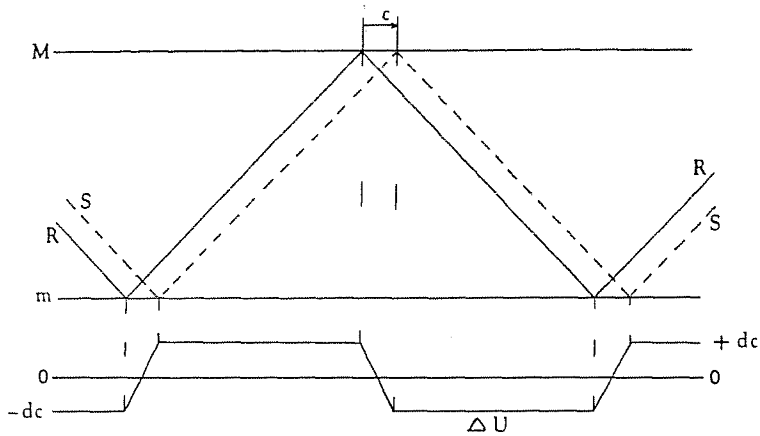


Fig. 58

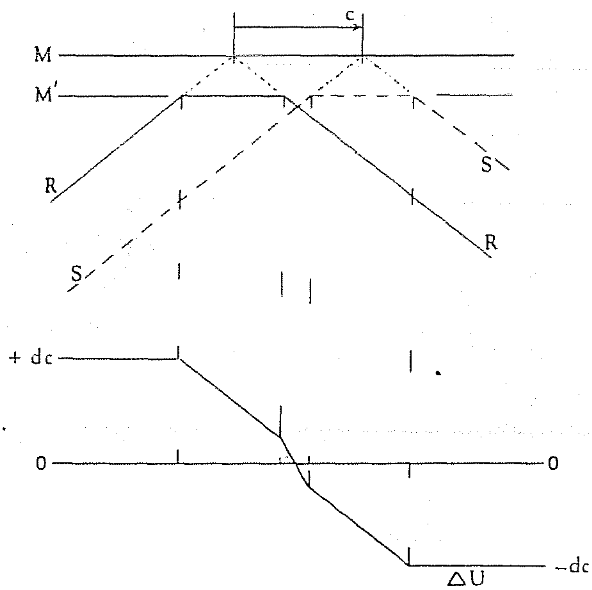


Fig. 59

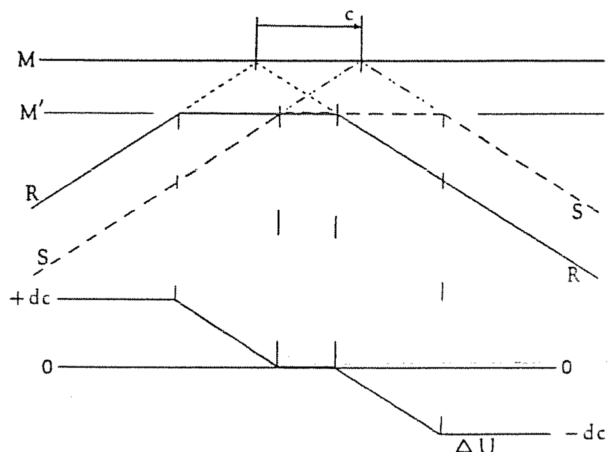

c
M
M′
R
S
S
R
+dc
0
0
−dc
ΔU

Fig. 60

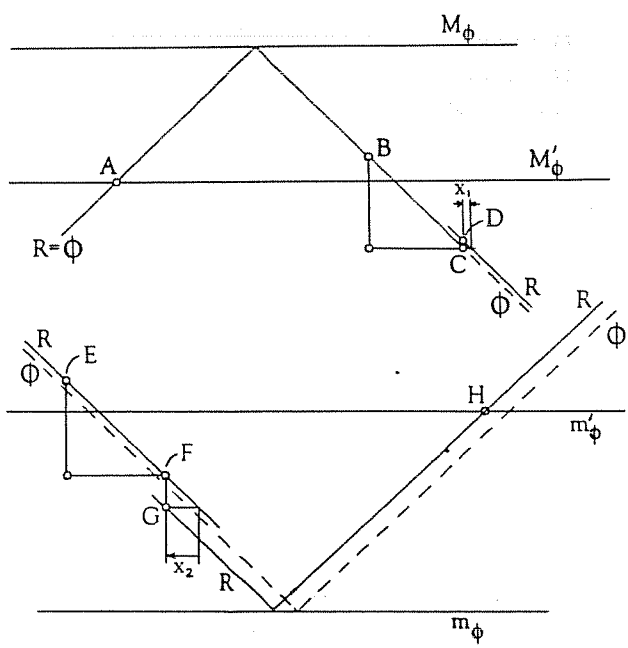

M_ϕ
A
B
M'_ϕ
x_1
D
R=Φ
C
R
Φ
R
Φ
R
E
Φ
H
m'_ϕ
F
G
x_2
R
m_ϕ

Fig. 61

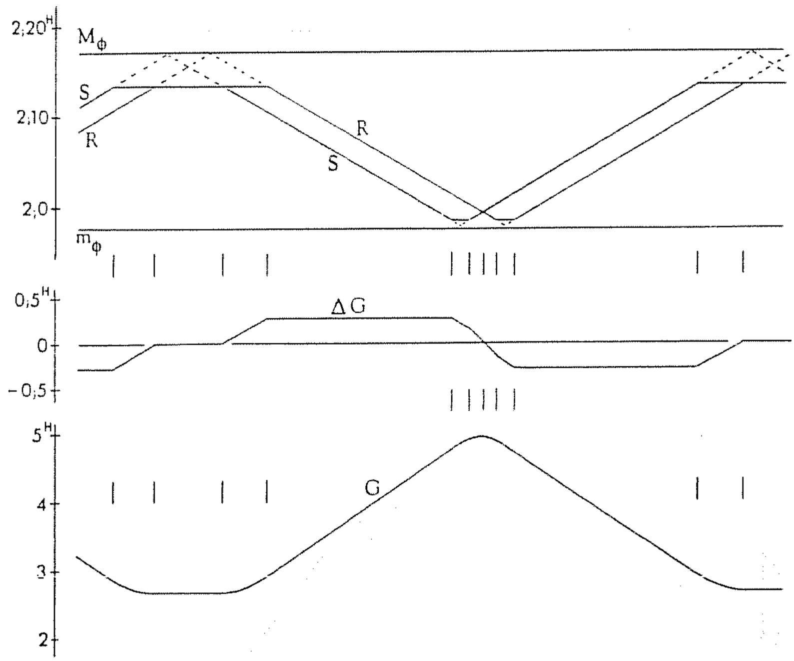


Fig. 62

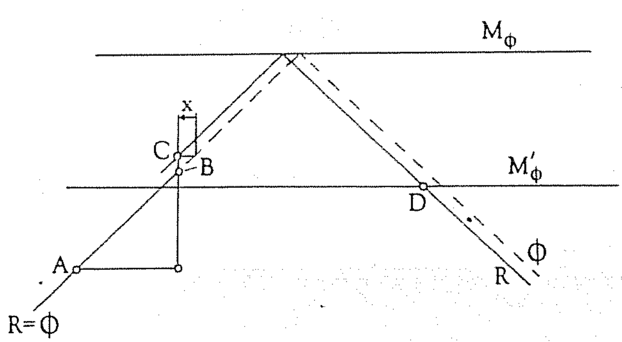


Fig. 63

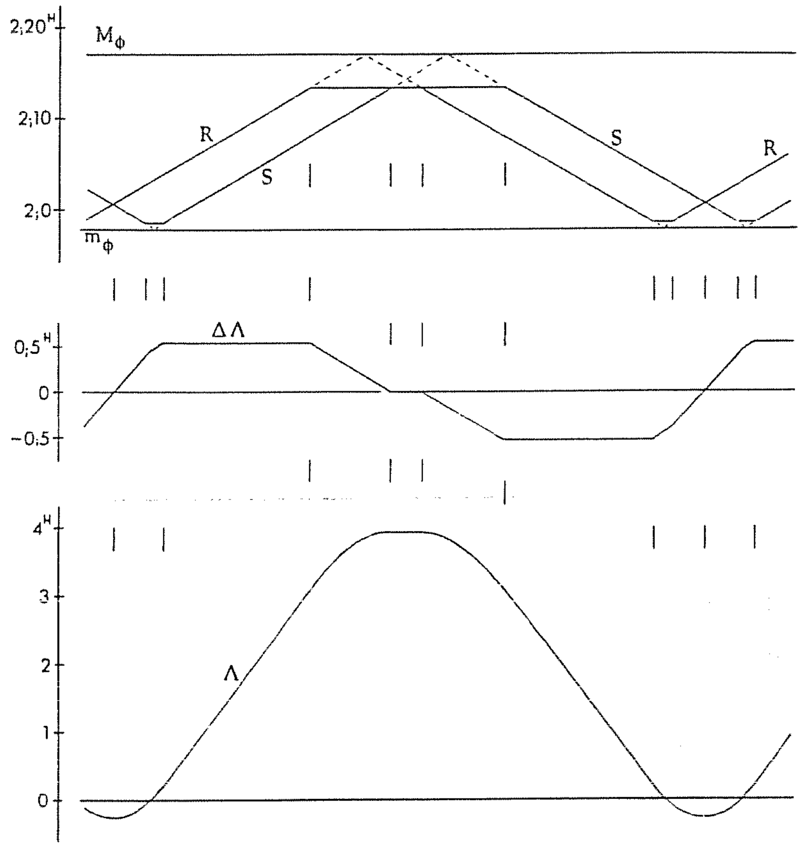


Fig. 64

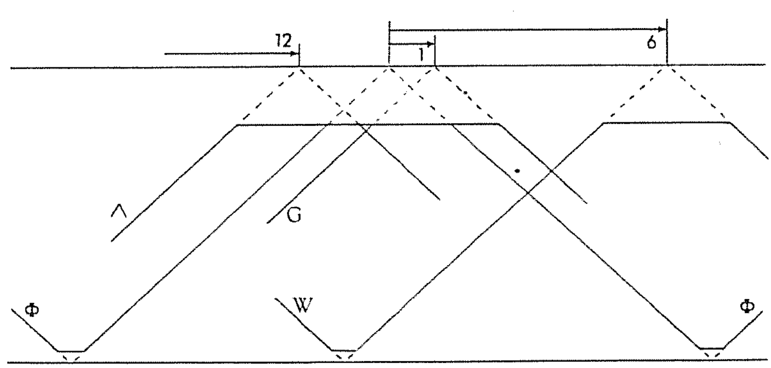


Fig. 65

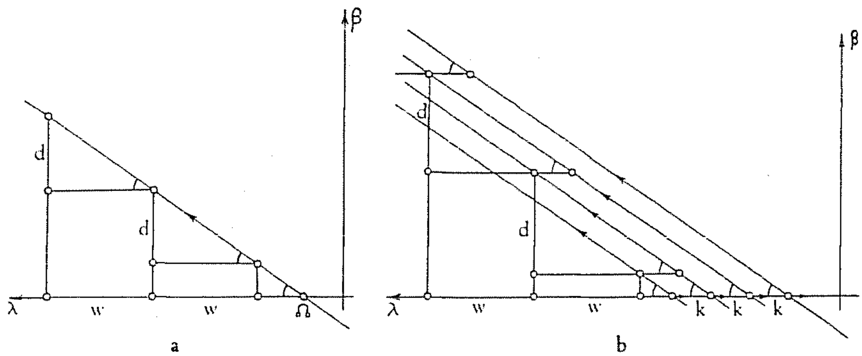


Fig. 66

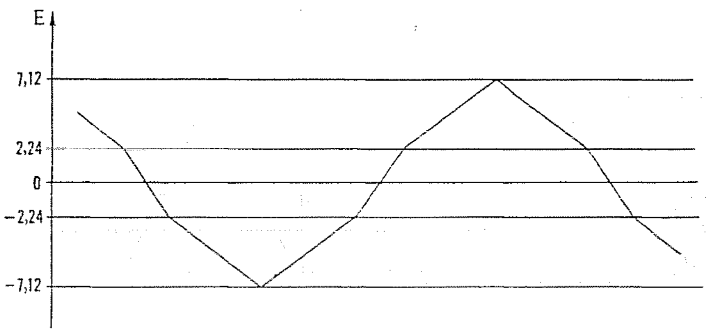


Fig. 67

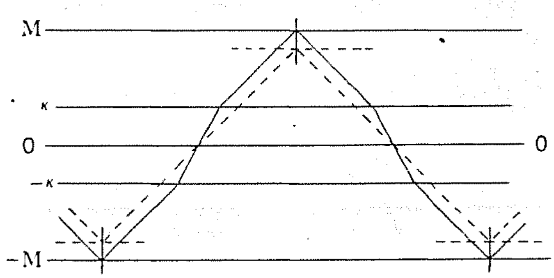


Fig. 68

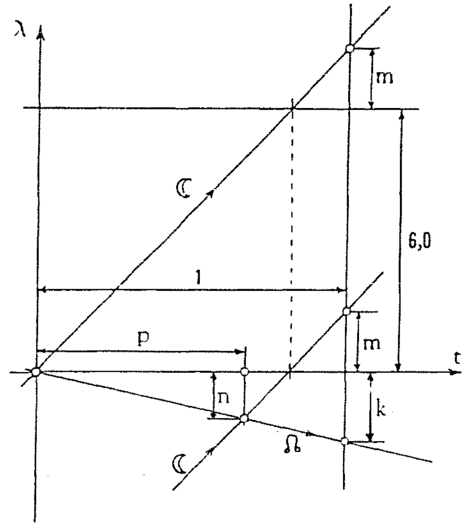
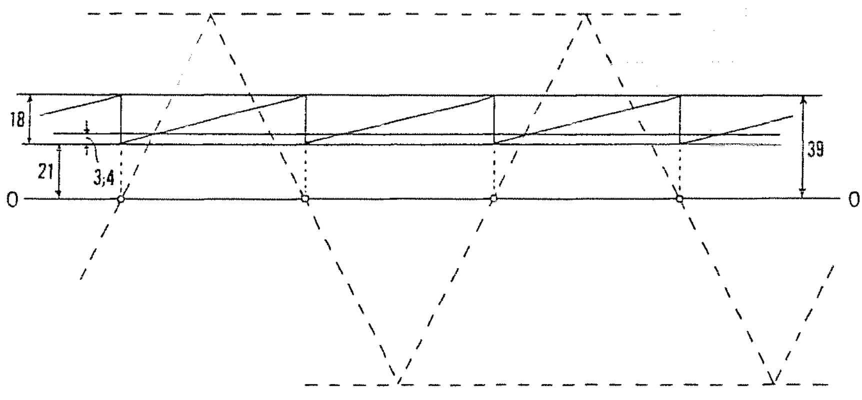


Fig. 69



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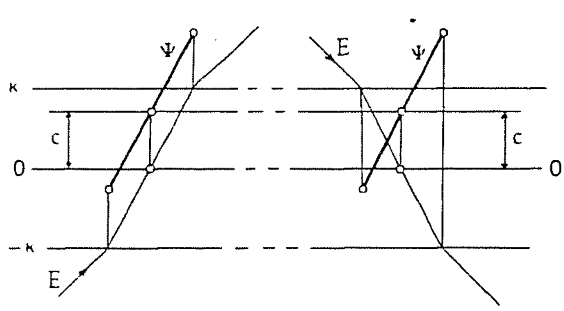

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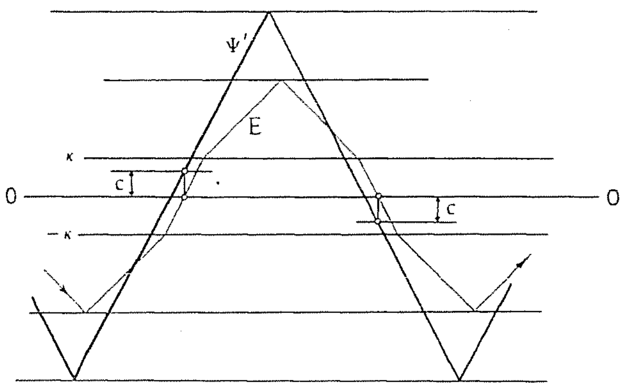

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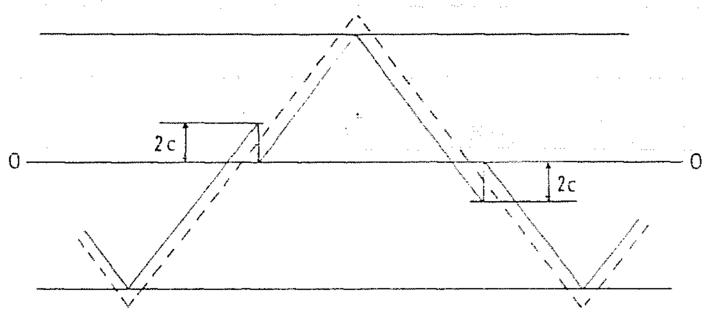

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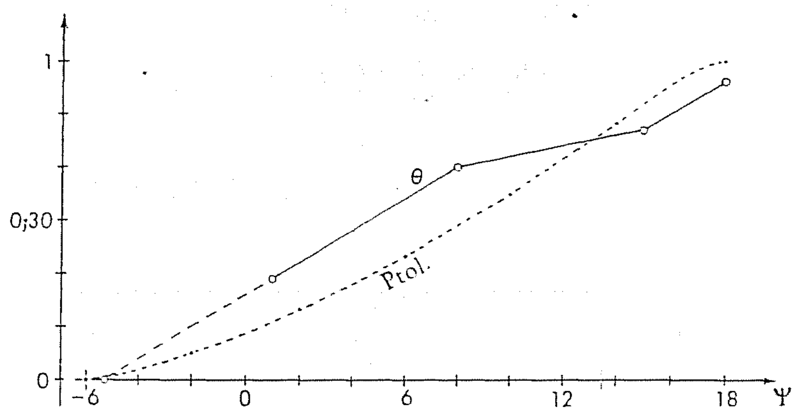

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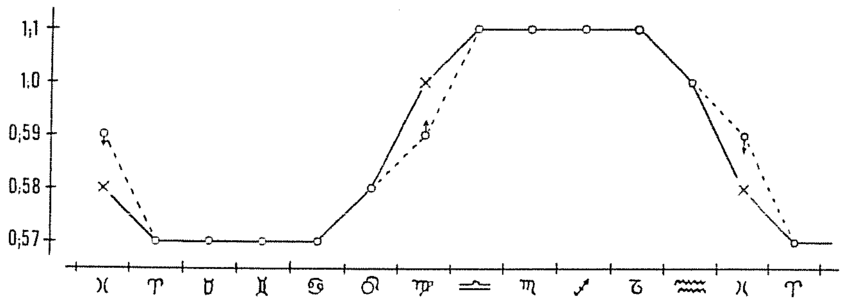

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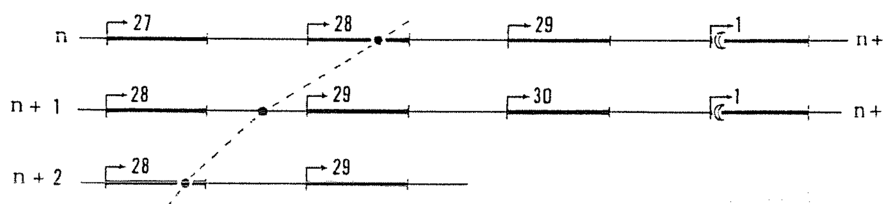

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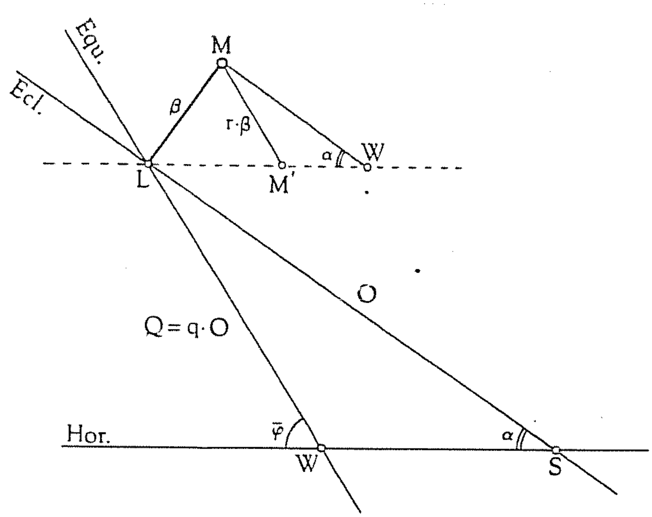

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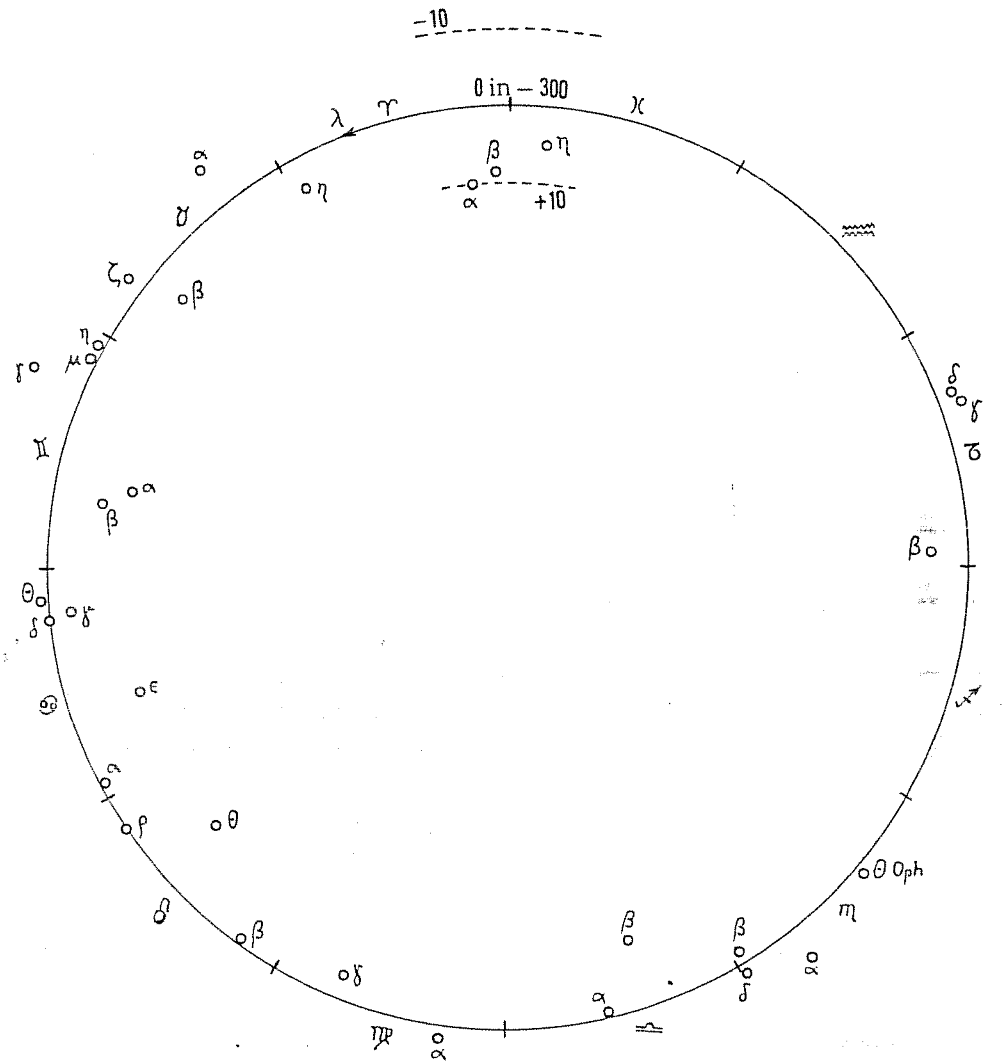


Fig. 78

Figures to Book IV

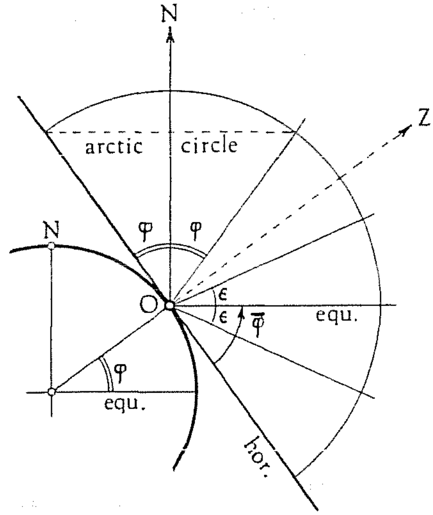


Fig. 1

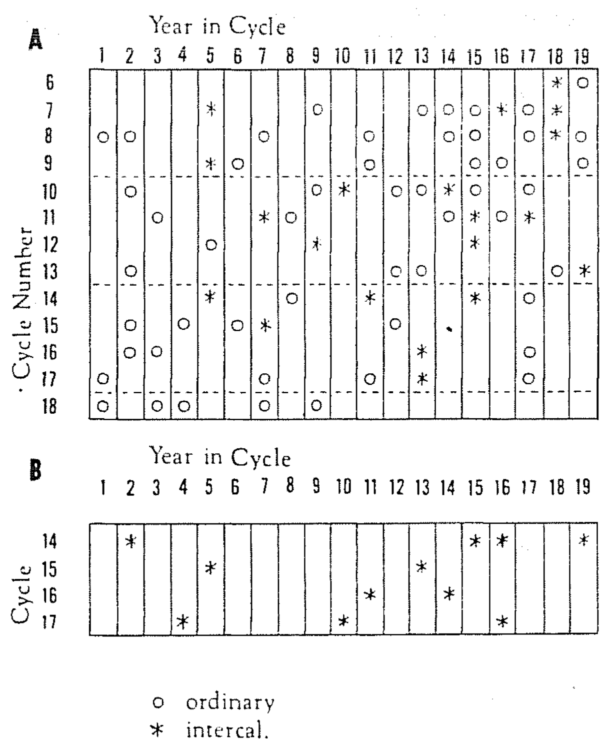


Fig. 2

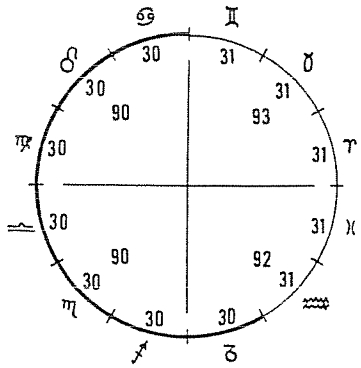


Fig. 3

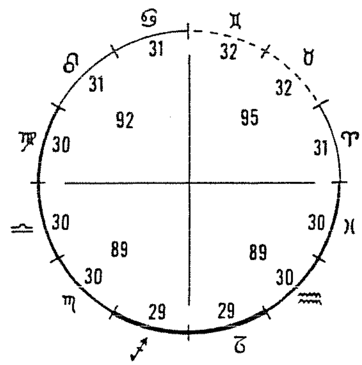
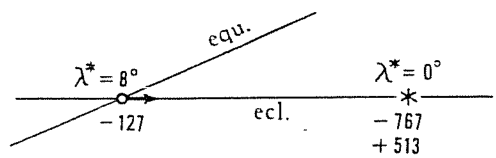


Fig. 4



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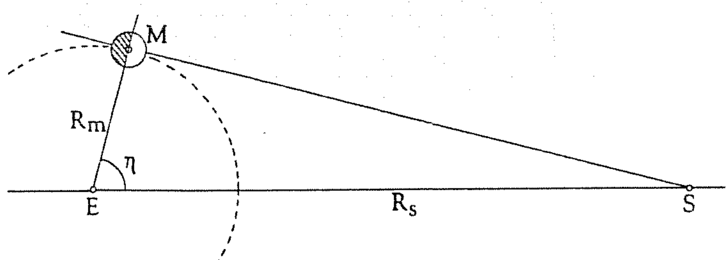

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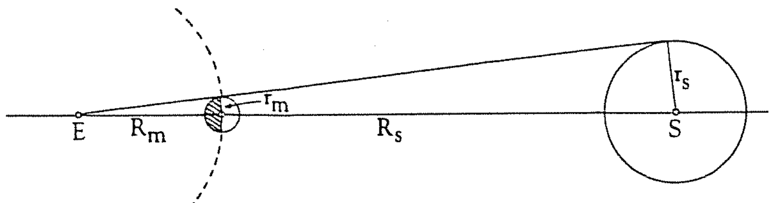

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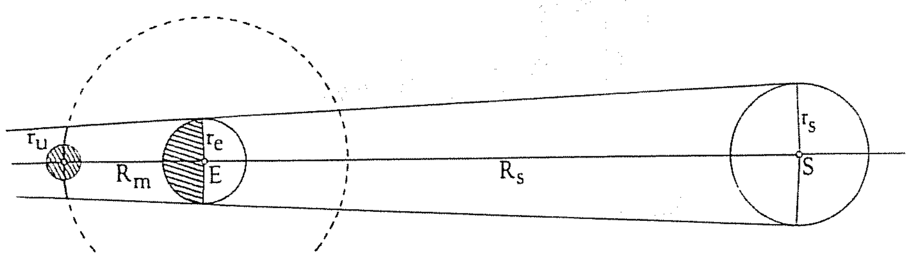

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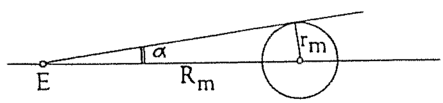

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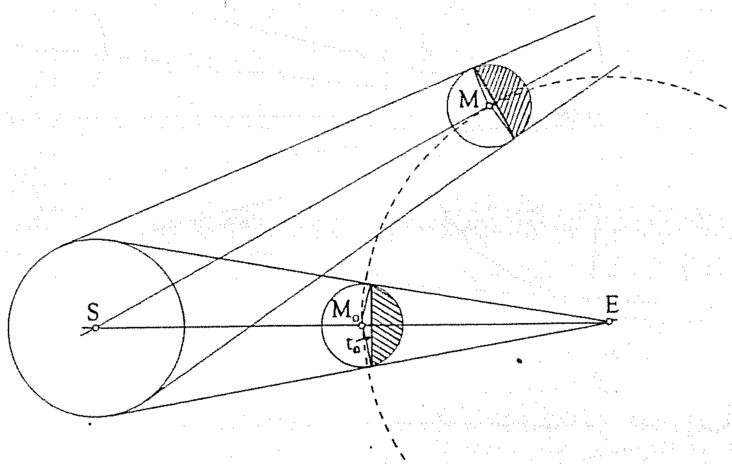


Fig. 10

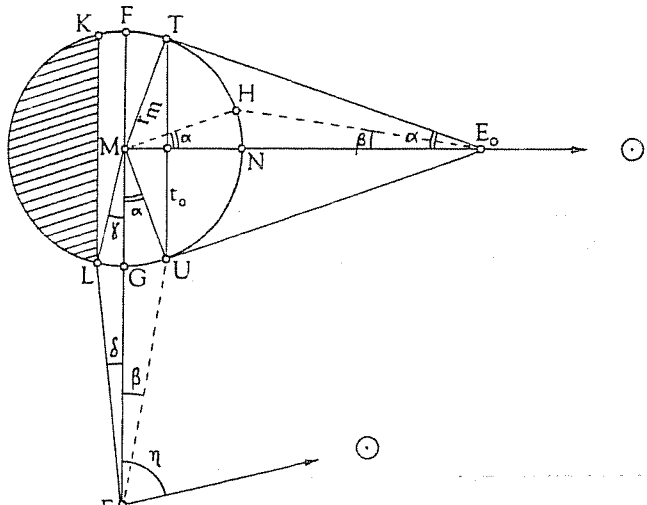


Fig. 11

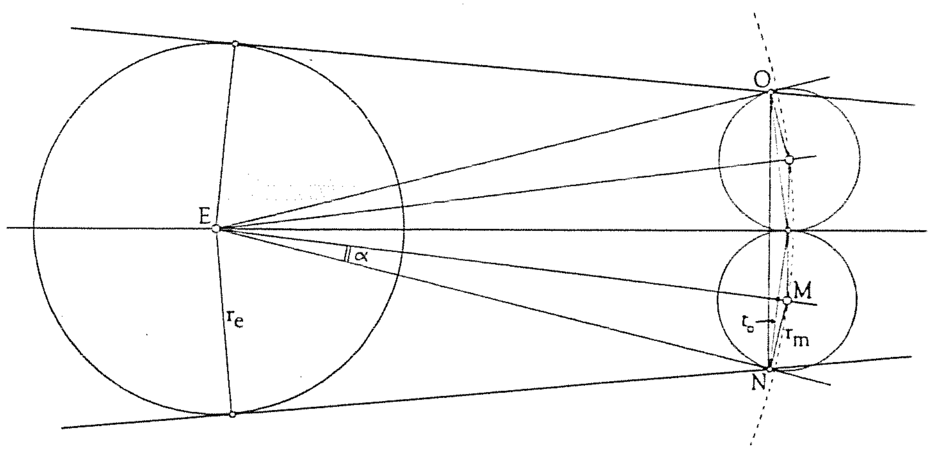


Fig. 12

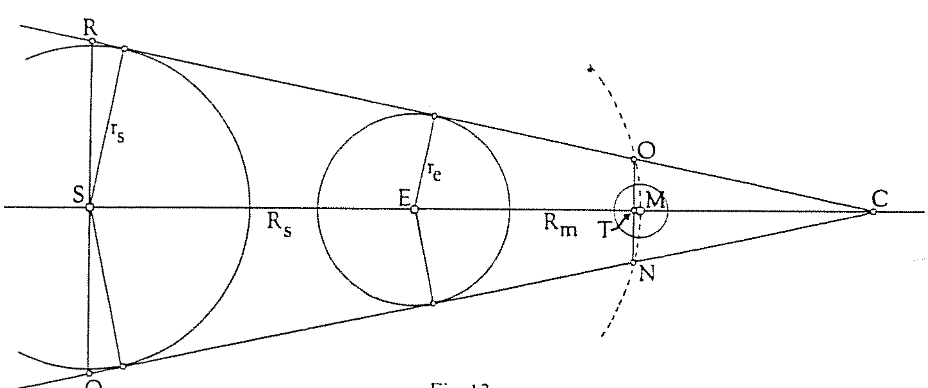


Fig. 13

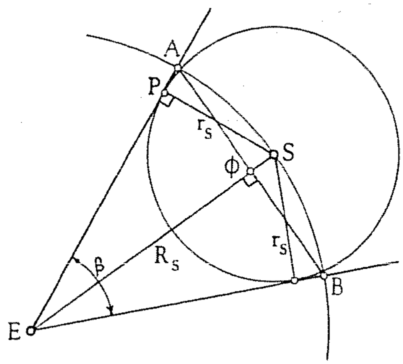


Fig. 14

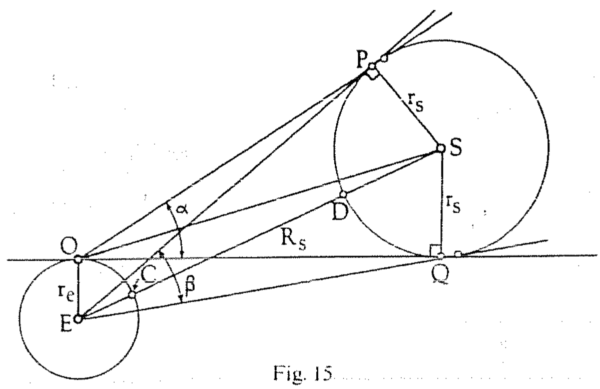


Fig. 15

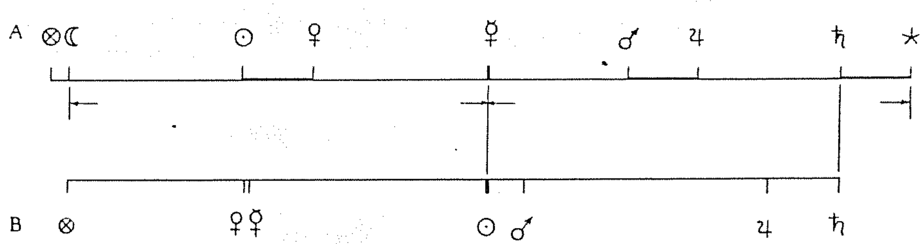
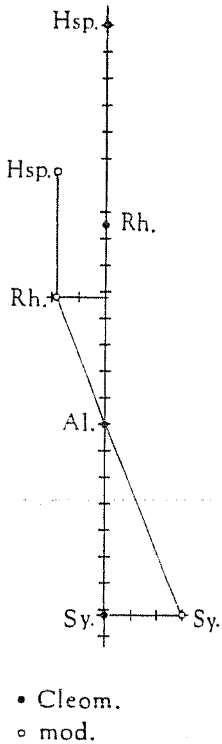


Fig. 16



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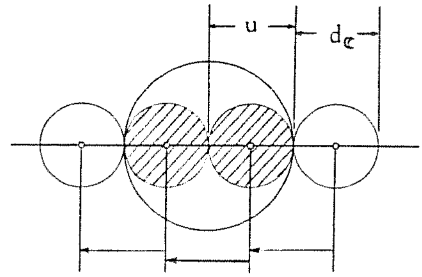

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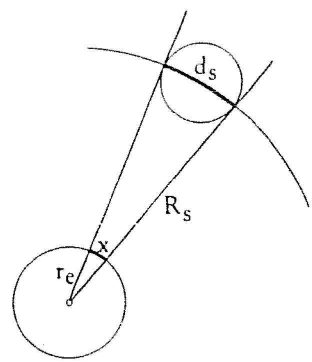

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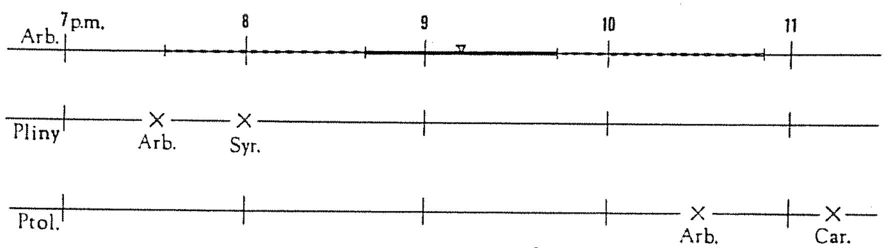
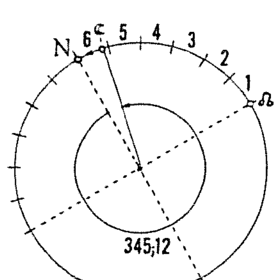


Fig. 20



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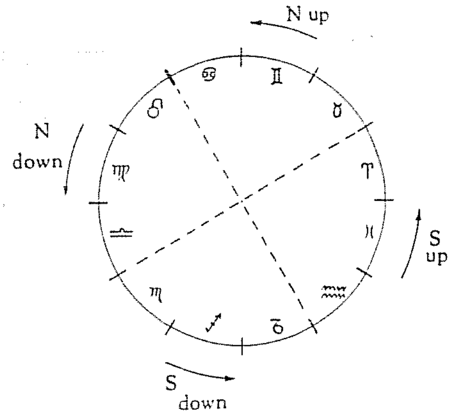

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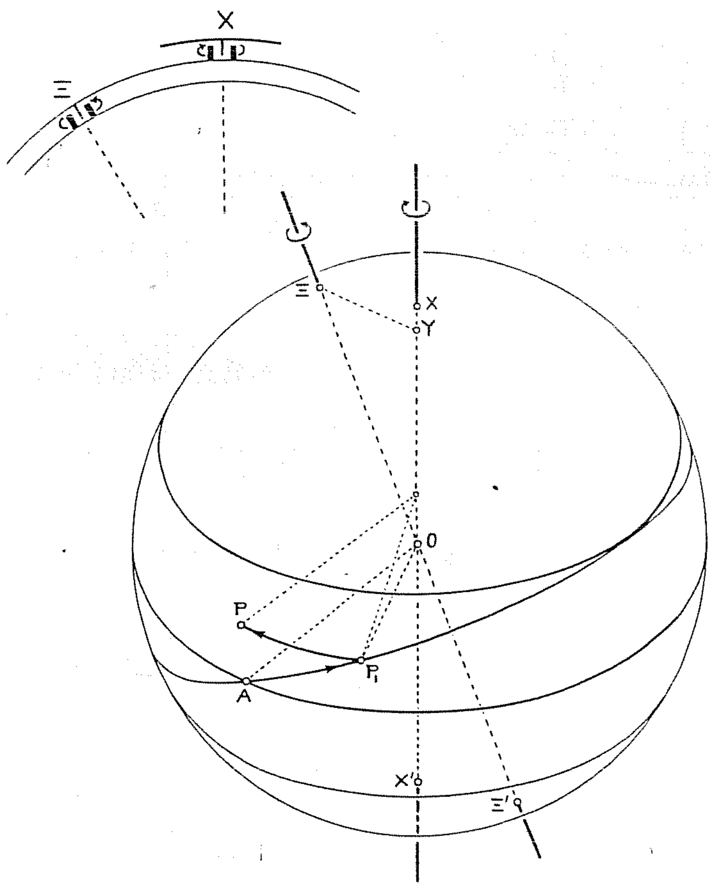

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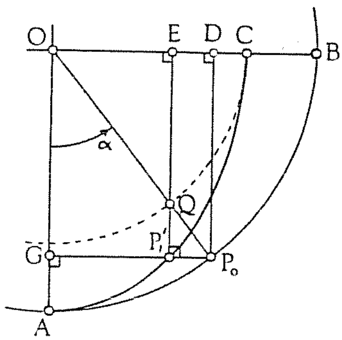

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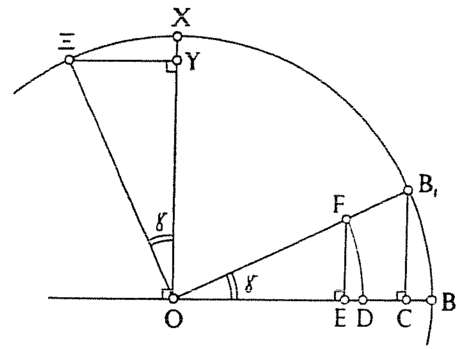

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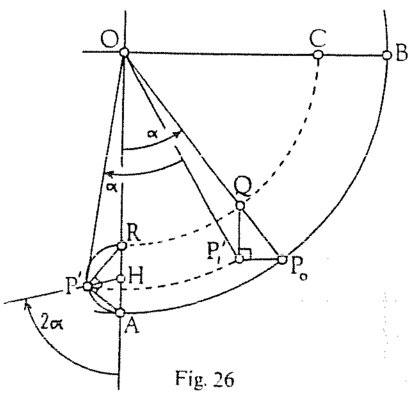

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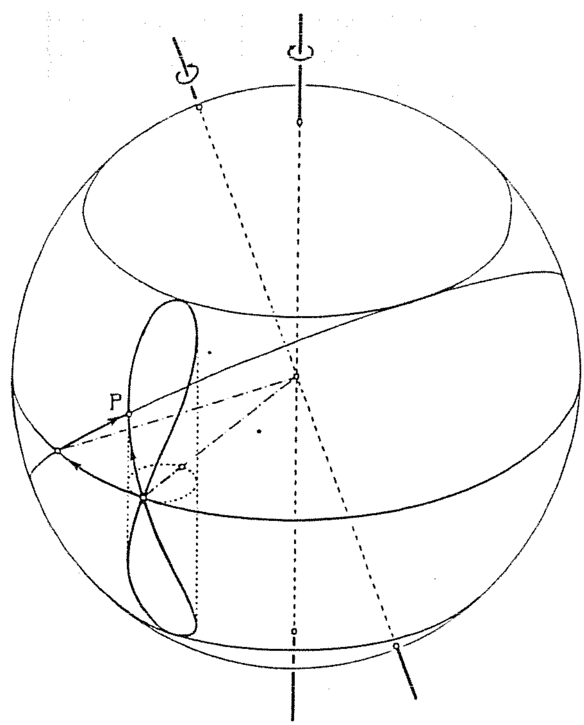

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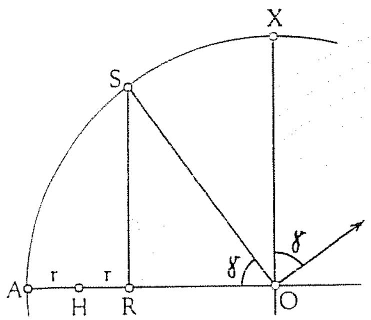

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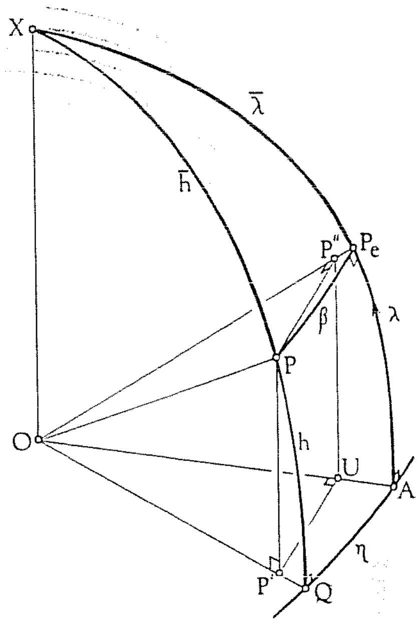

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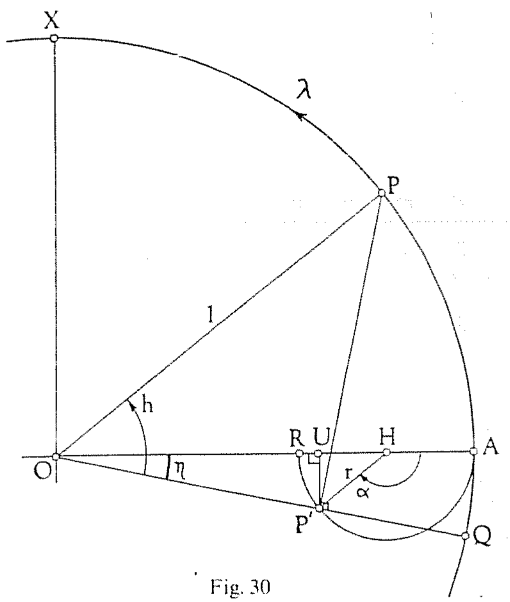

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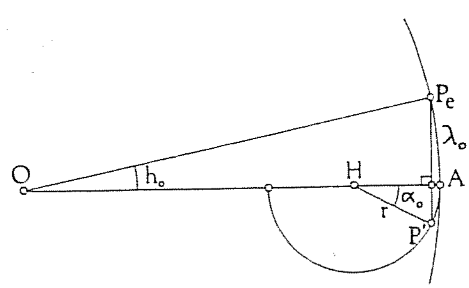

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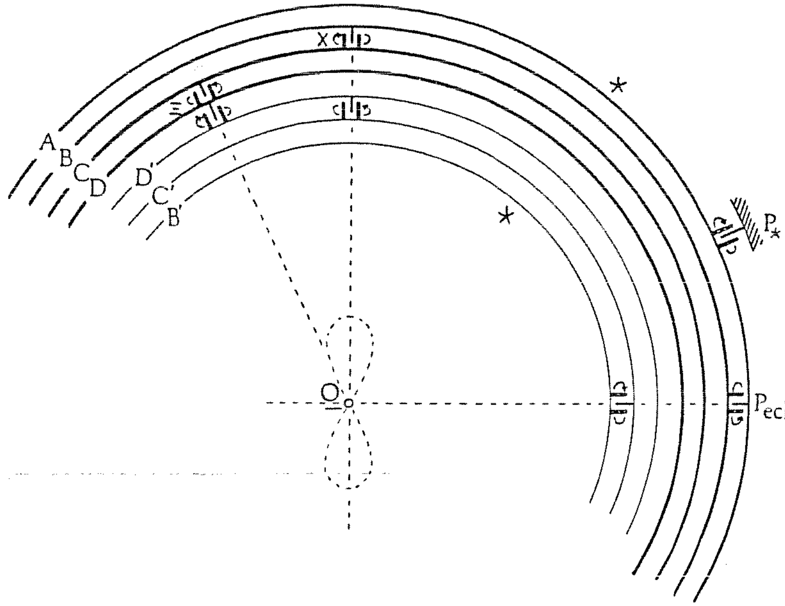

Fig. 32

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♉	♍	♑	♀	♄	☿	♂	♃	
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Fig. 33

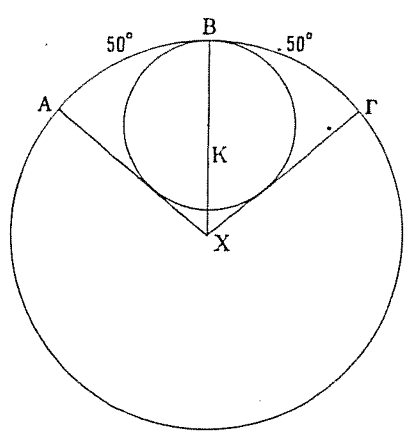


Fig. 34

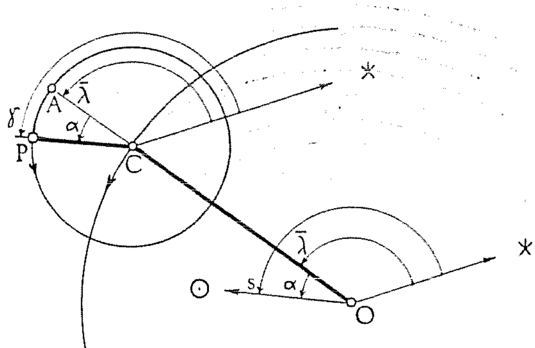
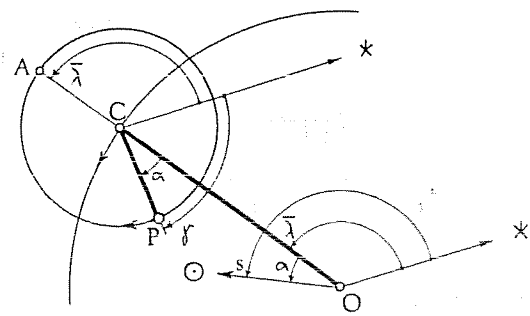


Fig. 35



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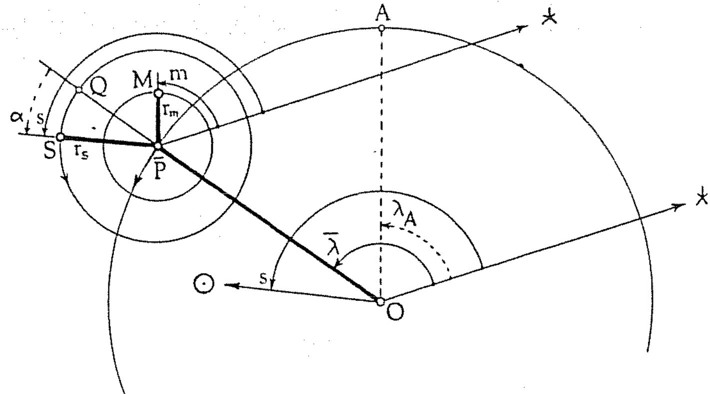

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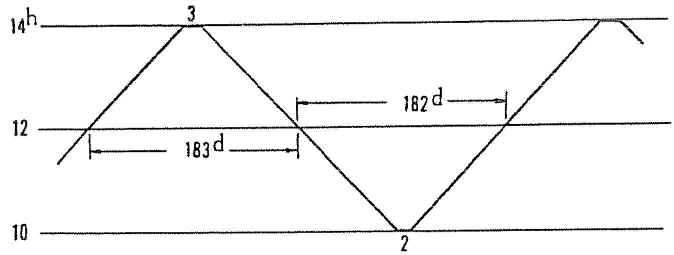

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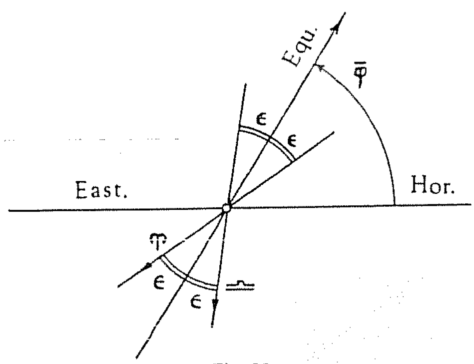

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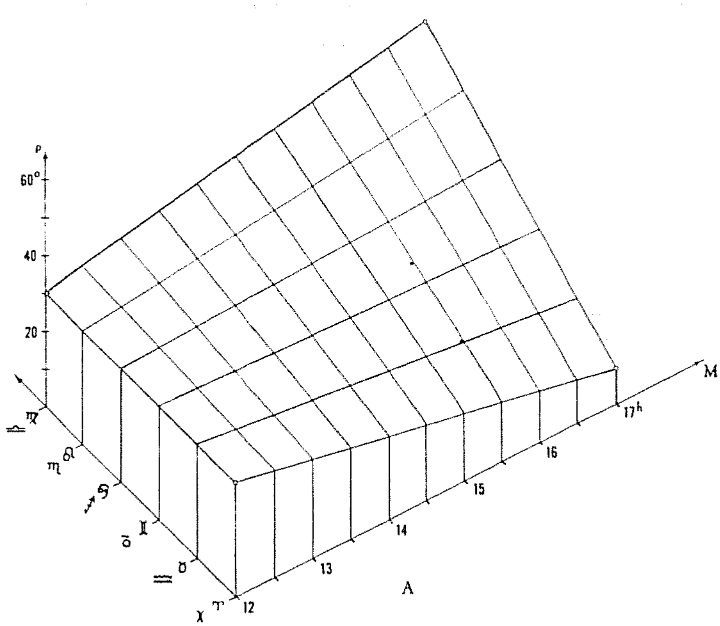

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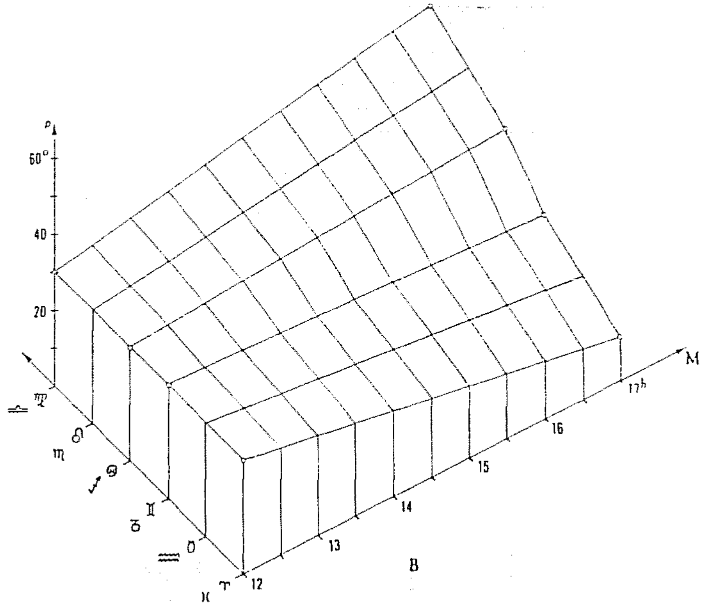

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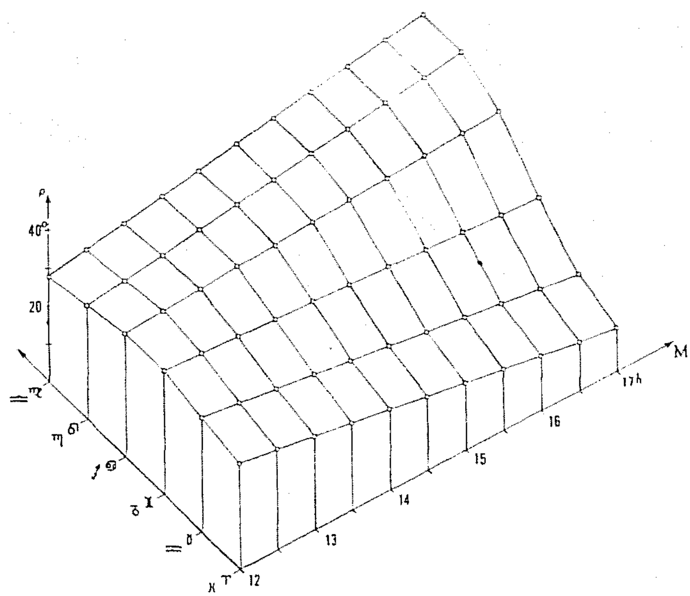

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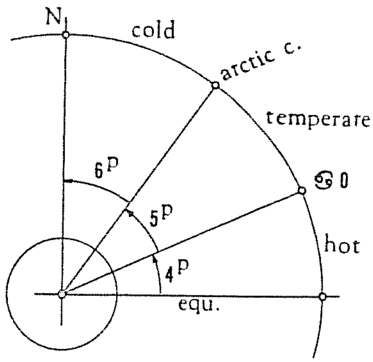

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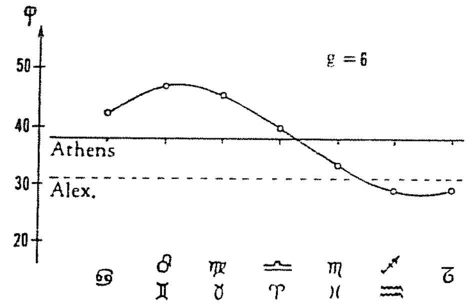

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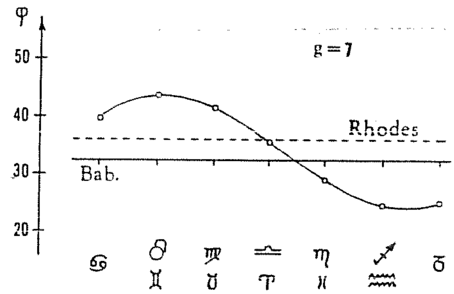

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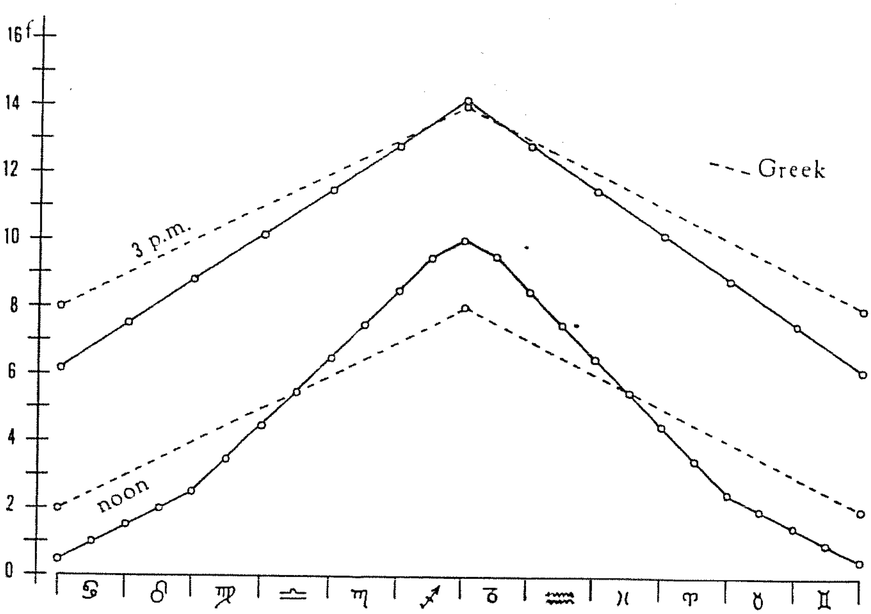

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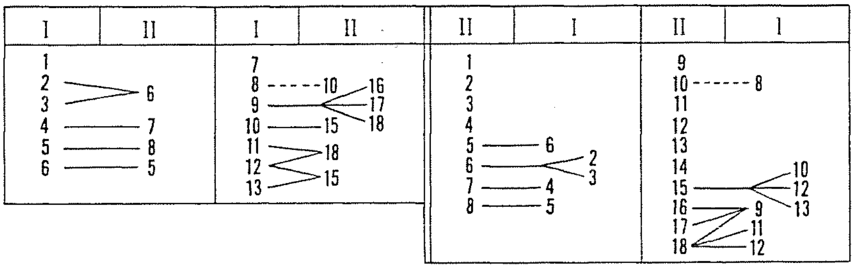
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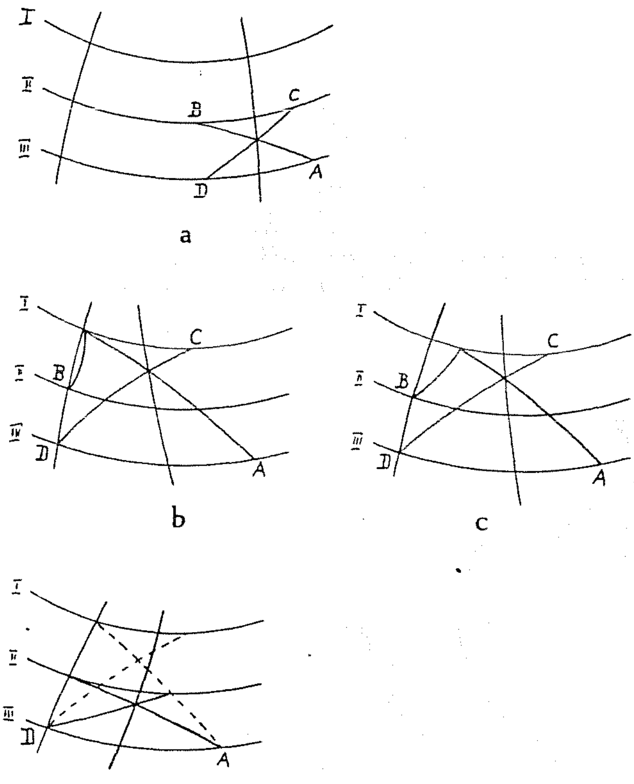
Fig. 47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D A
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Fig. 48

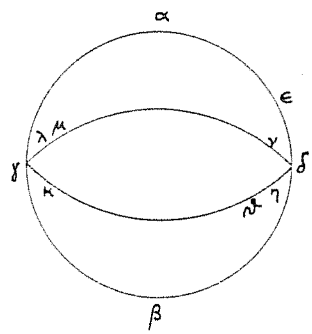


Fig. 49

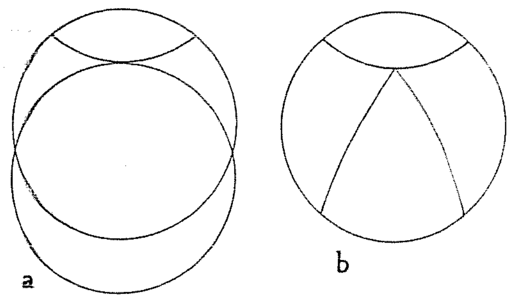


Fig. 50

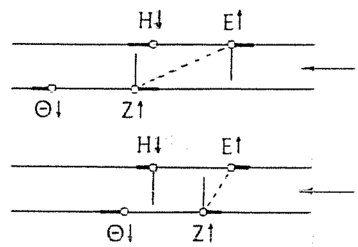


Fig. 51

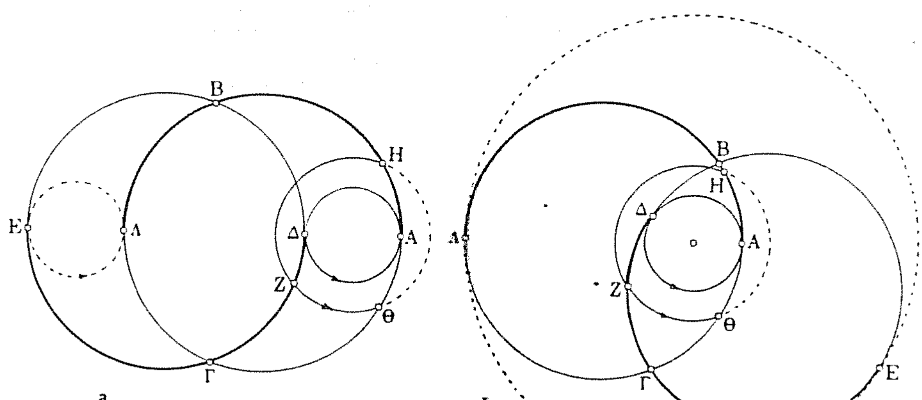


Fig. 52

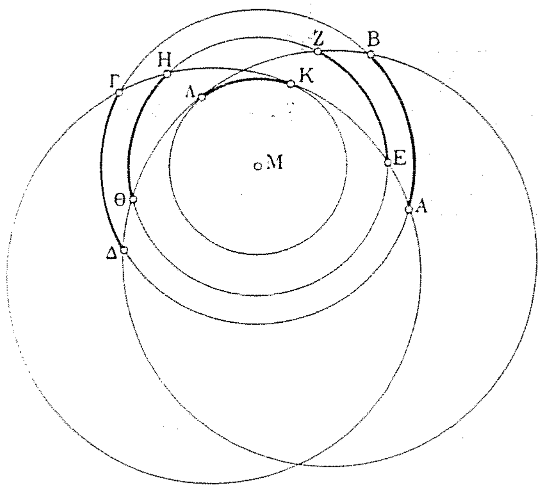
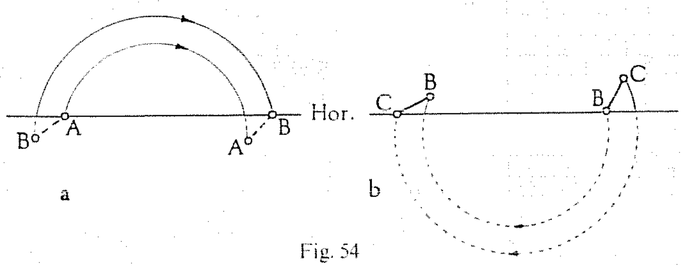


Fig. 53



a b

Fig. 54

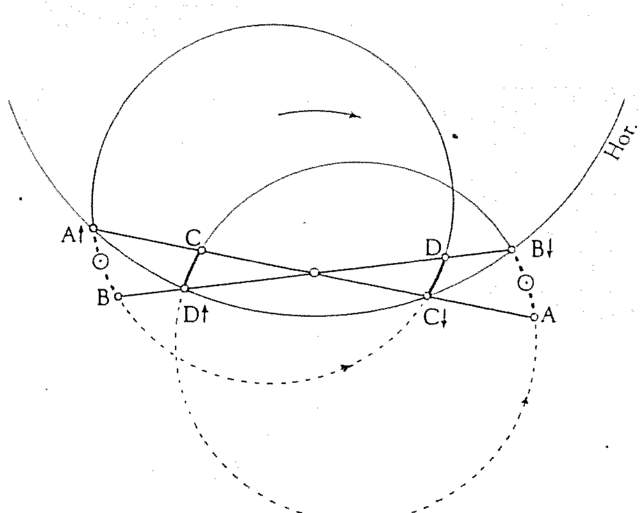
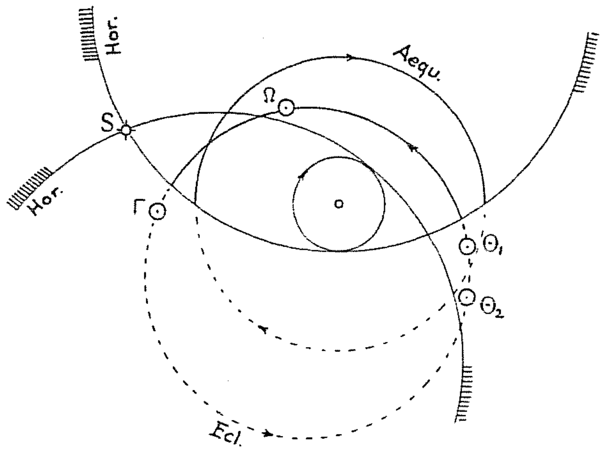


Fig. 55

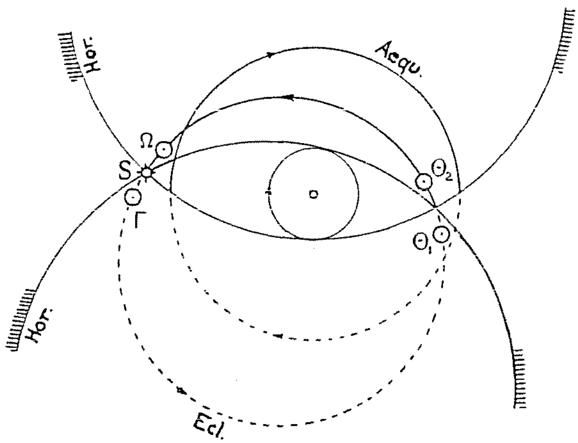
$\beta < 0$	$\star$	$\uparrow$	$\downarrow$
$\Omega \longrightarrow \Gamma$	$-$	$-$	$-$
$\Gamma \longrightarrow \Theta_2$	$+$	$+$	$-$
$\Theta_2 \longrightarrow \Theta_1$	$+$	$+$	$+$
$\Theta_1 \longrightarrow \Omega$	$+$	$-$	$+$
$\Omega \to \Gamma \to \Theta_2 \to \Theta_1 \to \Omega$			

a



$\beta = 0$	$\star$	$\uparrow$	$\downarrow$
$\Omega \longrightarrow \Gamma$	$-$	$-$	$-$
$\Gamma \longrightarrow \Theta_1$	$+$	$+$	$-$
$\Theta_1 \longrightarrow \Theta_2$	$+$	$-$	$-$
$\Theta_2 \longrightarrow \Omega$	$+$	$-$	$+$
$\Omega \to \Gamma \to \Theta_1 \to \Theta_2 \to \Omega$			

b



$\beta > 0$	$\star$	$\uparrow$	$\downarrow$
$\Omega \longrightarrow \Theta_1$	$+$	$+$	$-$
$\Theta_1 \longrightarrow \Theta_2$	$+$	$-$	$-$
$\Theta_2 \longrightarrow \Gamma$	$+$	$-$	$+$
$\Gamma \longrightarrow \Omega$	$+$	$+$	$+$
$\Omega \to \Theta_1 \to \Theta_2 \to \Gamma \to \Omega$			

c

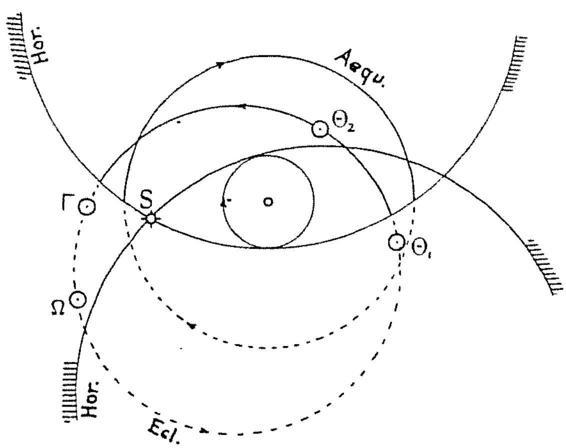


Fig. 56

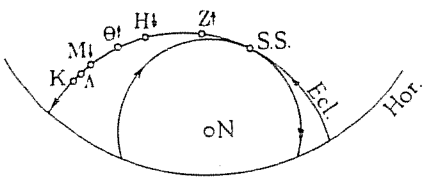


Fig. 57

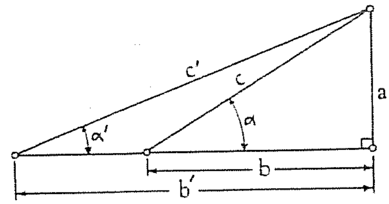


Fig. 58

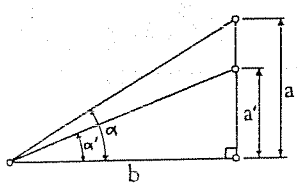


Fig. 59

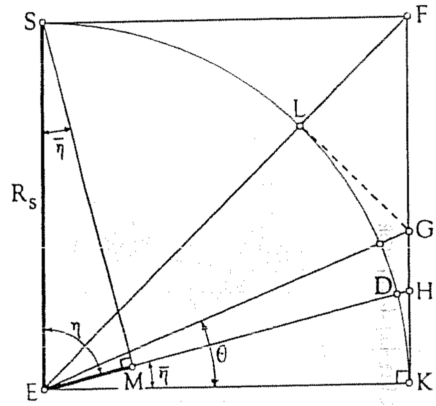


Fig. 60

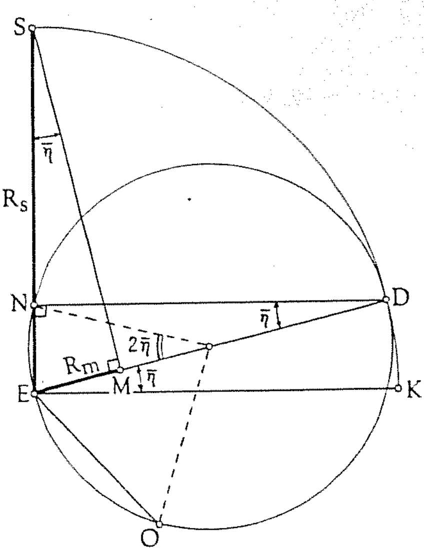


Fig. 61

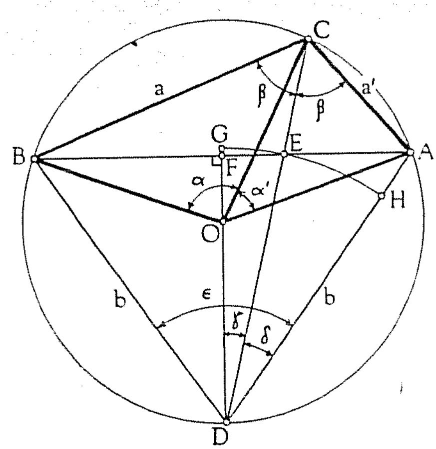


Fig. 62

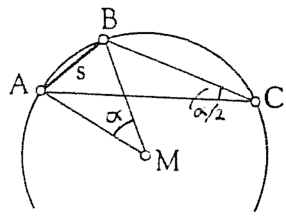


Fig. 63

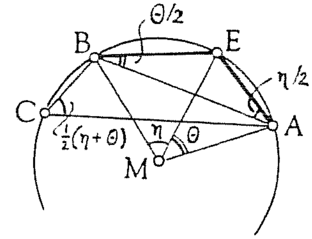


Fig. 64

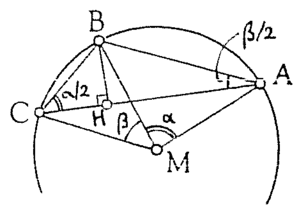


Fig. 65

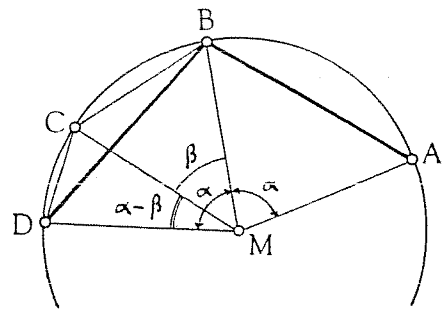


Fig. 66

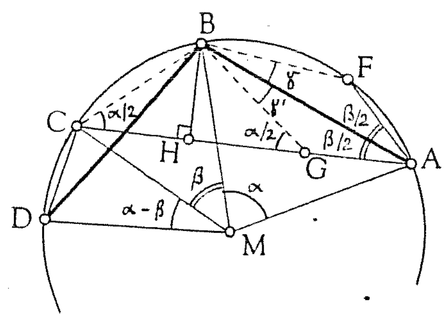


Fig. 67

Figures to Book V

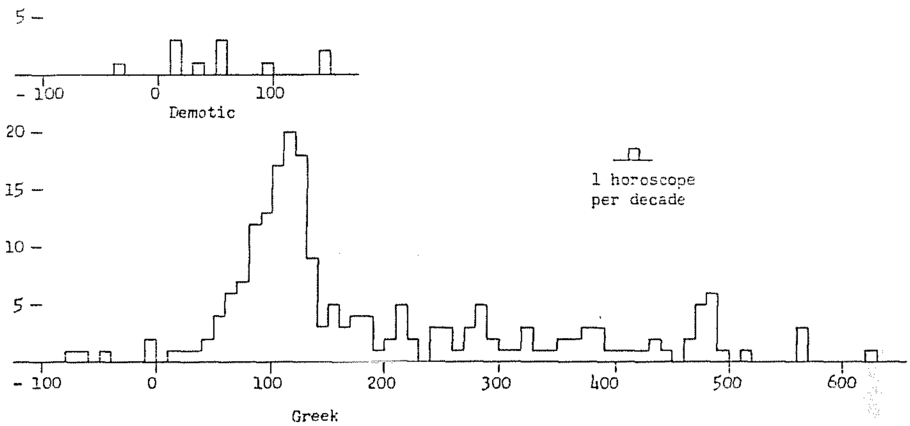


Fig. 1

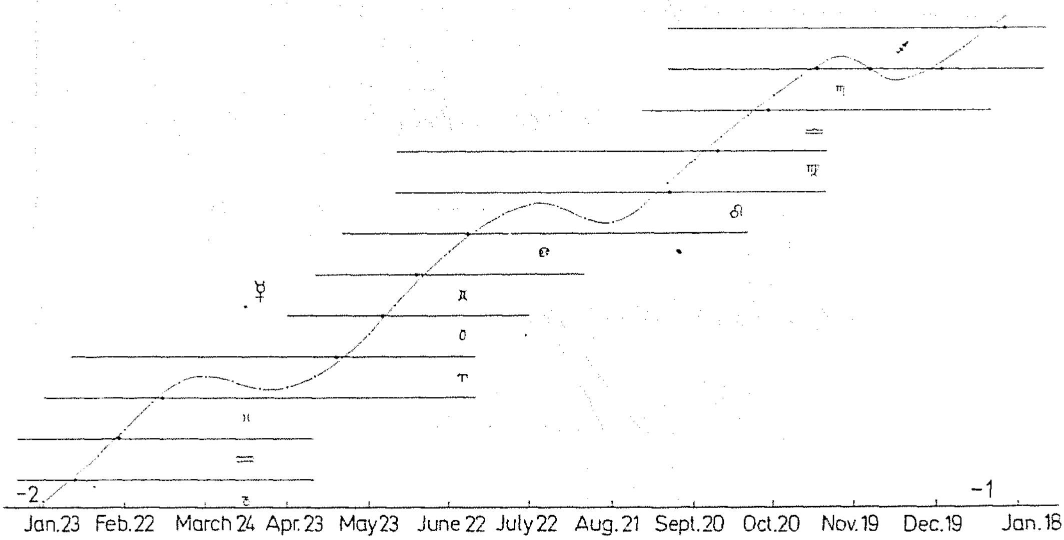


Fig. 2

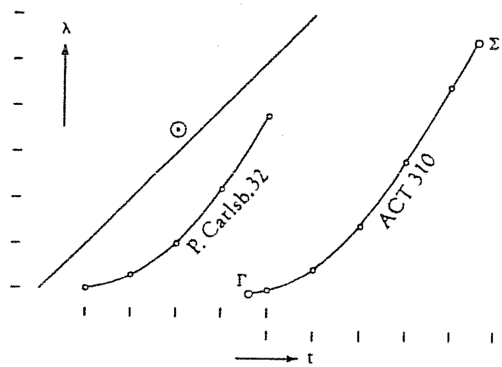

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Fig. 3

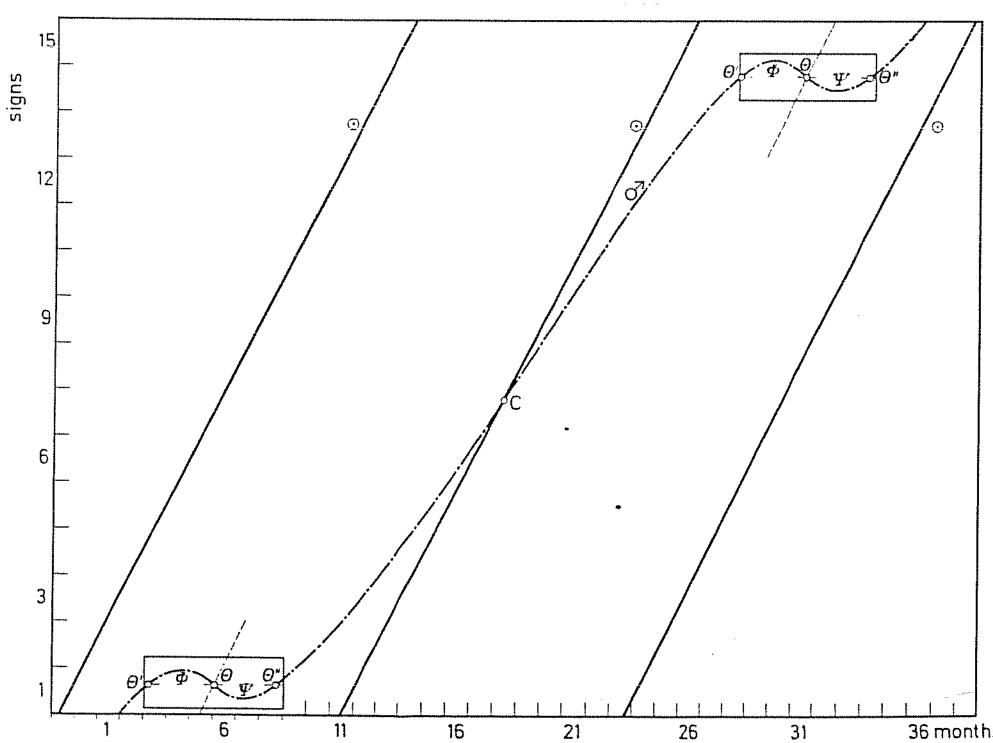
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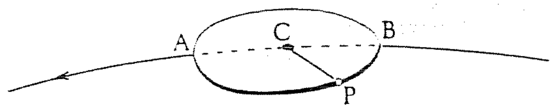
Fig. 4



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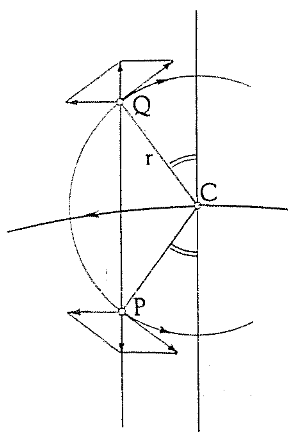


Fig. 6

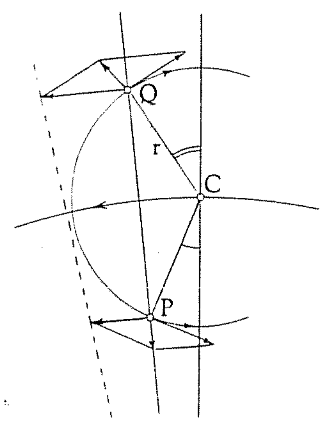


Fig. 7

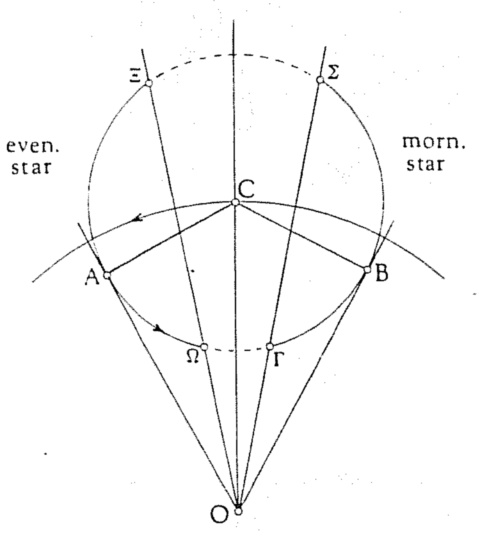


Fig. 8

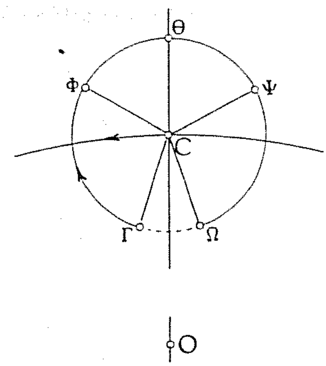


Fig. 9

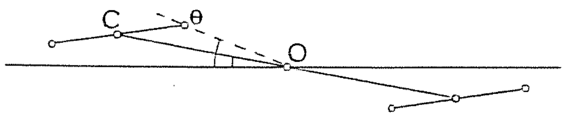


Fig. 10

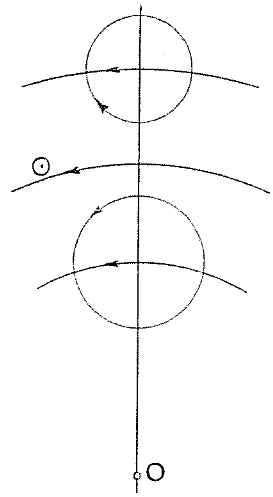


Fig. 12

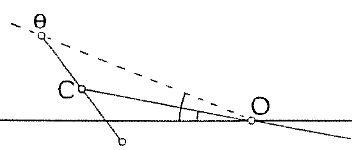


Fig. 11

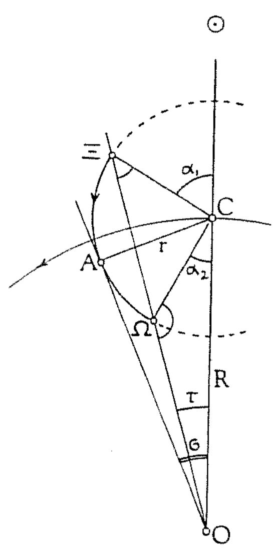


Fig. 13

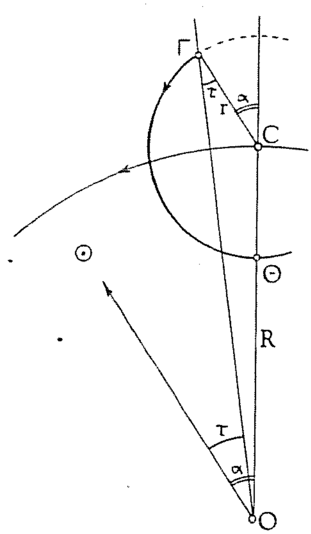


Fig. 14

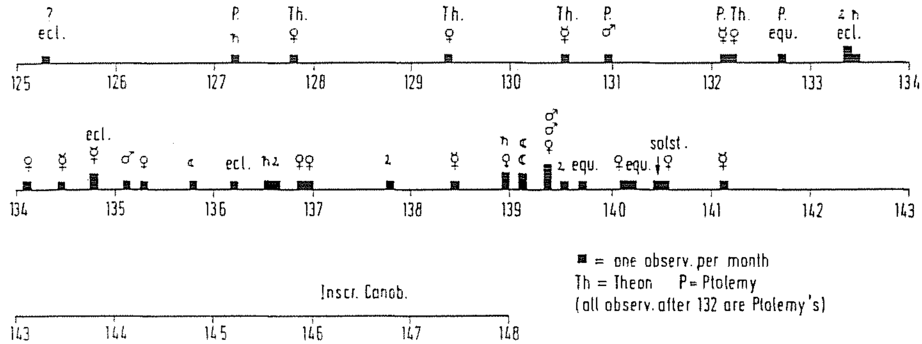

■ = one observ. per month
Th = Theon P = Ptolemy
(all observ. after 132 are Ptolemy's)
Inscr. Canob.

Fig. 15

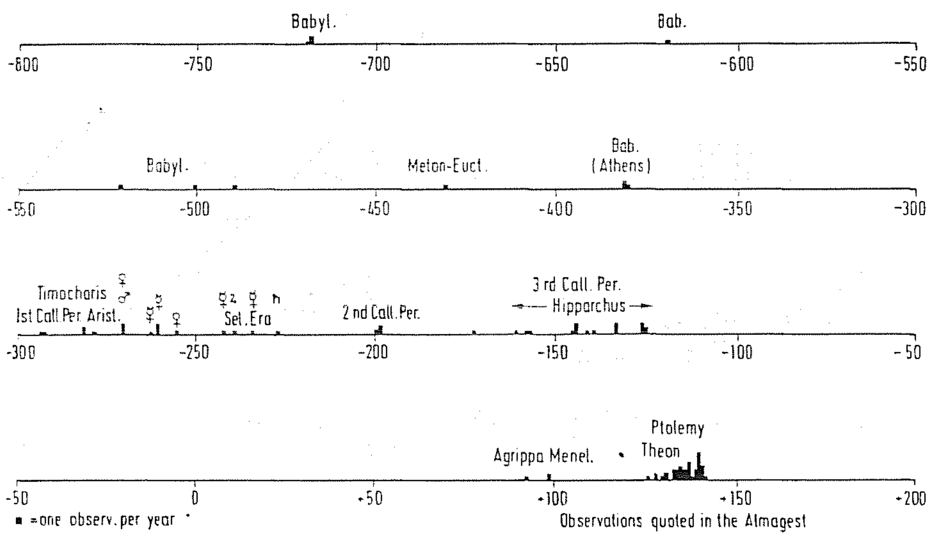

Babyl.
Bab.
Meton-Euct.
Bab. (Athens)
Timocharis
1st Call.Per. Arist.
Sel. Era
2nd Call.Per.
3rd Call. Per.
Hipparchus
Agrippa Menel.
Ptolemy
Theon
■ = one observ. per year
Observations quoted in the Almagest

Fig. 16

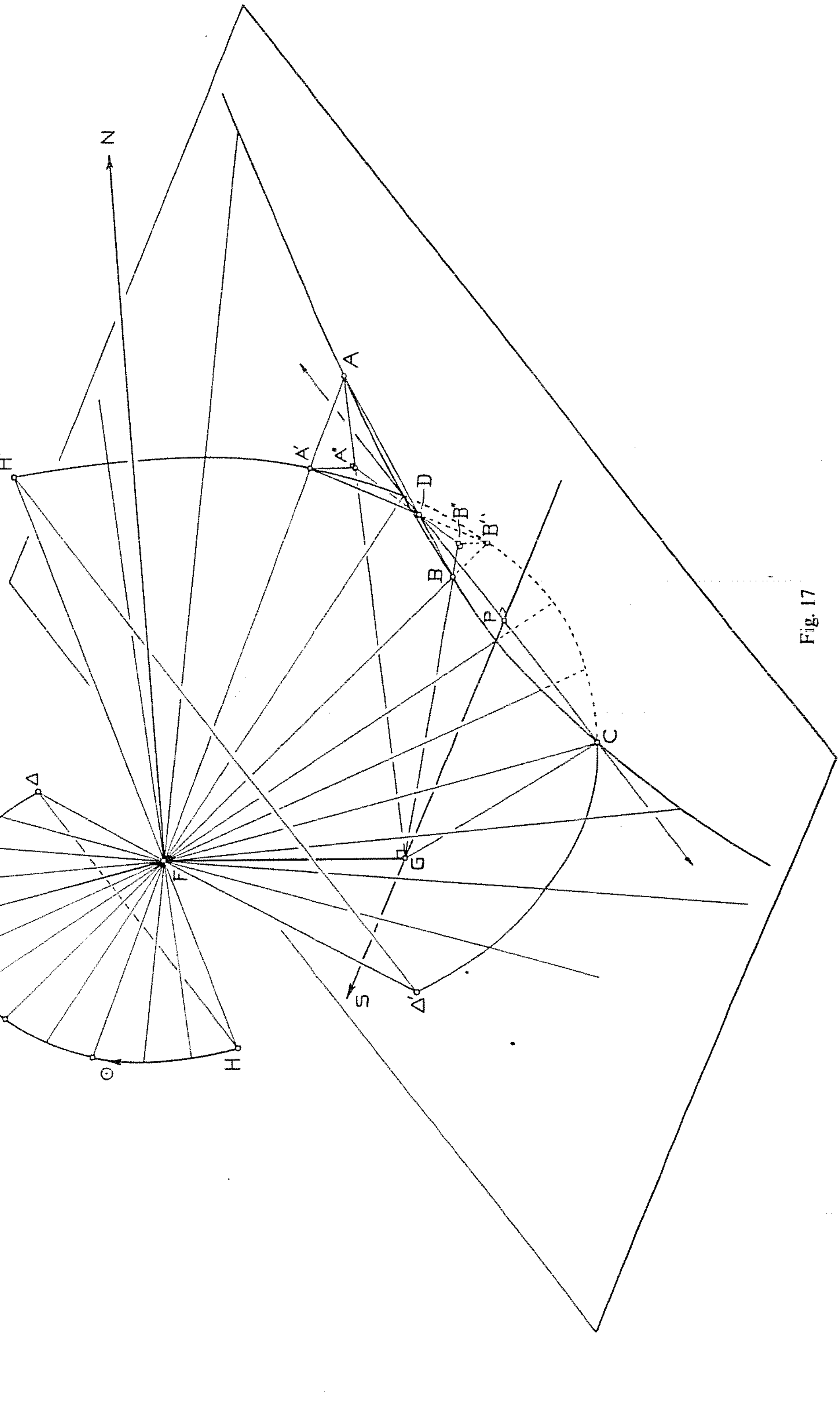
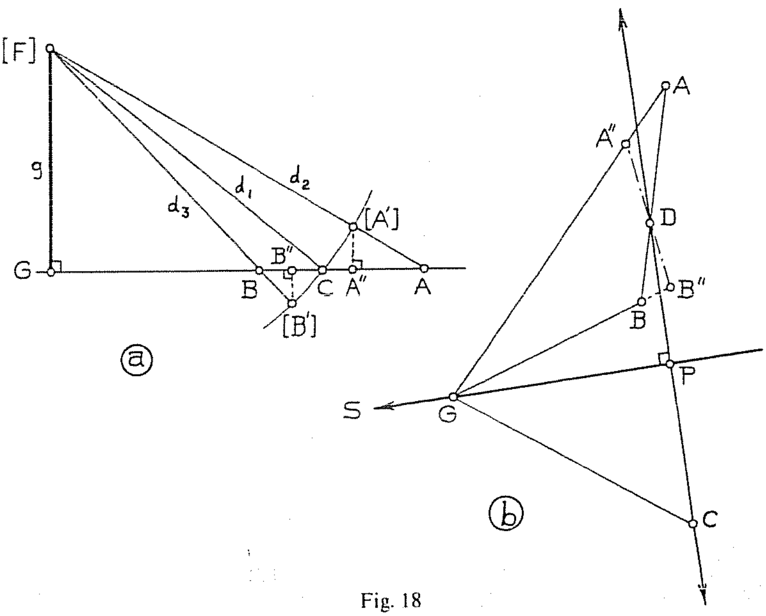


Fig. 17


[F]
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d_1
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B''
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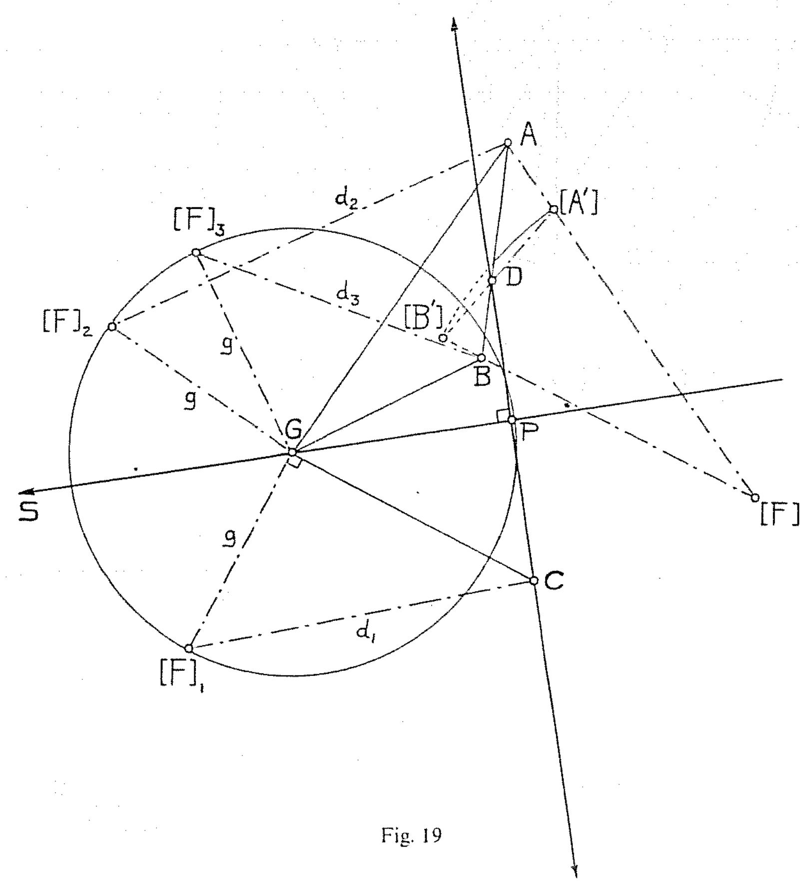
Fig. 18


A
d_2
[A']
$[F]_3$
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d_3
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d_1
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Fig. 19

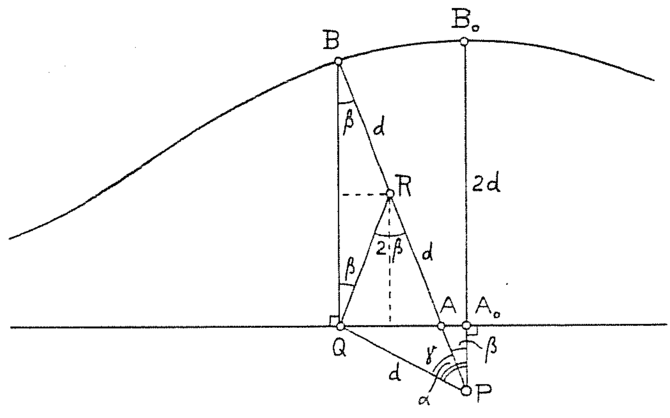


Fig. 20

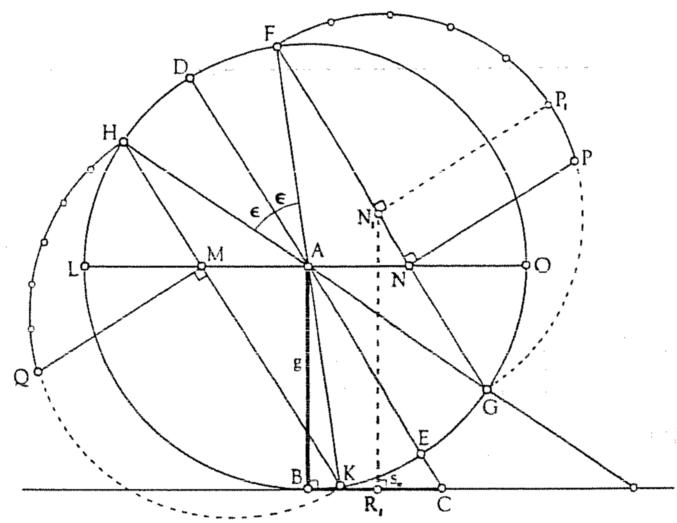


Fig. 21

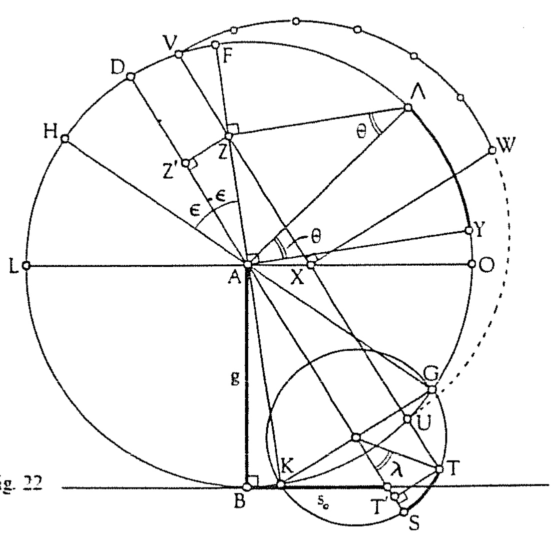


Fig. 22

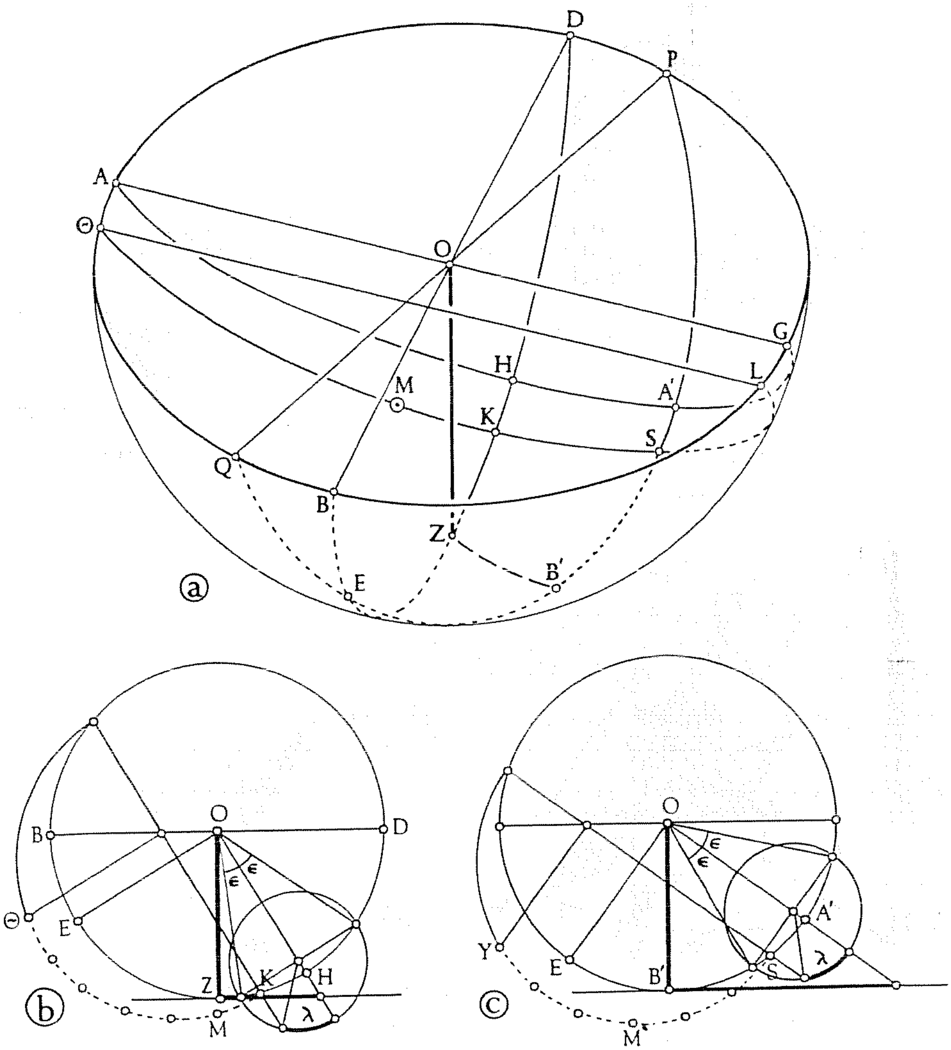
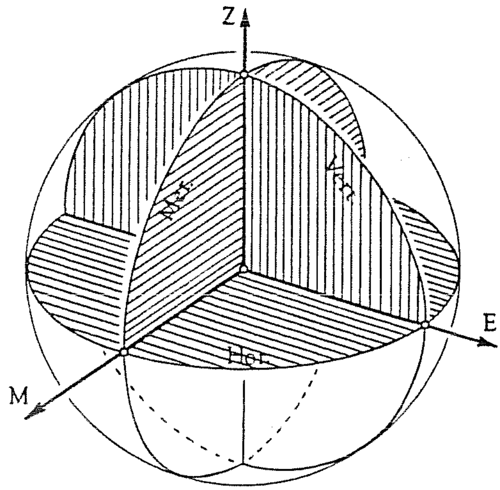


Fig. 23



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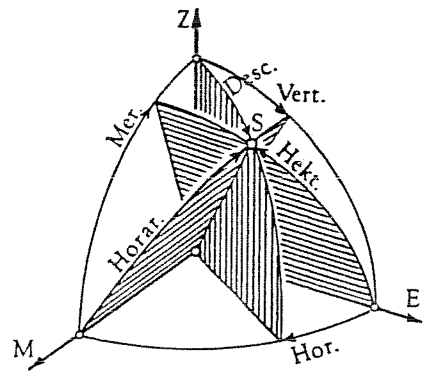

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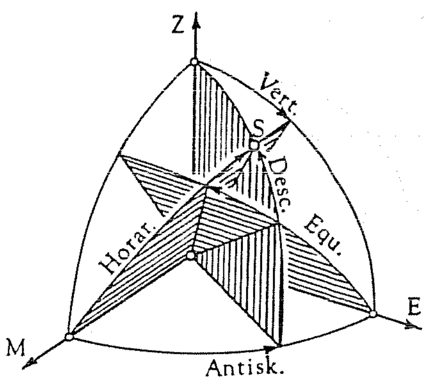

Fig. 26

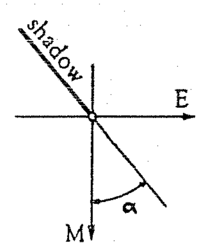

Fig. 27

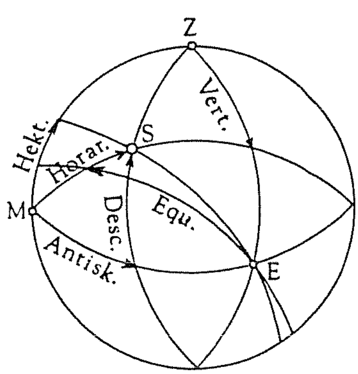

Fig. 28

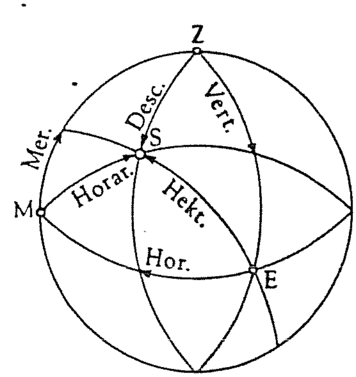

Fig. 29

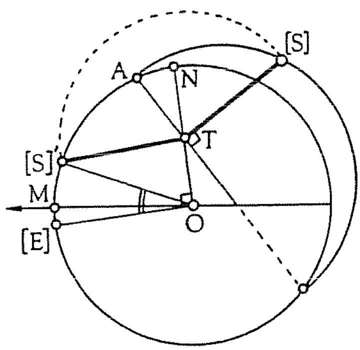

Fig. 30

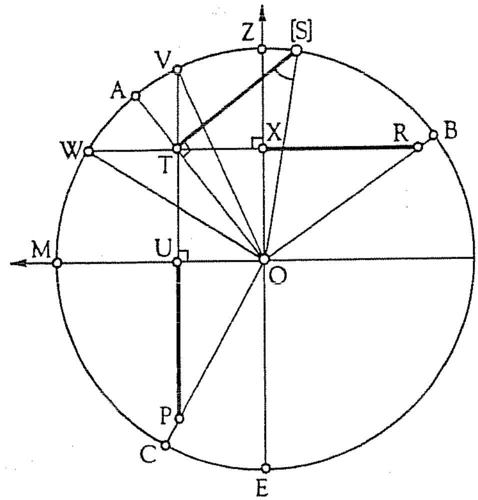

Fig. 31

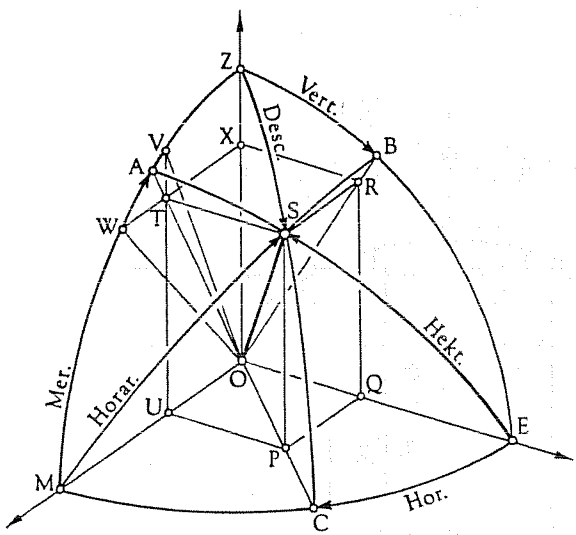

Fig. 32

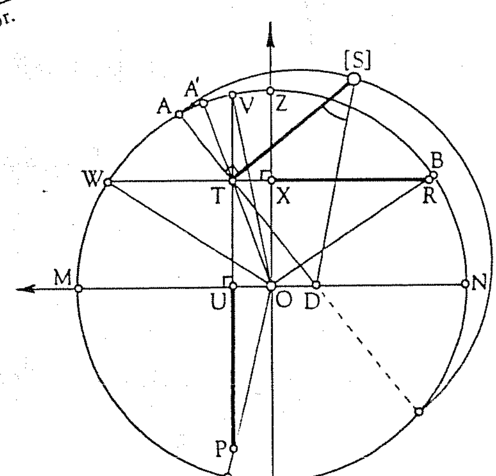

Fig. 33

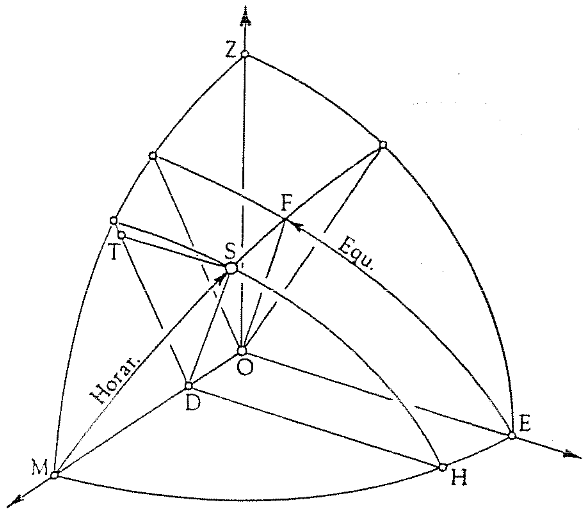

Fig. 34

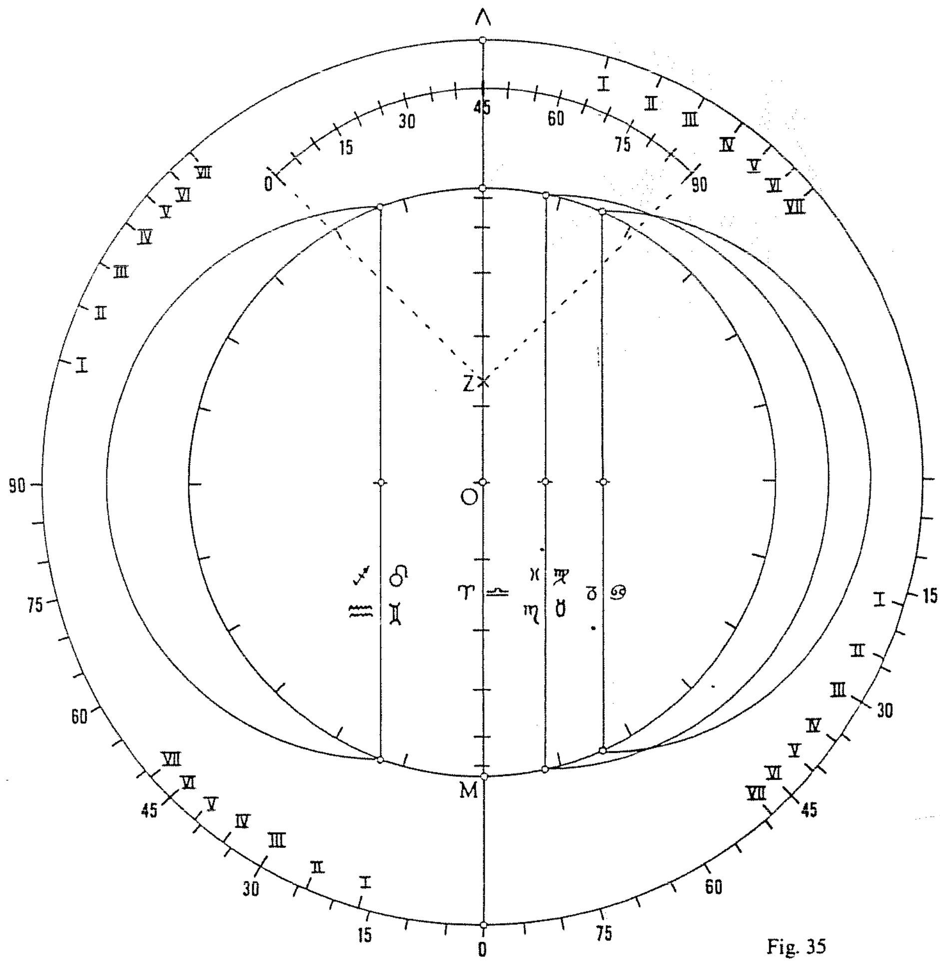

Fig. 35

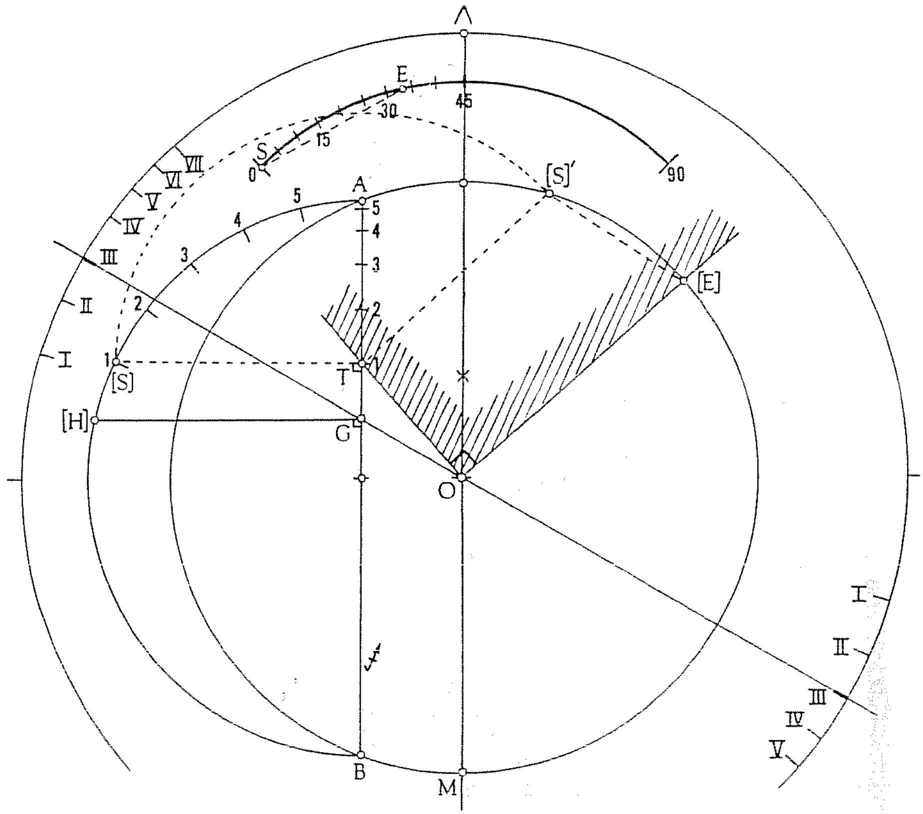

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Fig. 36

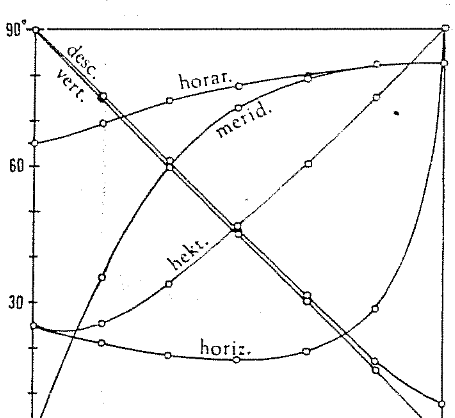

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Fig. 37

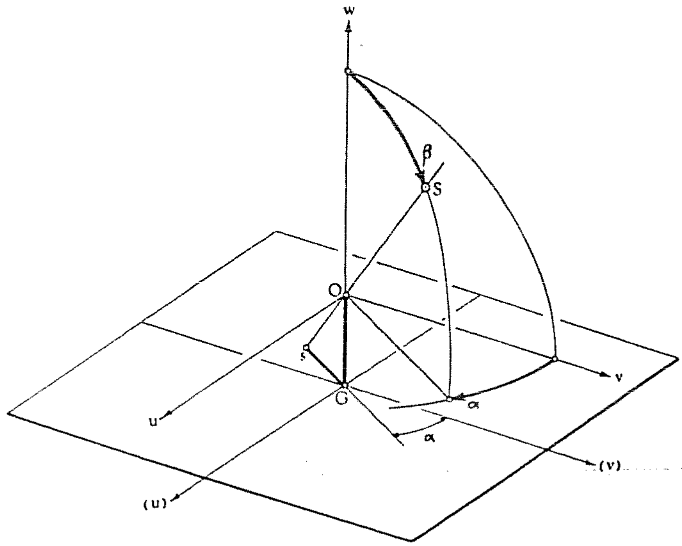

Fig. 38

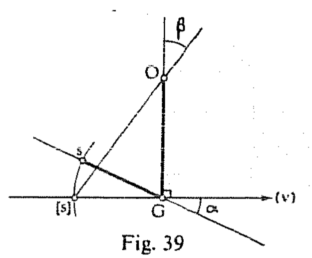

Fig. 39

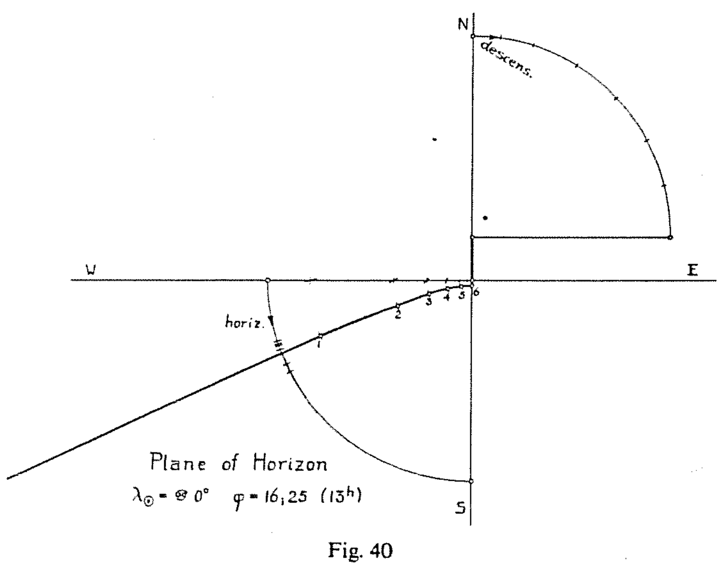

Fig. 40

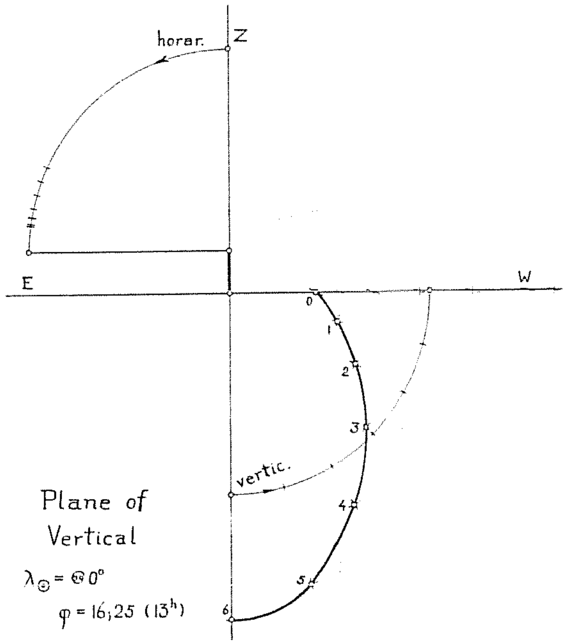
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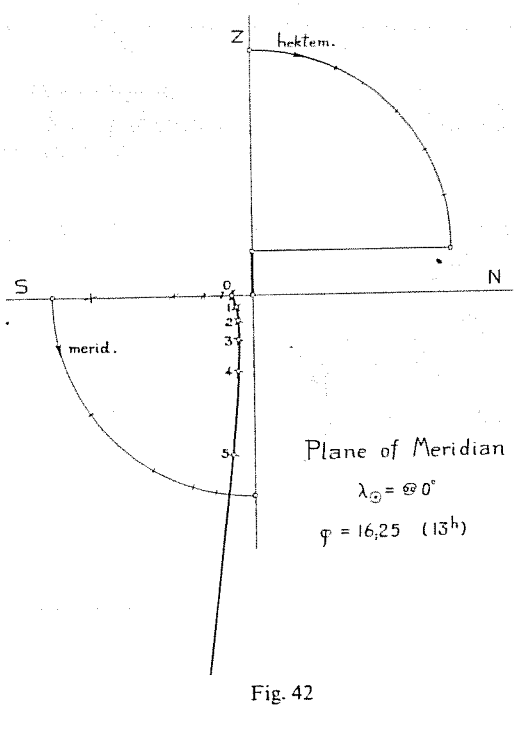
Fig. 41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merid.
Plane of Meridian
$\lambda_{\odot} = ♋ 0°$
$\varphi = 16;25$ (13ʰ)

Fig. 42

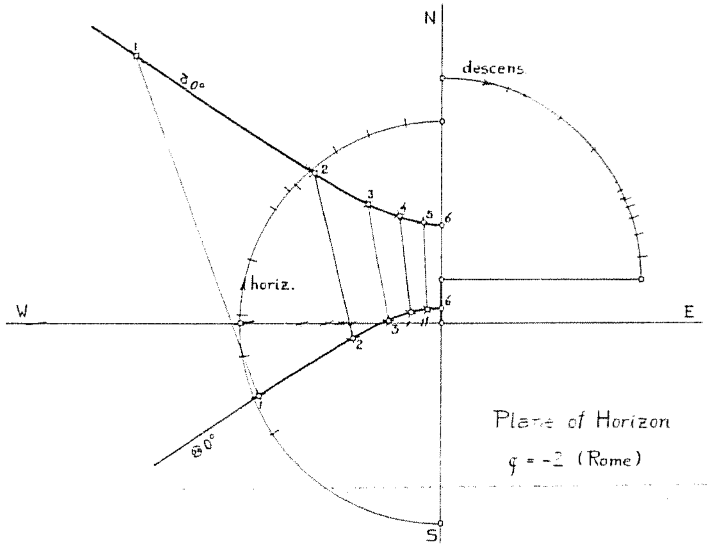


Fig. 43

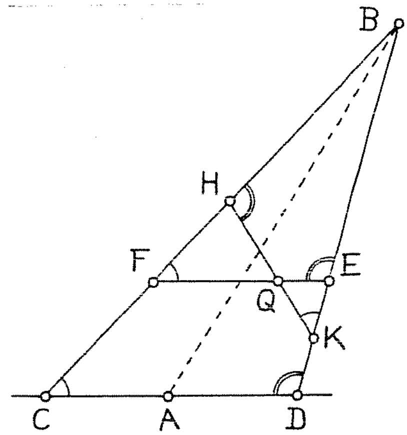


Fig. 44

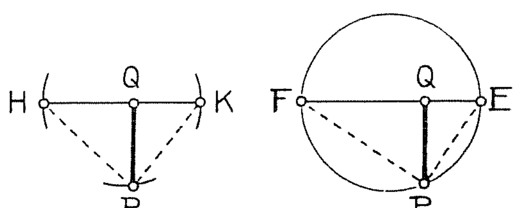


Fig. 45

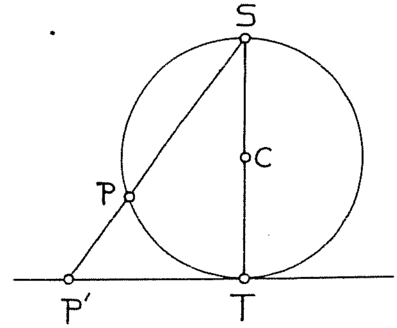


Fig. 46

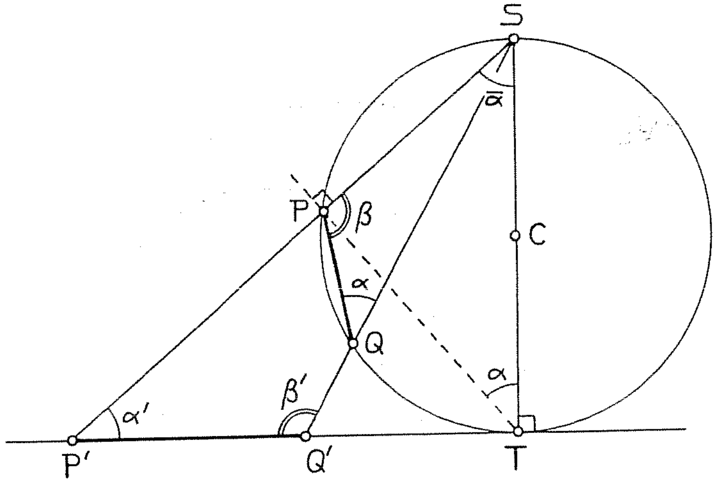


Fig. 47

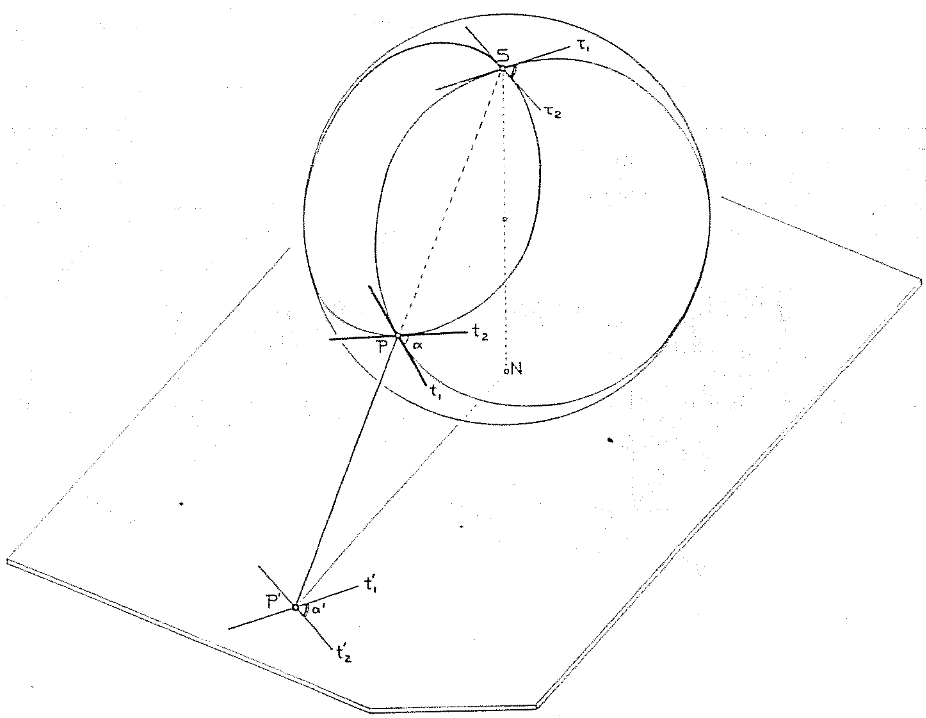


Fig. 48

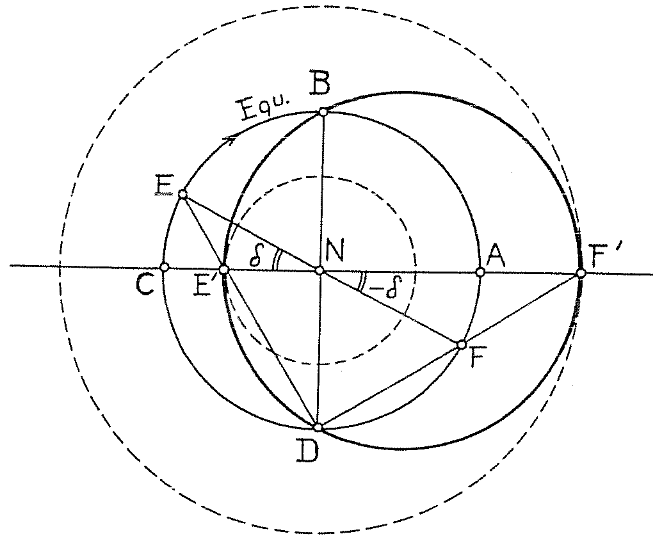


Fig. 49

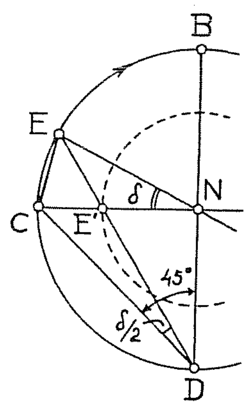


Fig. 50

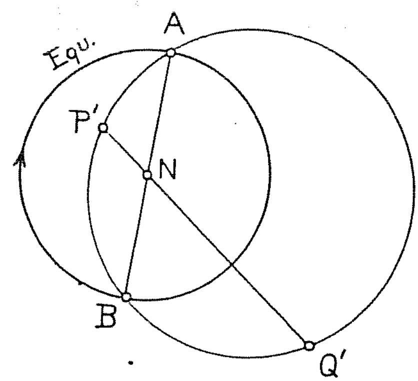


Fig. 51

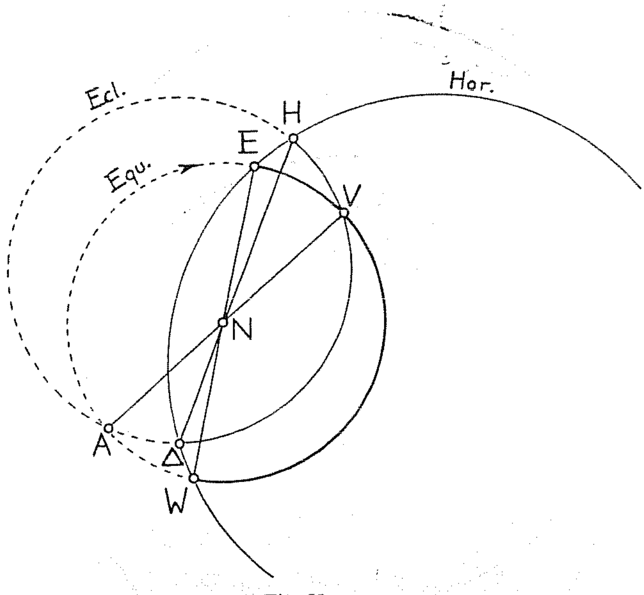


Fig. 52

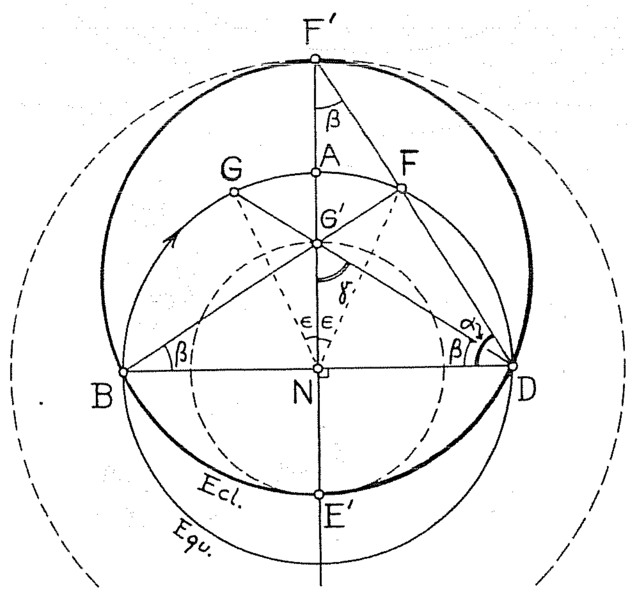
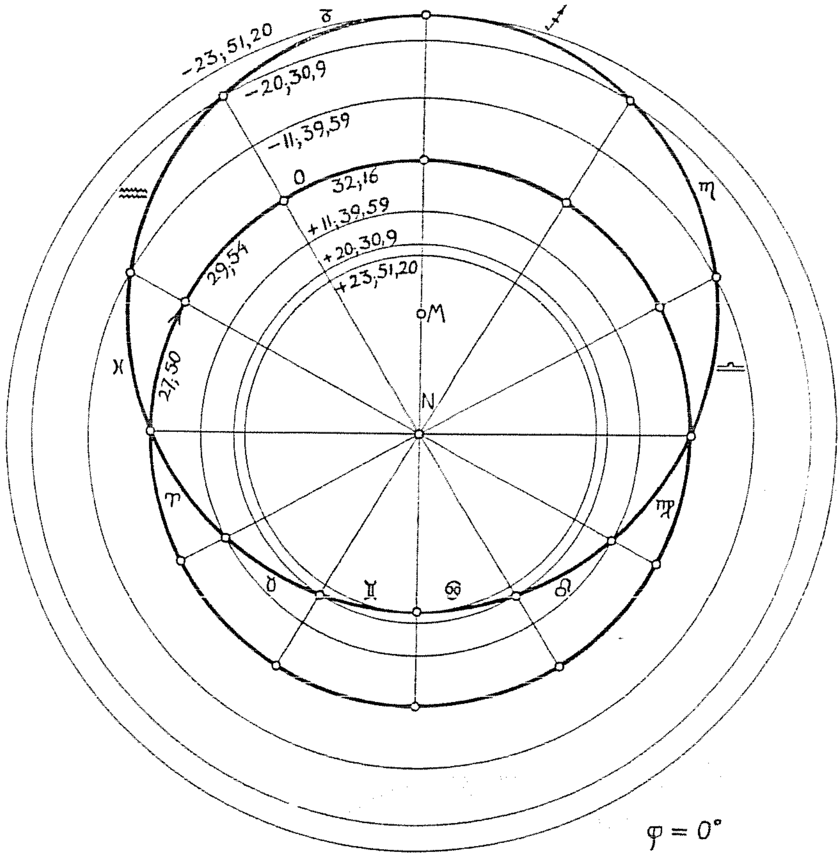


Fig. 53


♑
♐
−23;51,20
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−11;39,59
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32;16
+11;39,59
+20;30,9
+23;51,20
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27;50
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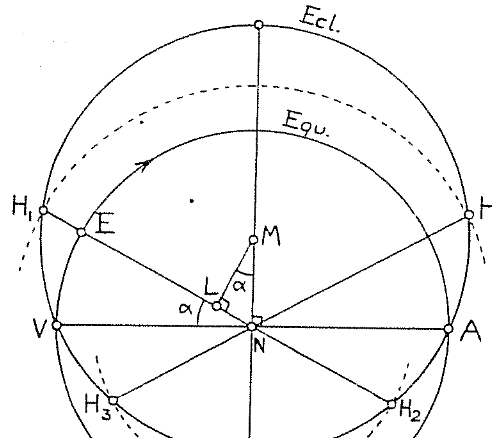
♌
$\varphi = 0°$

Fig. 54


Ecl.
Equ.
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N
A
H_3
H_2
H_4

Fig. 55

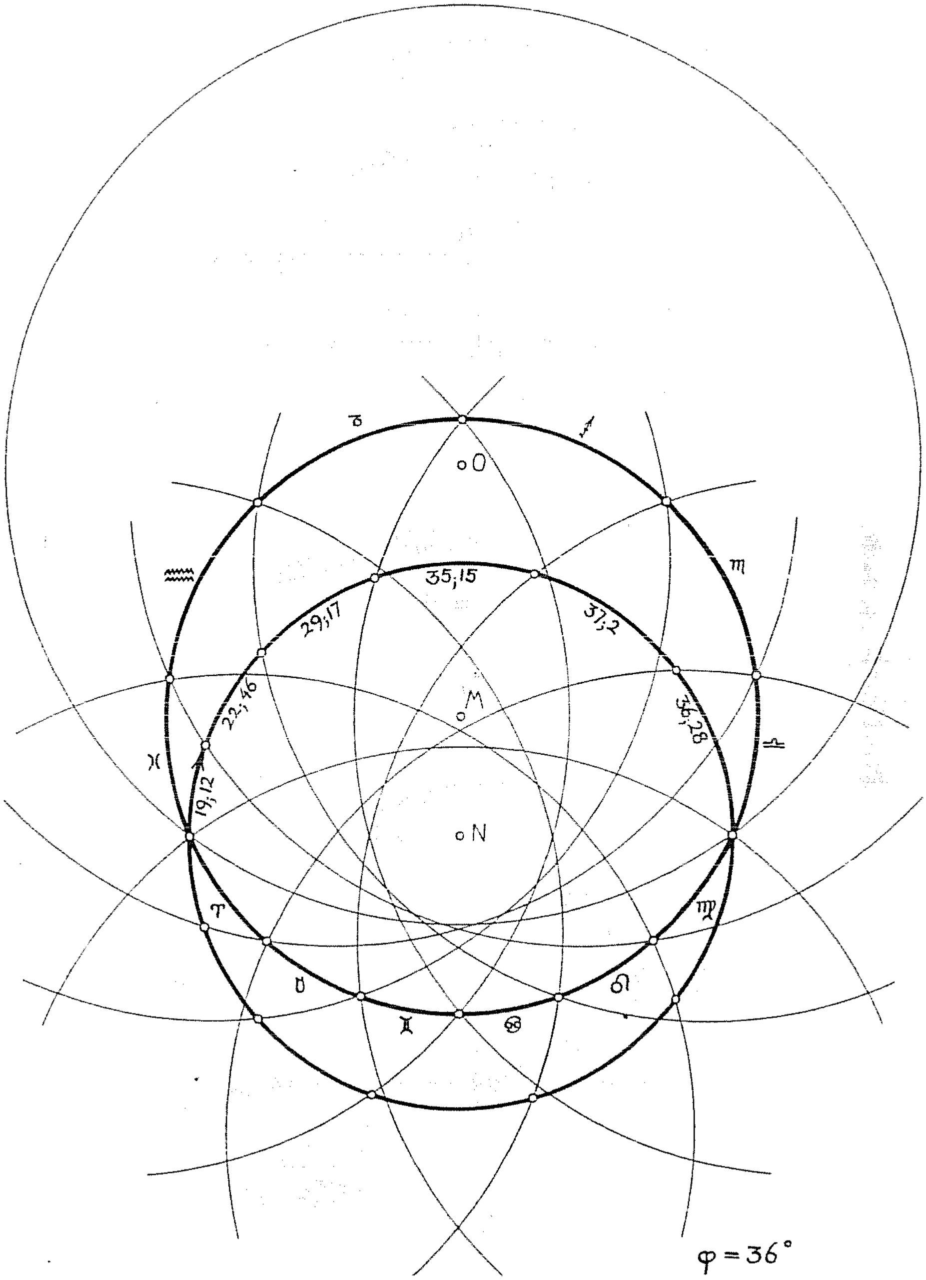


Fig. 56

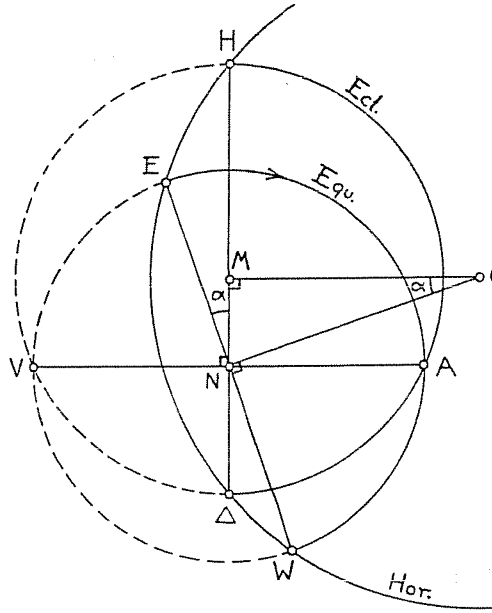


Fig. 57

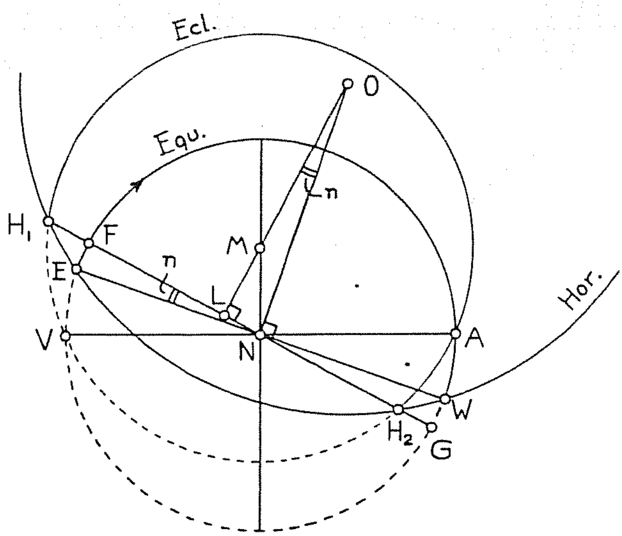


Fig. 58

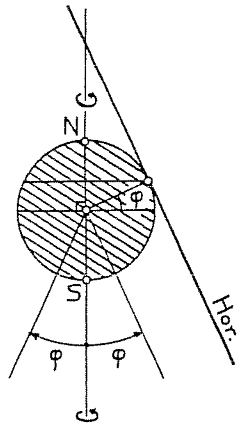


Fig. 59

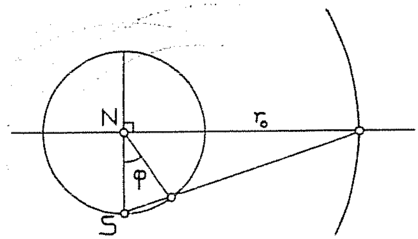


Fig. 60

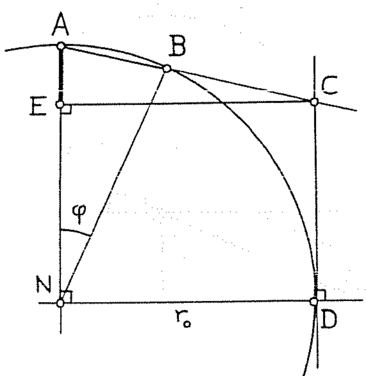


Fig. 61

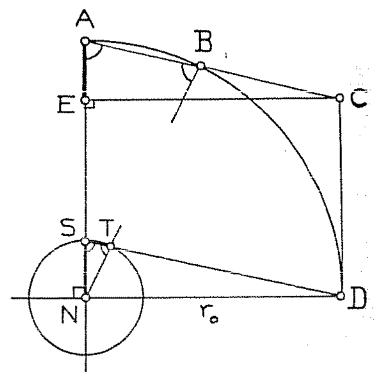


Fig. 62

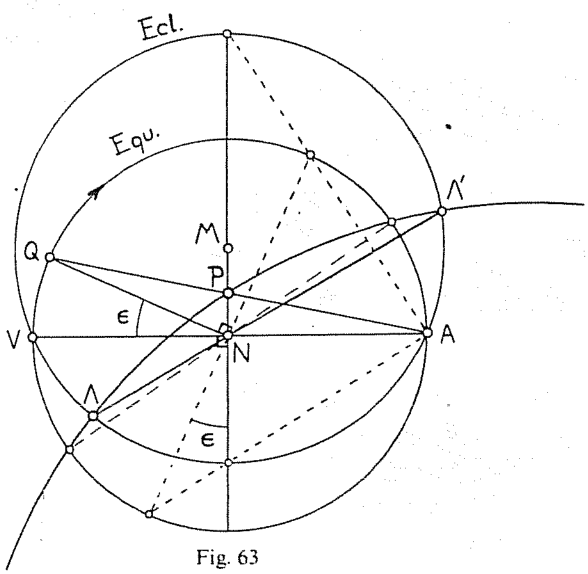


Fig. 63

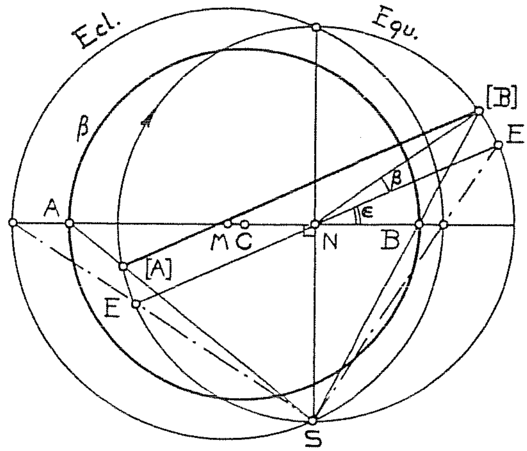


Fig. 64

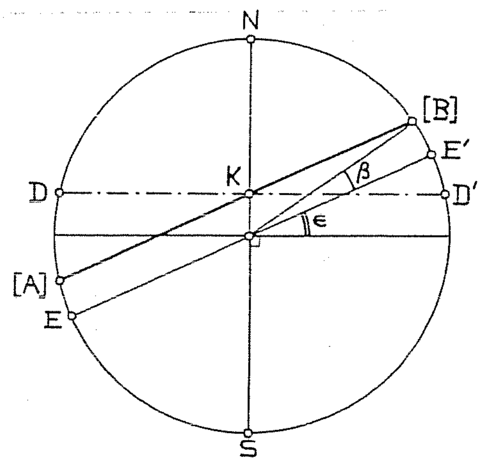


Fig. 65

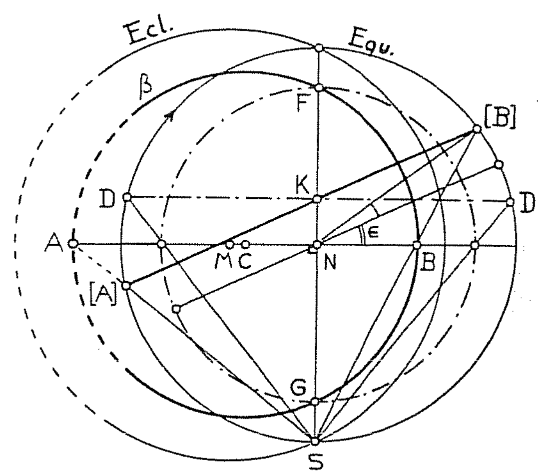
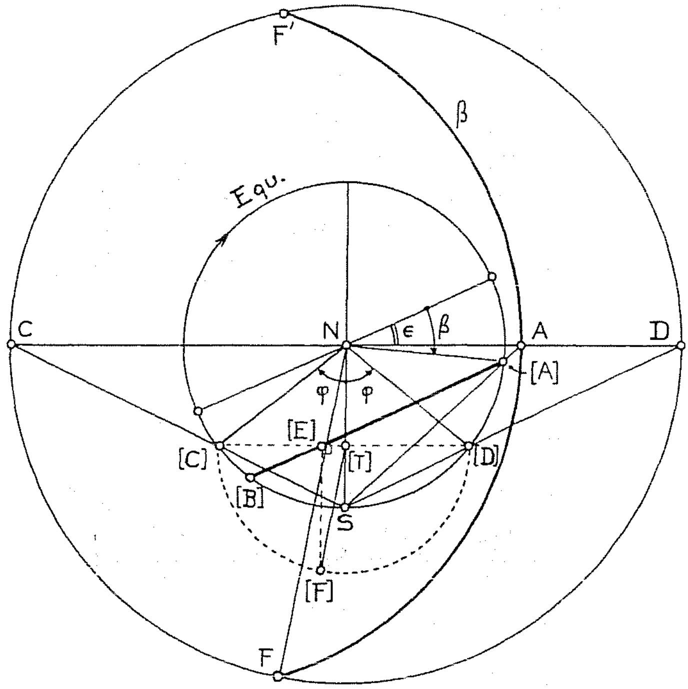


Fig. 66


F'
β
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β
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[A]
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φ
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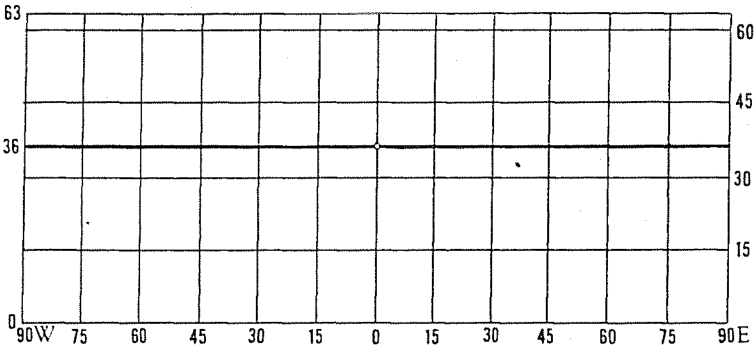
F

Fig. 67


63
36
0
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90E

Fig. 68

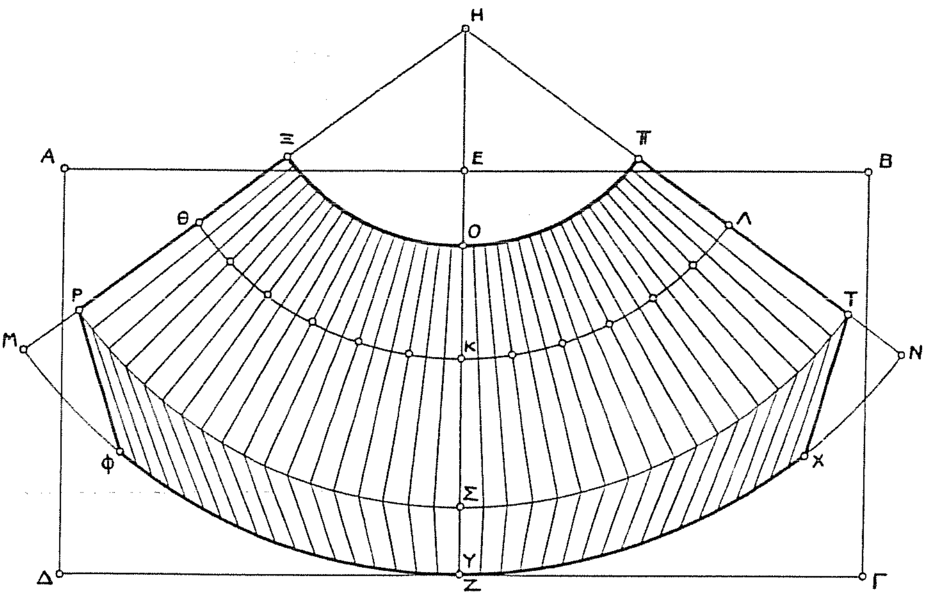

Fig. 69

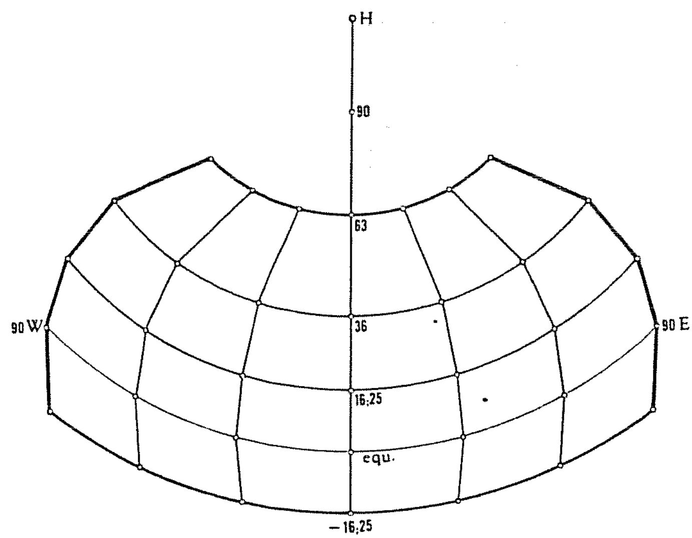

Fig. 70

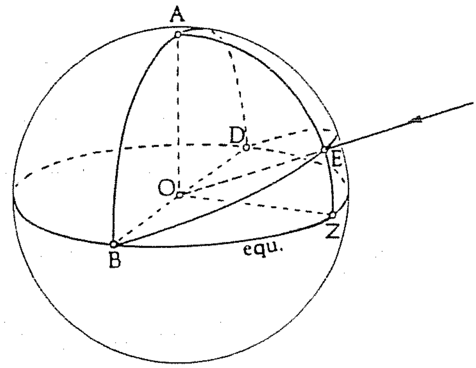

Fig. 71

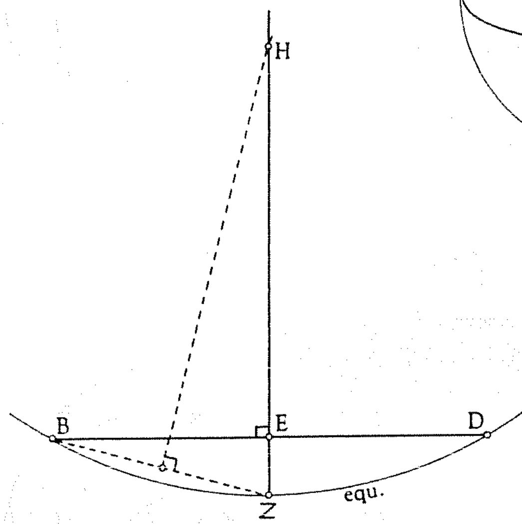

Fig. 72

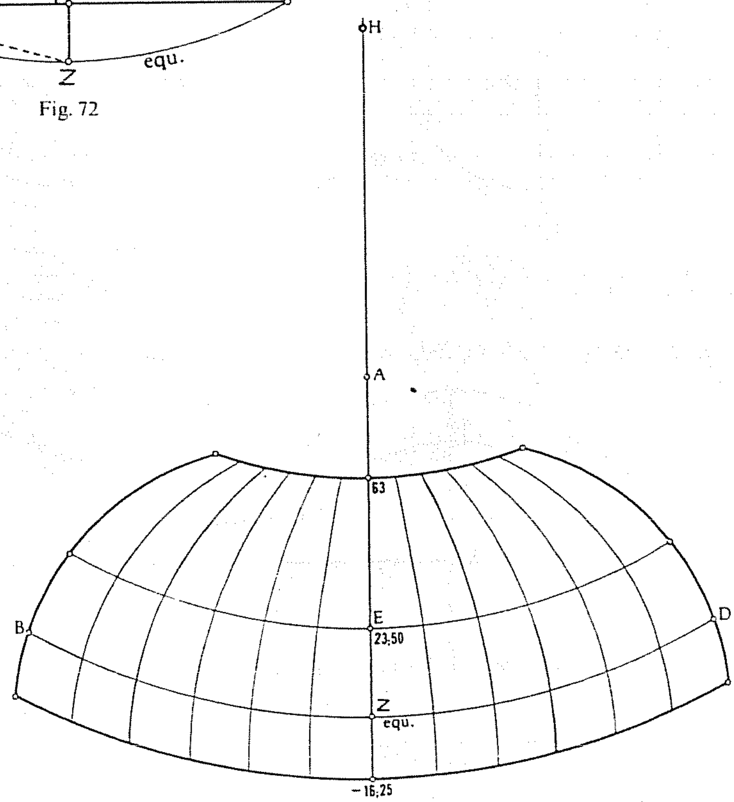

Fig. 73

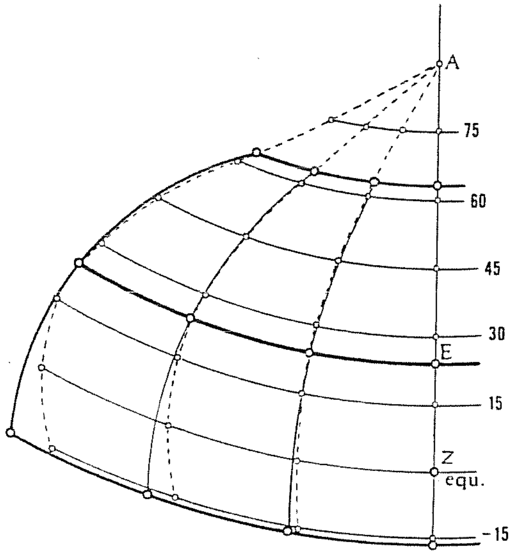

Fig. 74

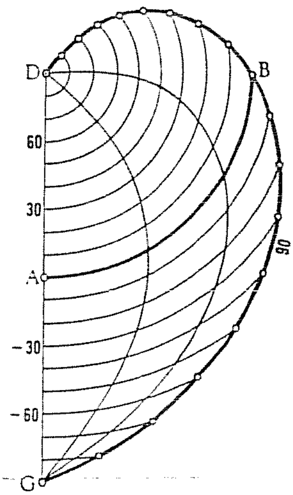

Fig. 75

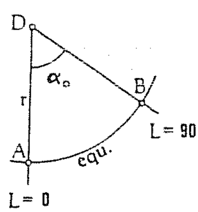

Fig. 76

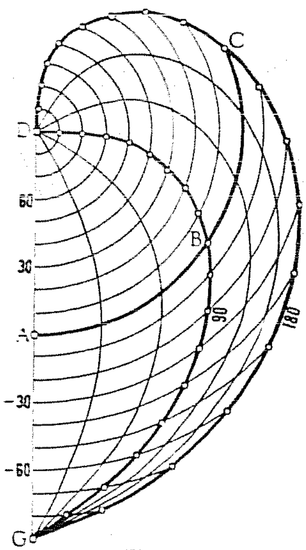

Fig. 77

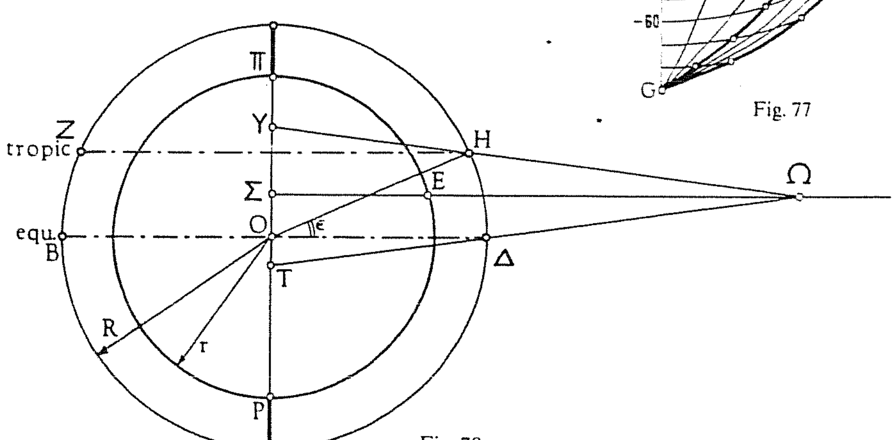

Fig. 78

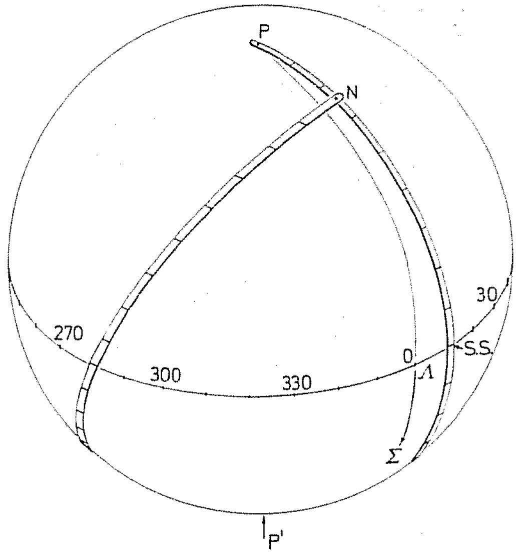
P
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Fig. 79

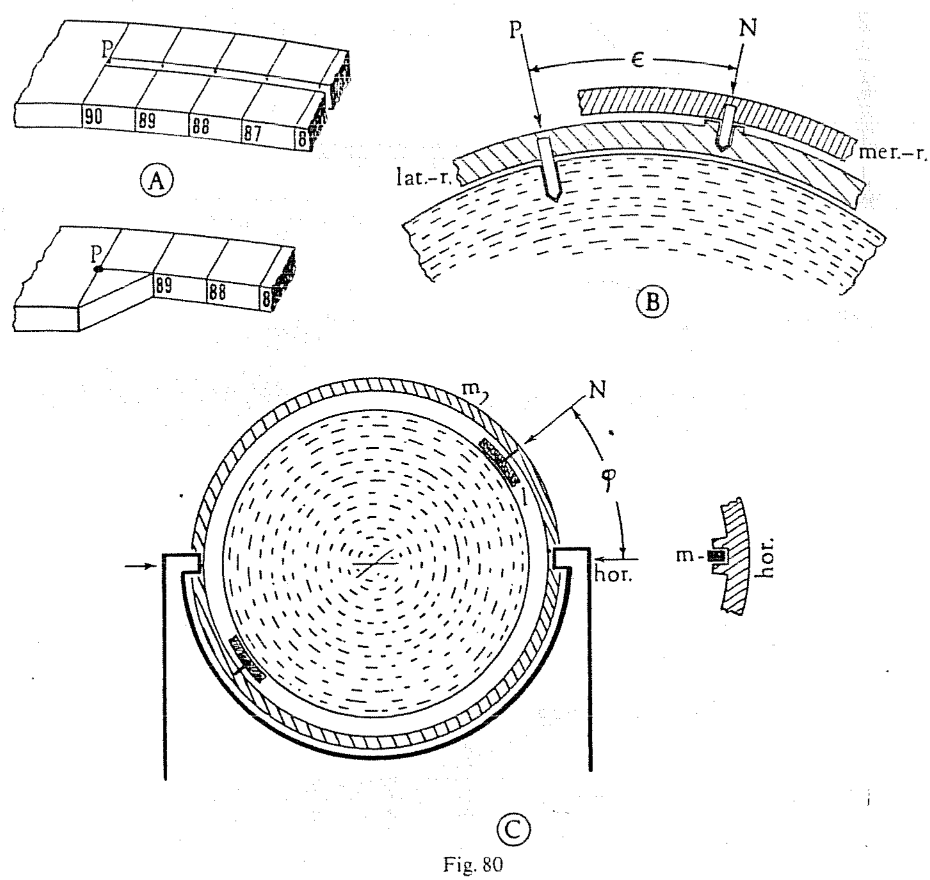
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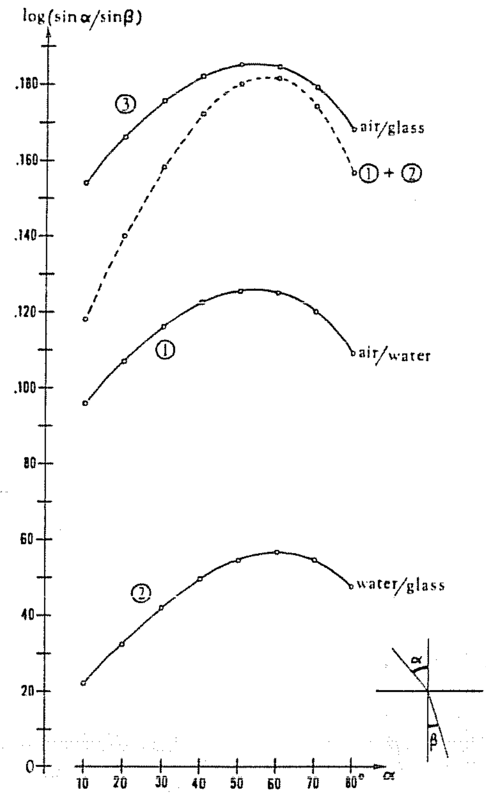
Fig. 80


log(sin α/sin β)
.180
.160
.140
.120
.100
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0
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30
40
50
60
70
80°
α
③
air/glass
① + ②
①
air/water
②
water/glass
α
β

Fig. 81

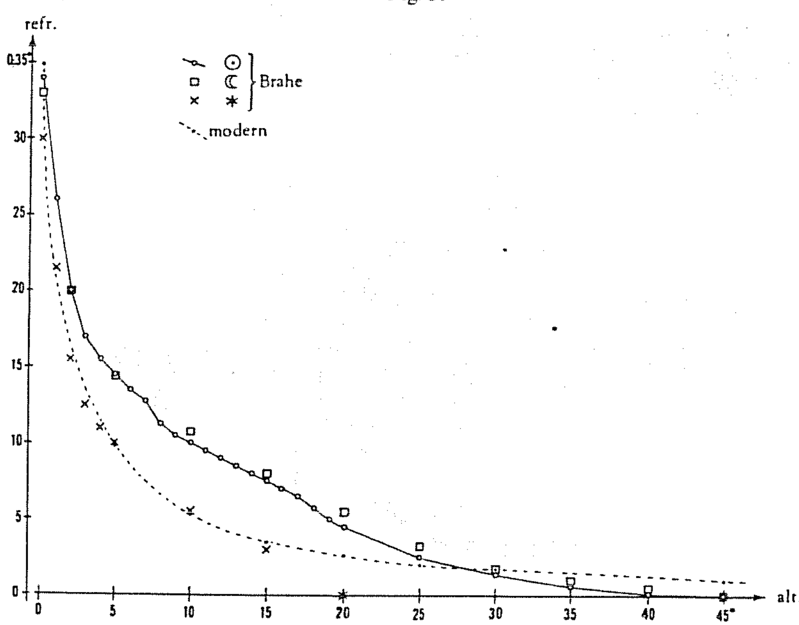
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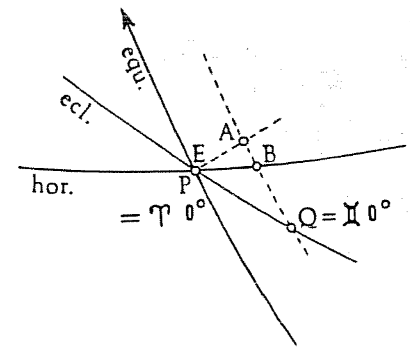
Fig. 82


equ.
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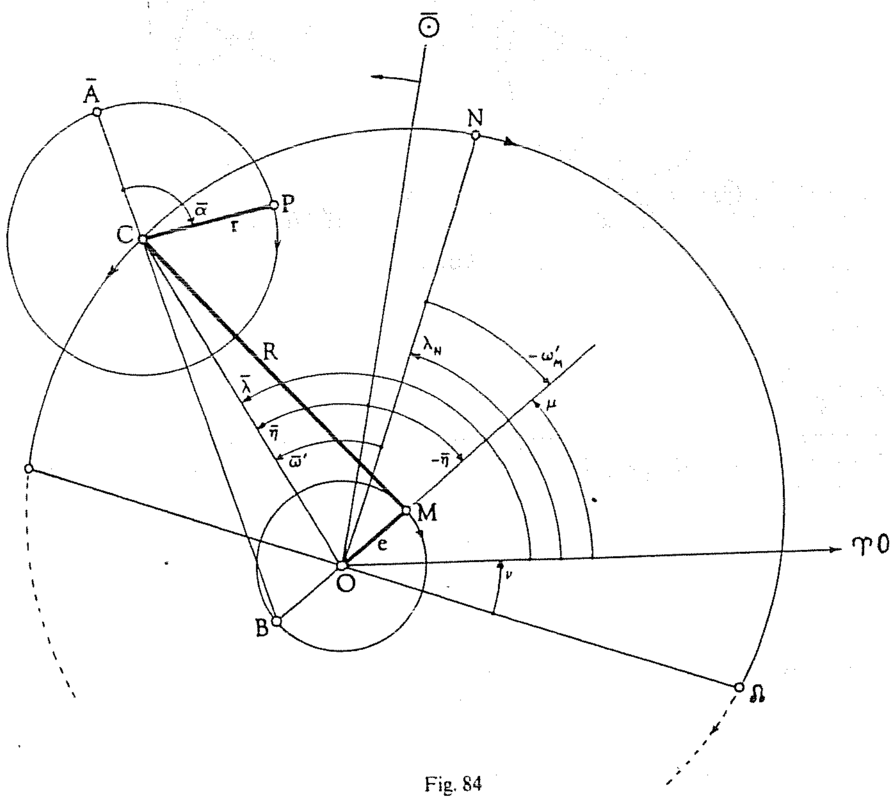
hor.
A
E
B
P
$= \Upsilon\ 0°$
$Q = \text{♊}\ 0°$

Fig. 83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$\bar{\eta}$
$\bar{\omega}'$
λ_N
$-\omega_M'$
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$-\bar{\eta}$
M
e
O
B
ν
$\Upsilon 0$
☊

Fig. 84

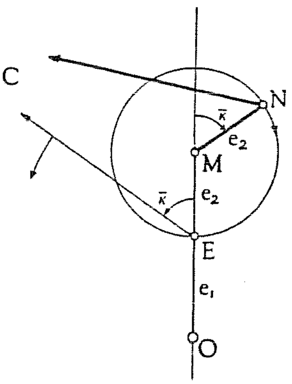


Fig. 85

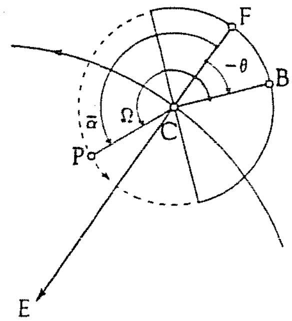


Fig. 87

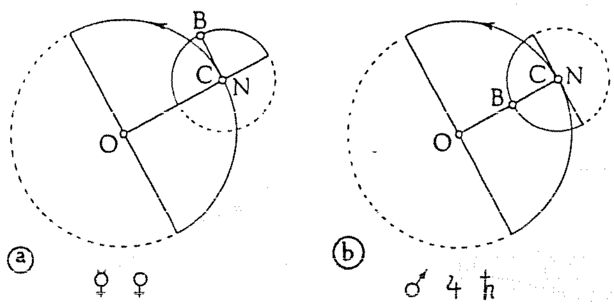


Fig. 86

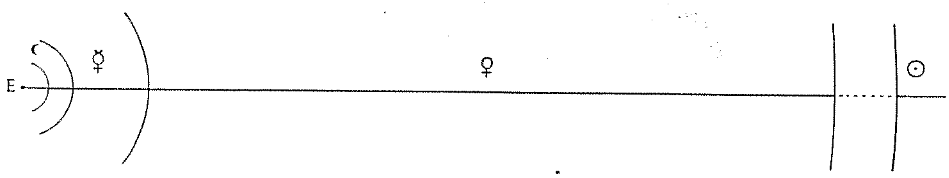


Fig. 88

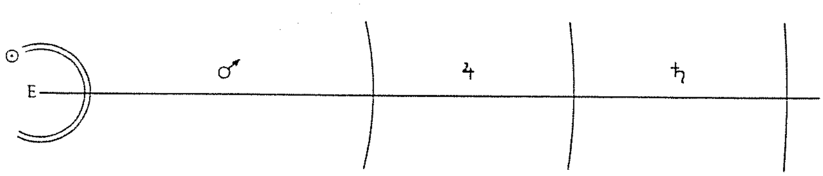


Fig. 89

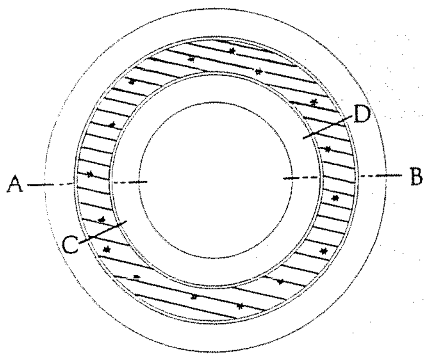


Fig. 90

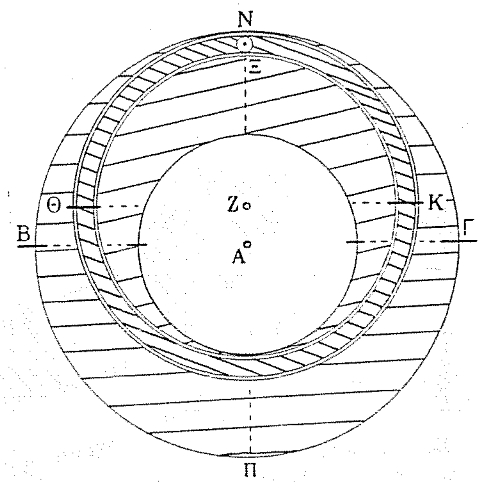


Fig. 91

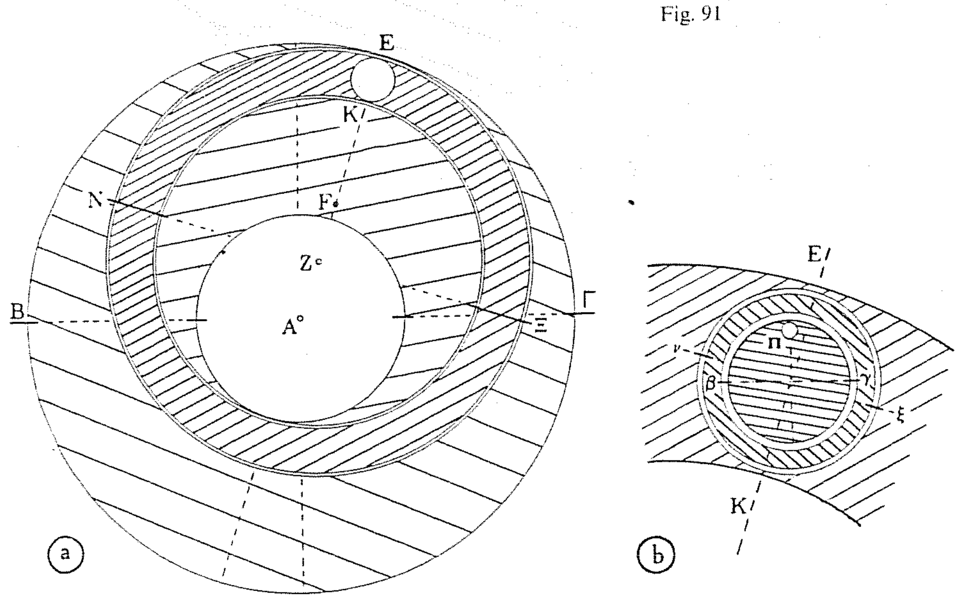


Fig. 92

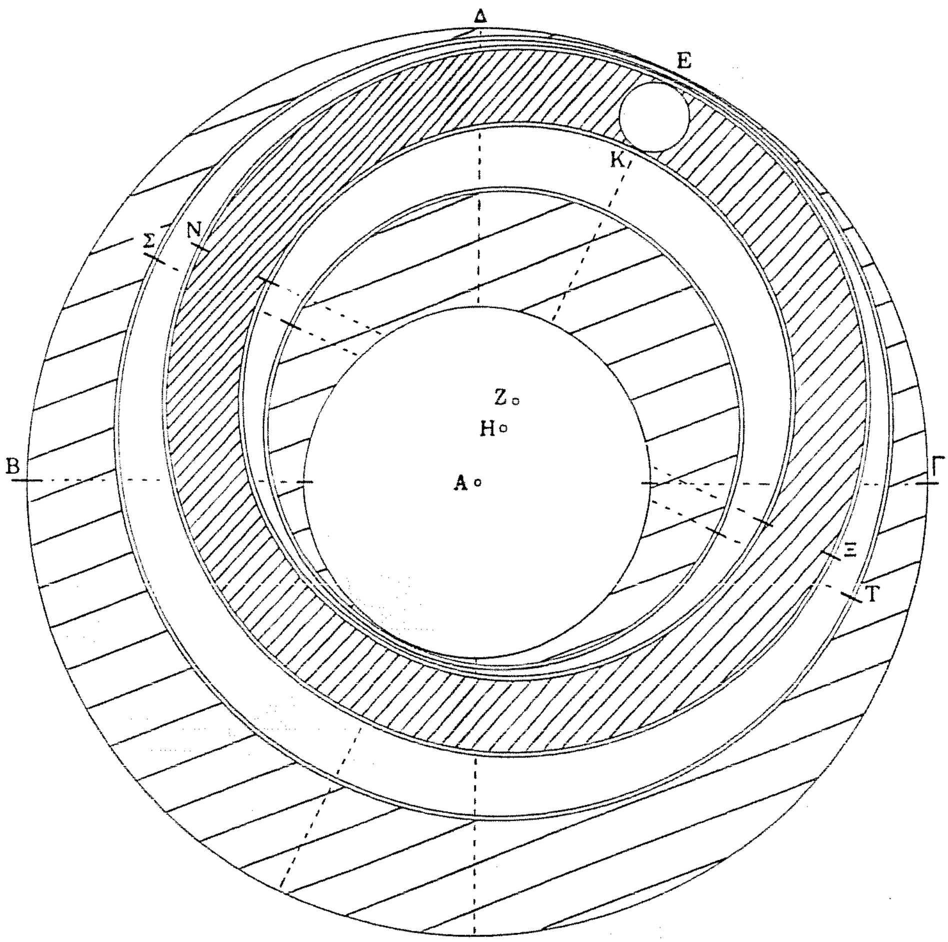


Fig. 93

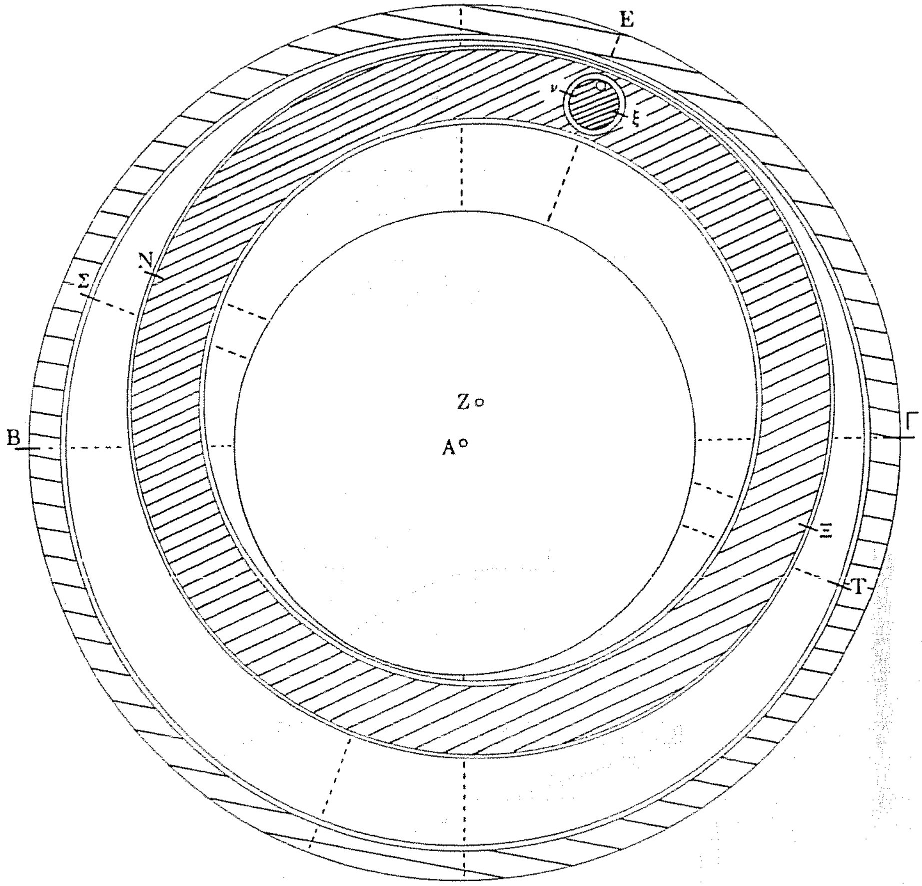


Fig. 94

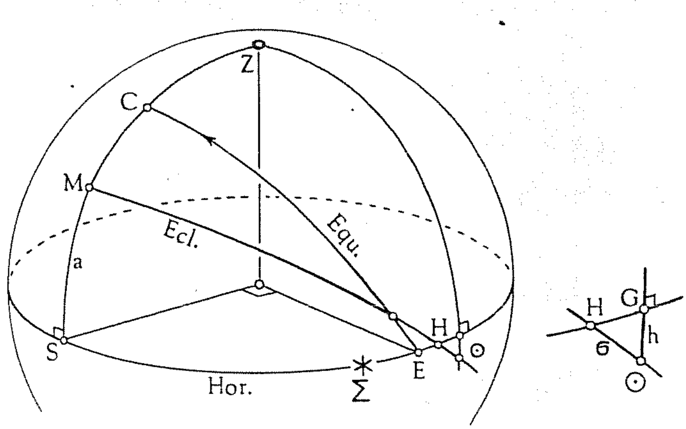


Fig. 95

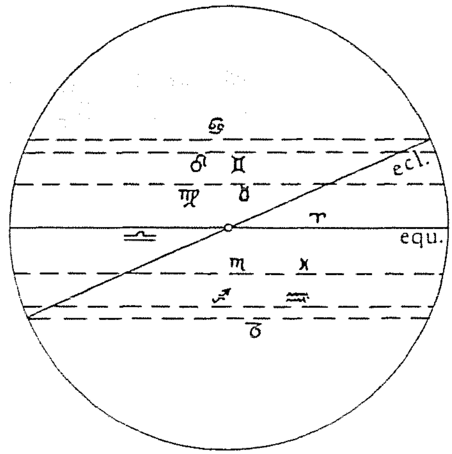


Fig. 96

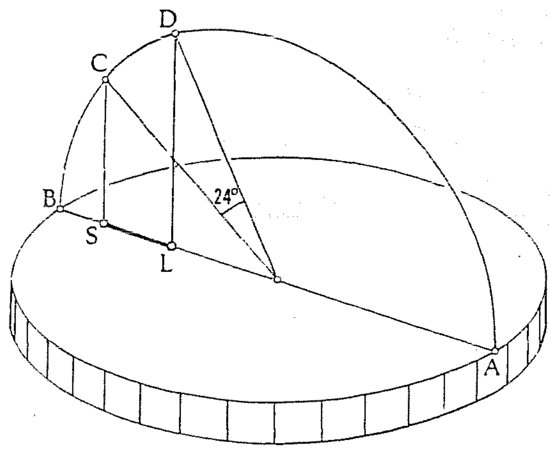


Fig. 97

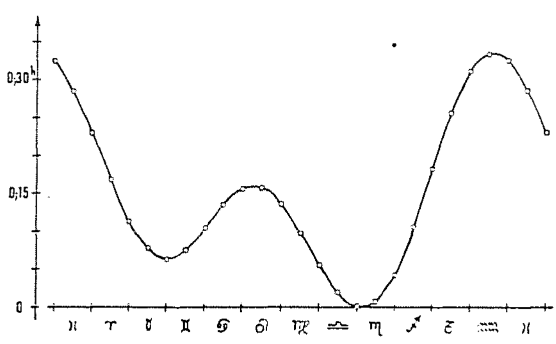


Fig. 98

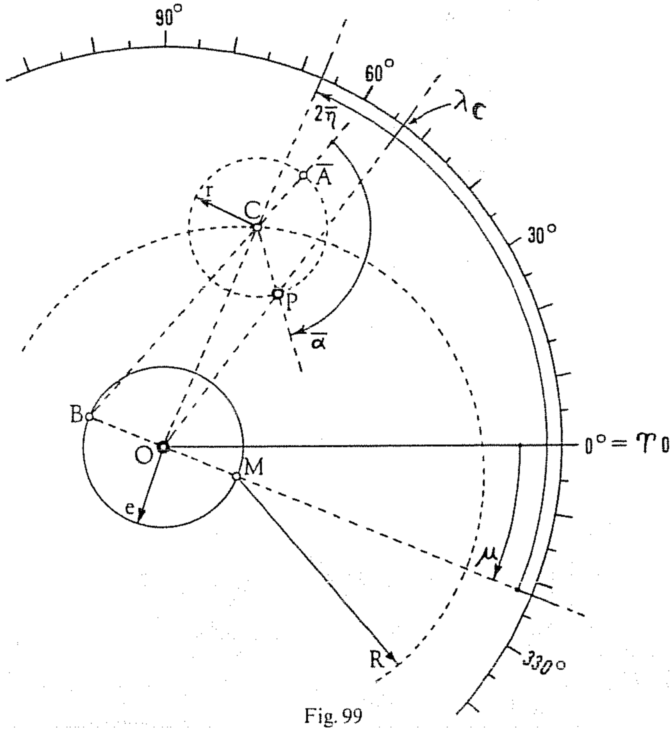


Fig. 99

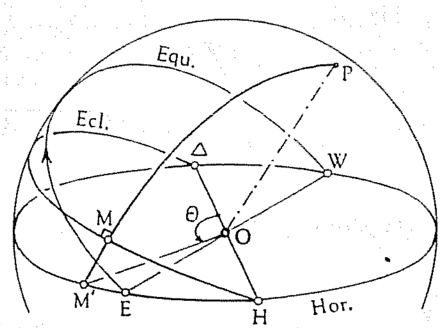


Fig. 100

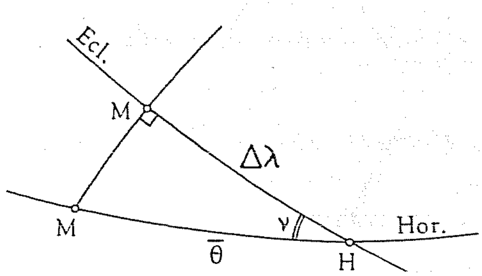


Fig. 101

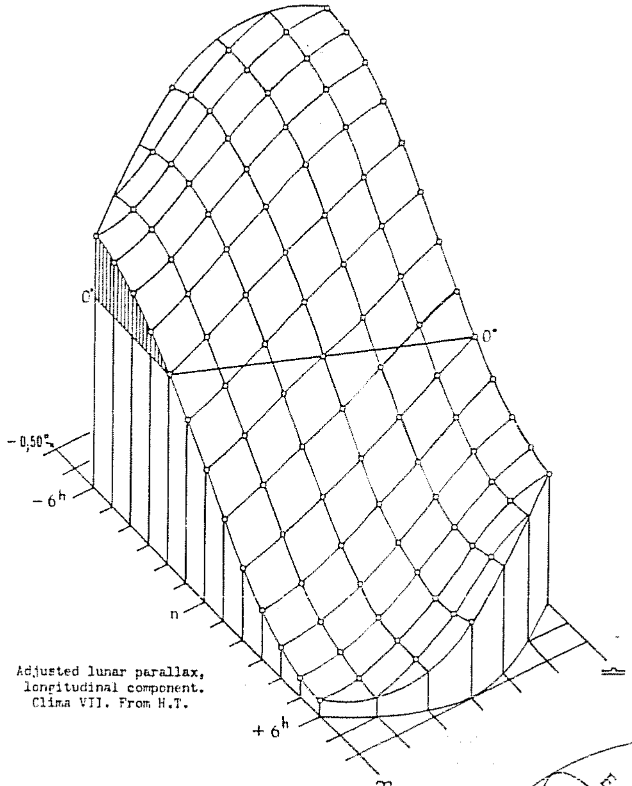


Fig. 102

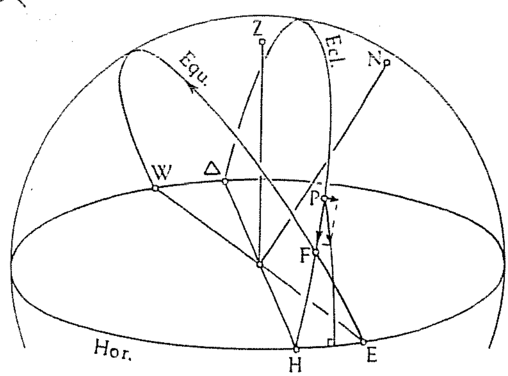
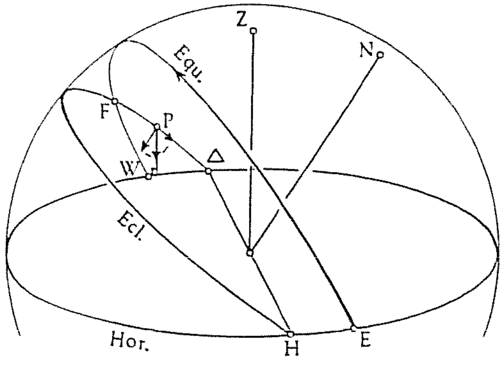


Fig. 103



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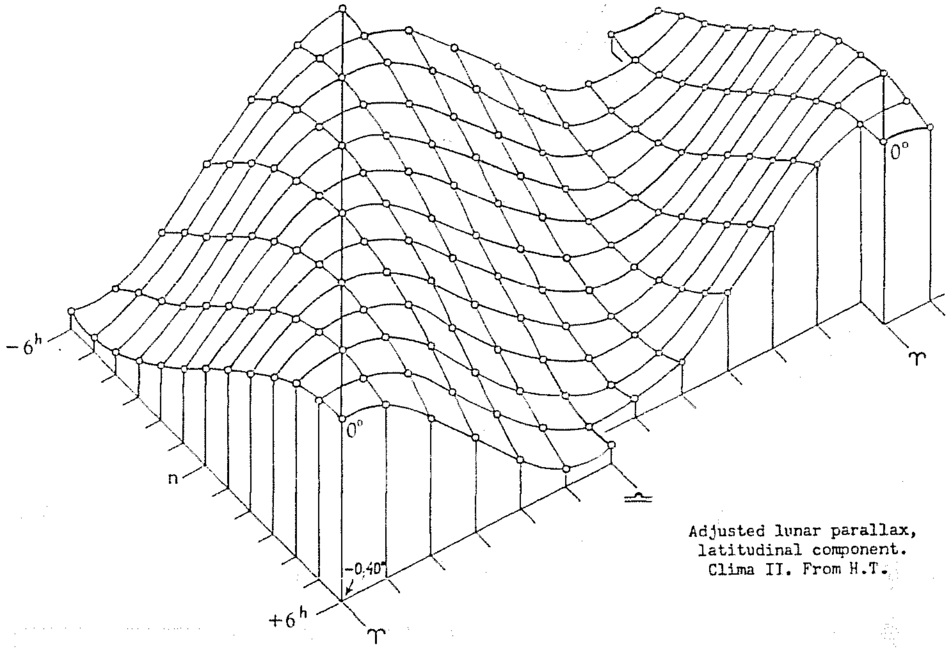


Fig. 105

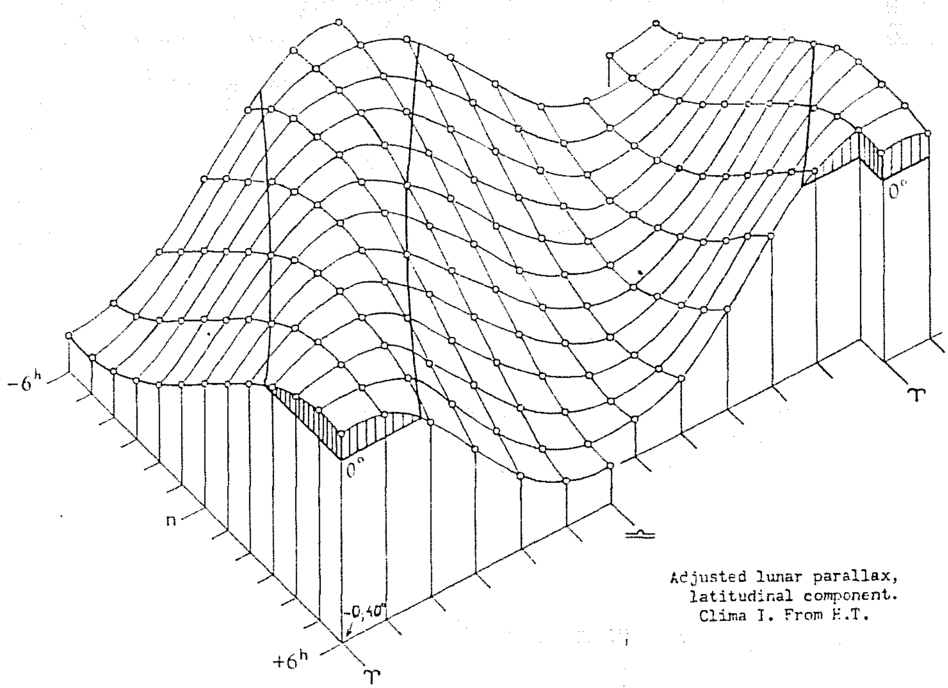
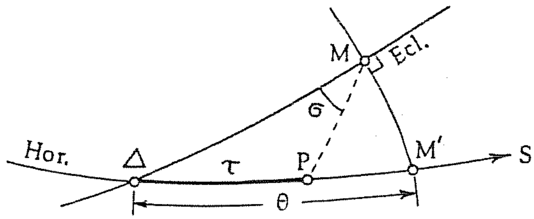


Fig. 106



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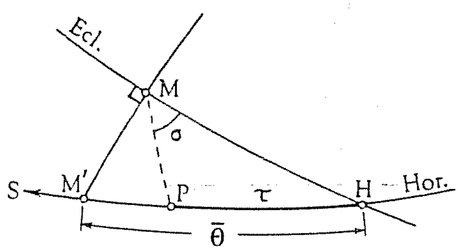

Fig. 108

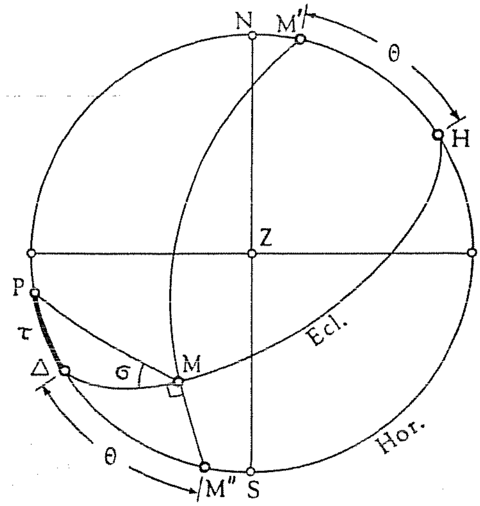

Fig. 109

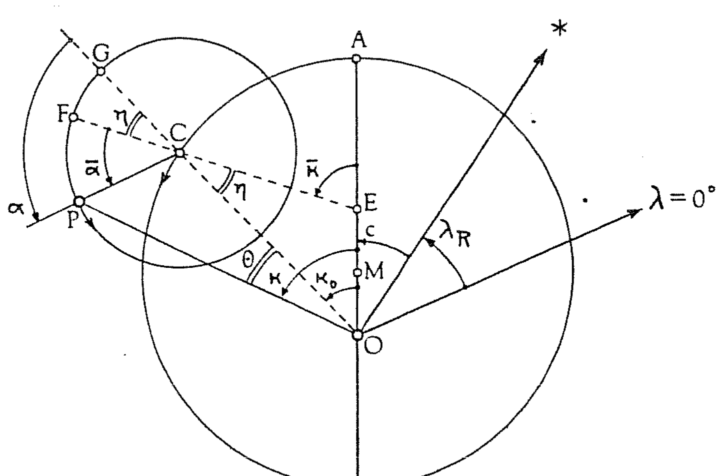

Fig. 110

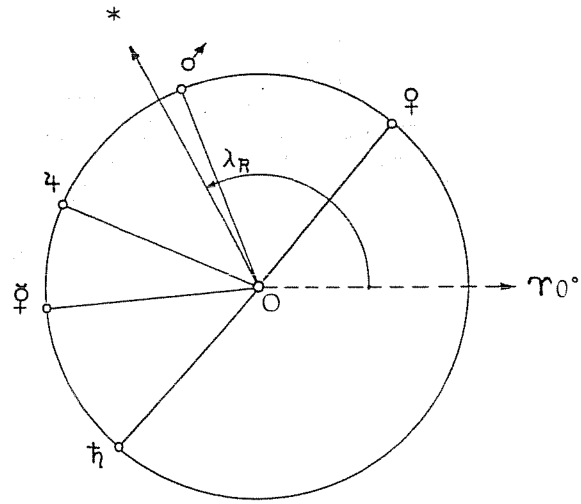

Fig. 111

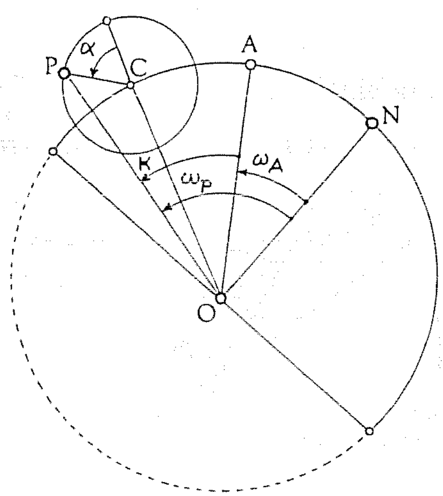

Fig. 112

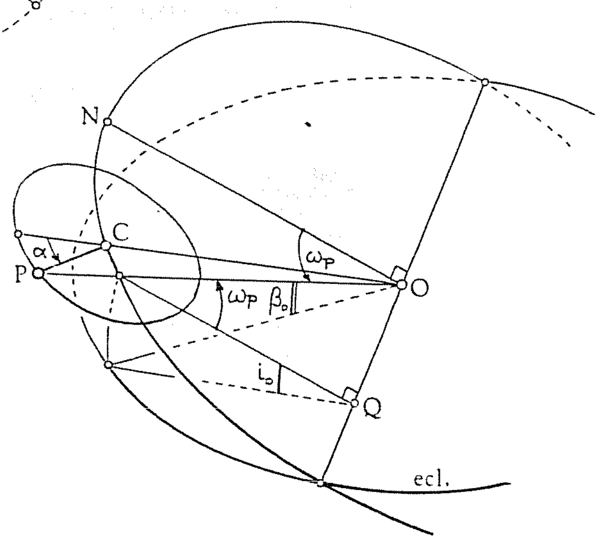

Fig. 113

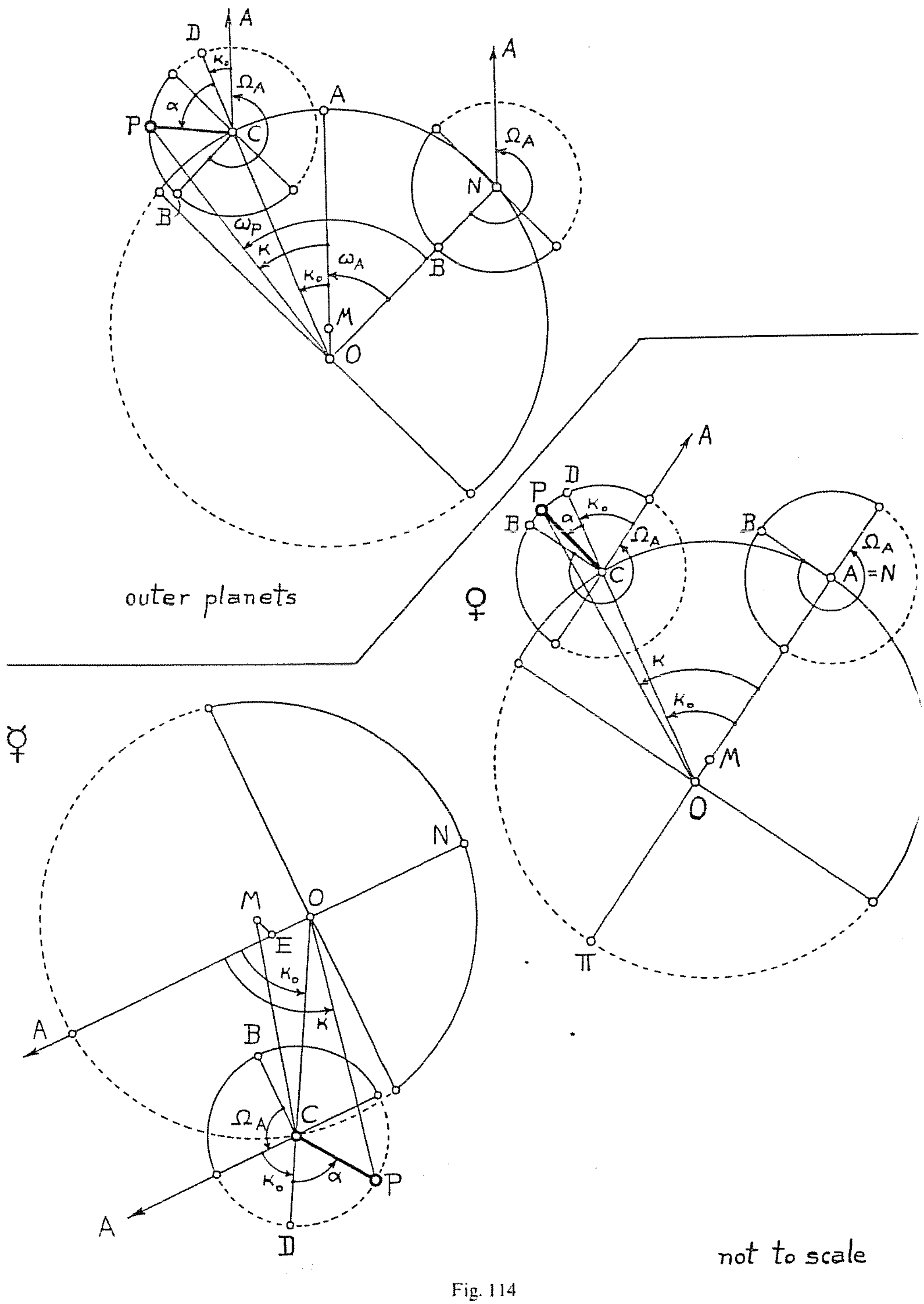
A
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Fig. 114

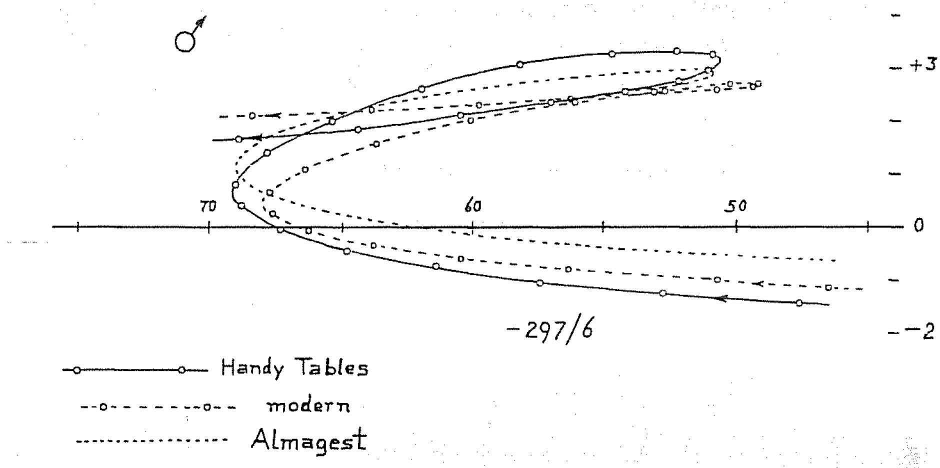
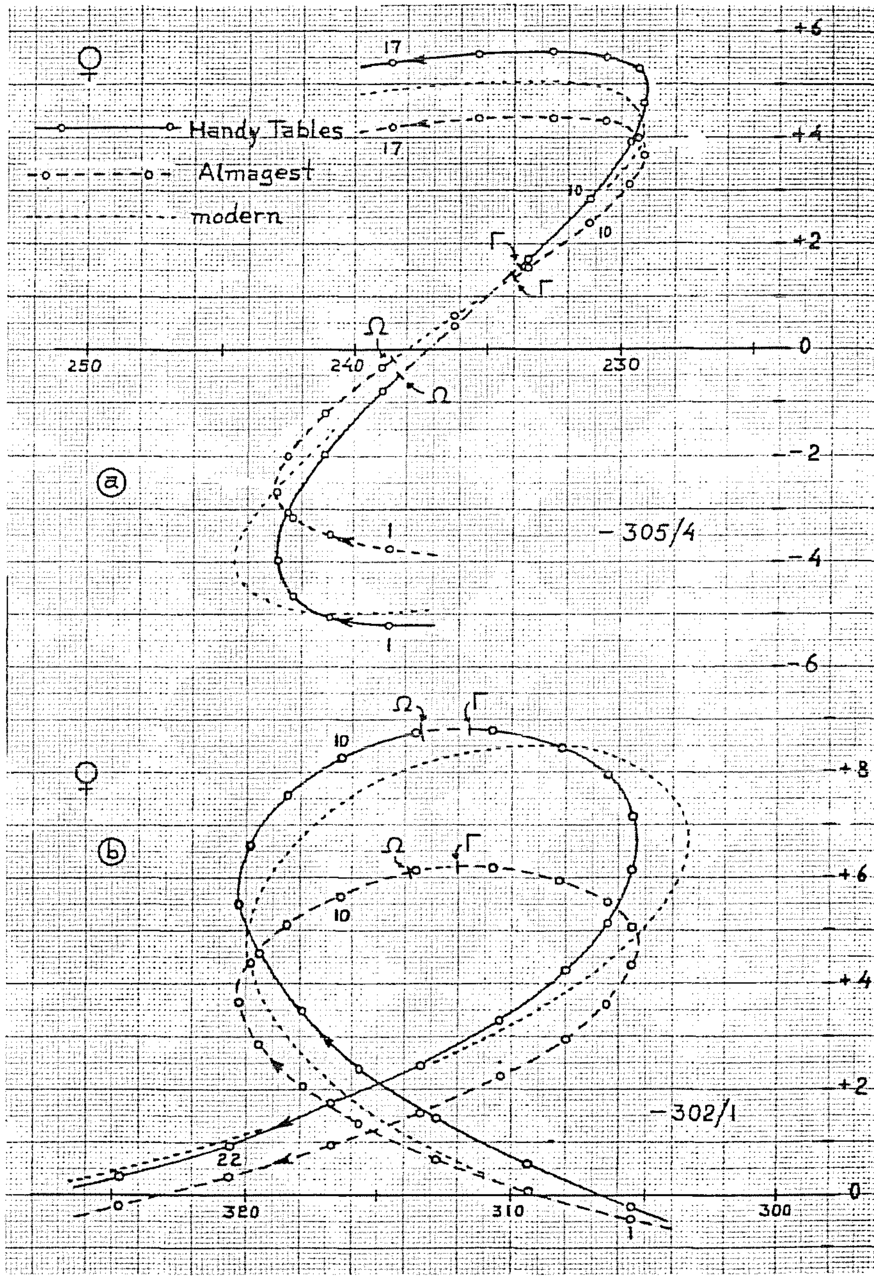


Fig. 115



♀
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Fig. 116

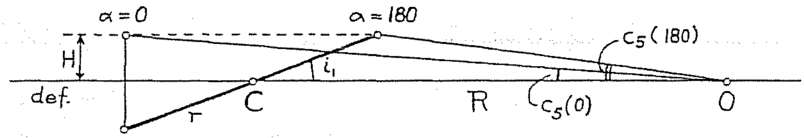

Fig. 117

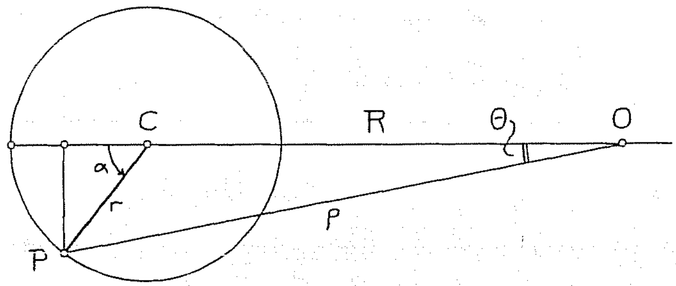

Fig. 118

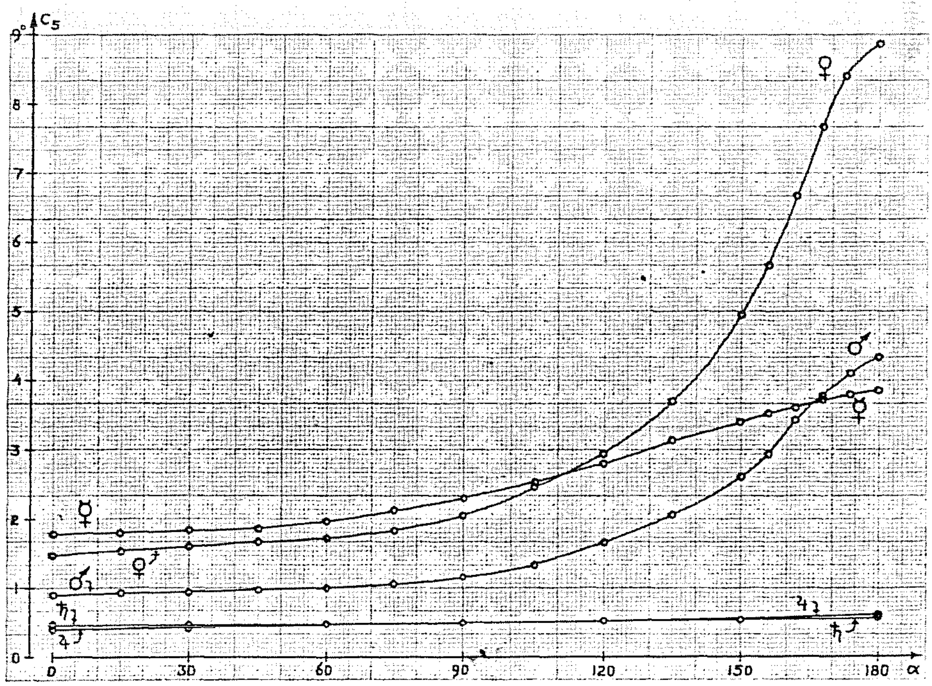

Fig. 119

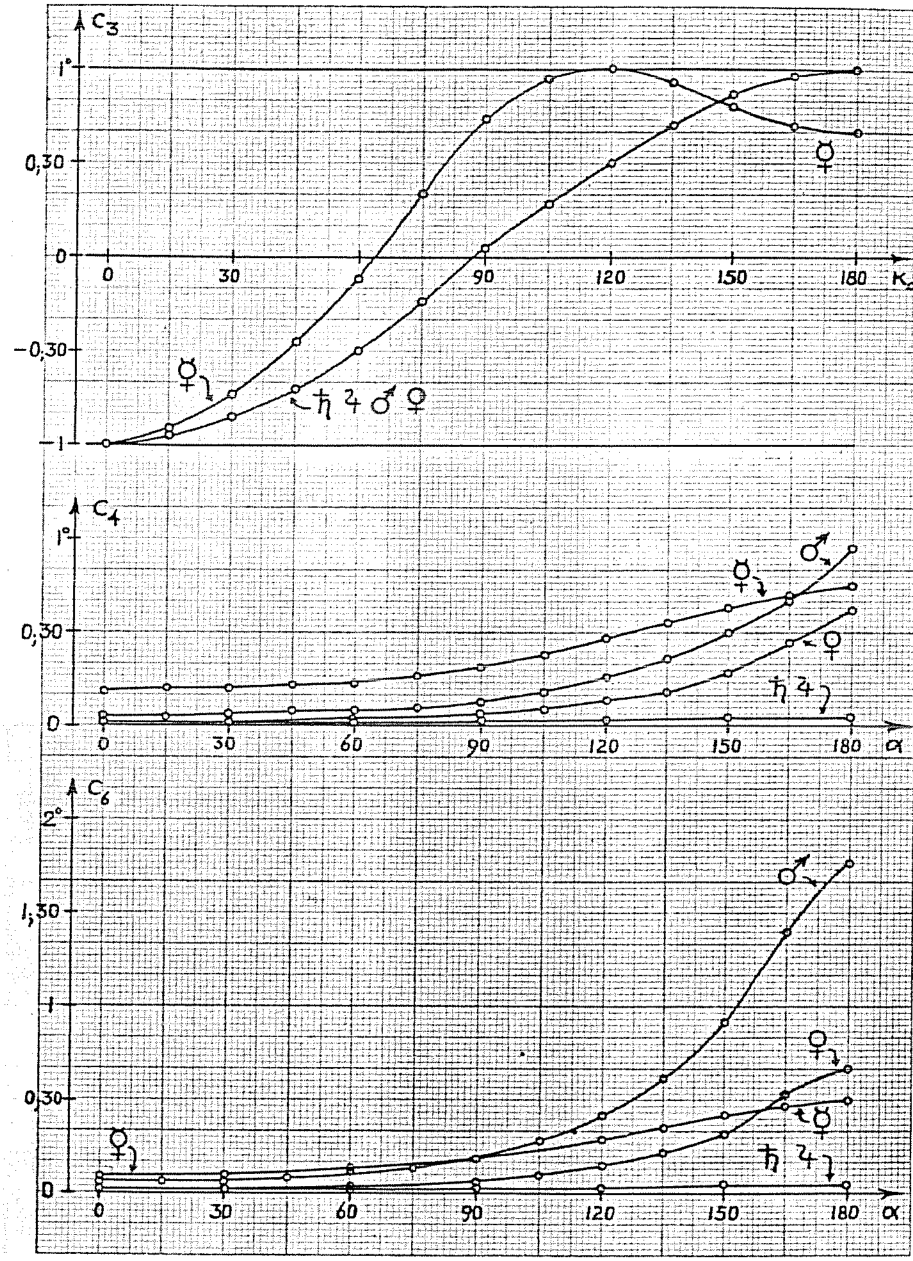

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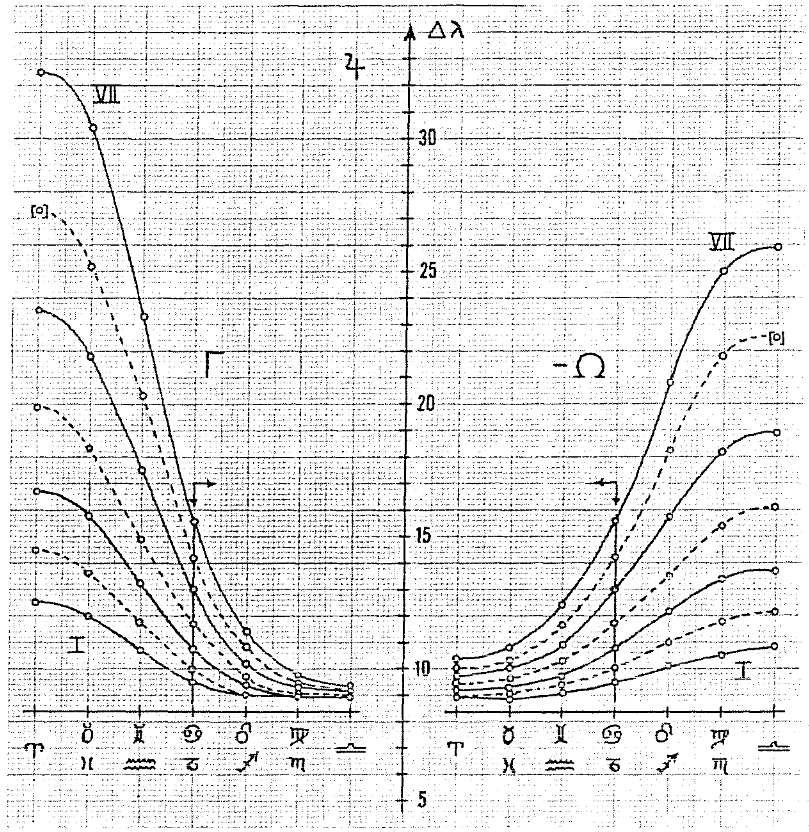
Fig. 120



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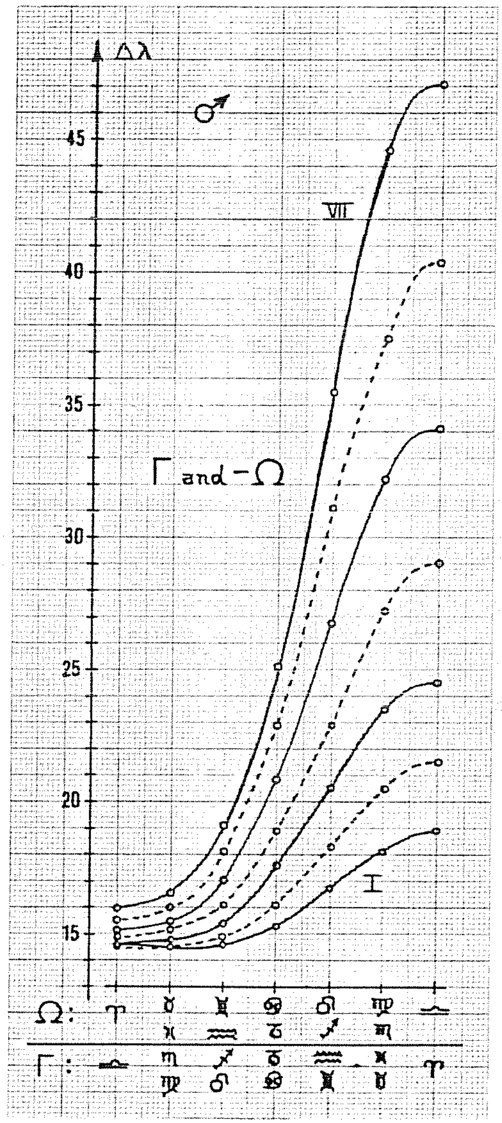


Fig. 122

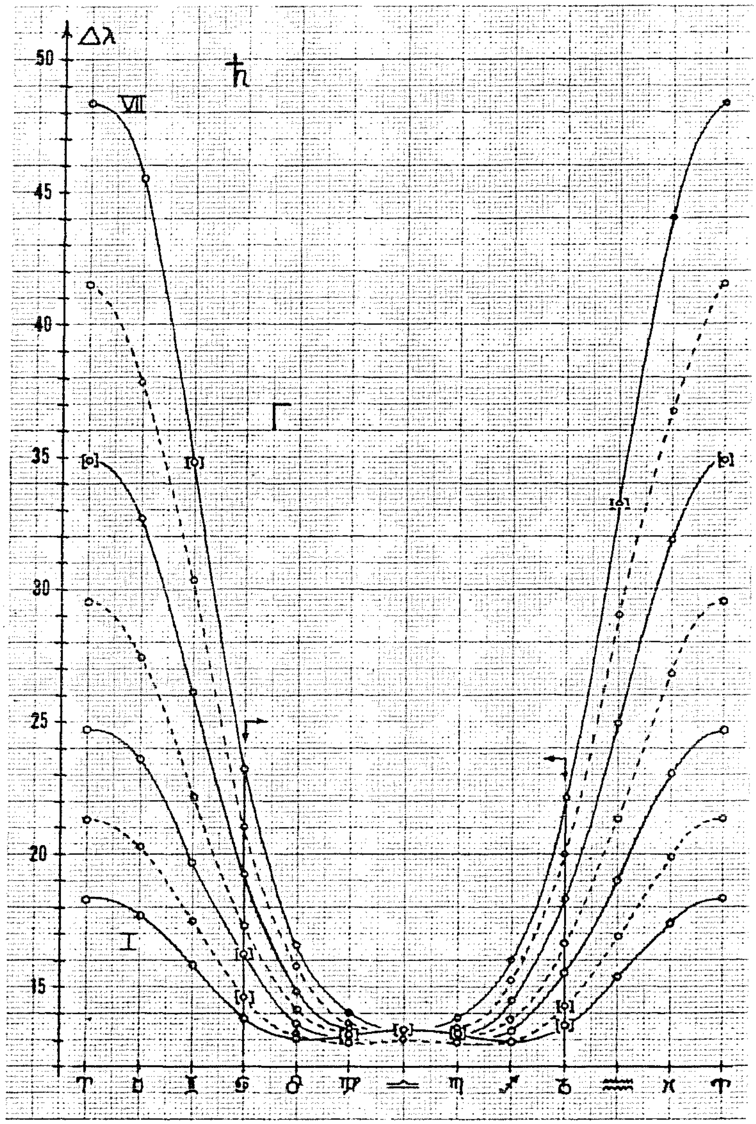
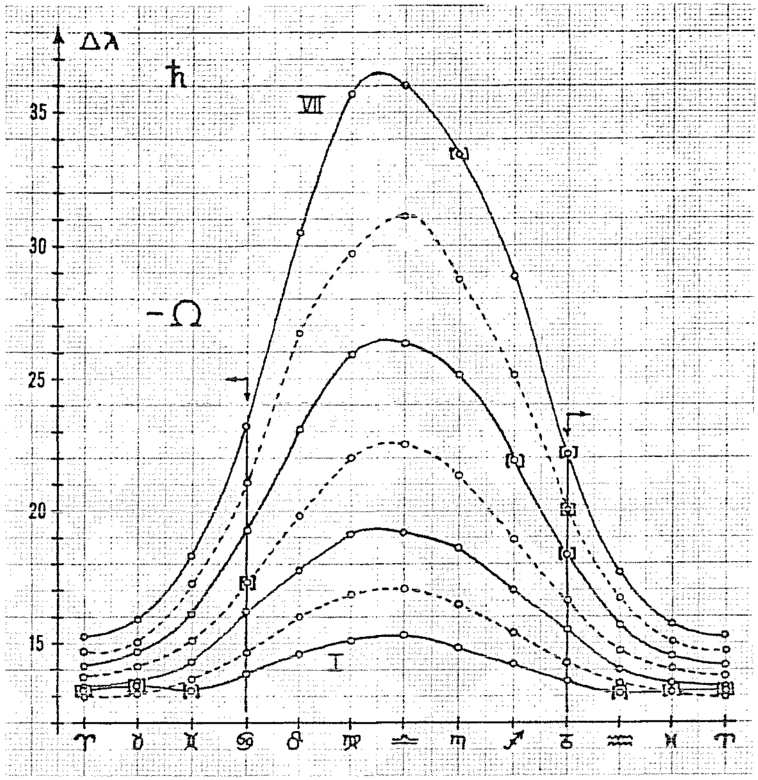


Fig. 123



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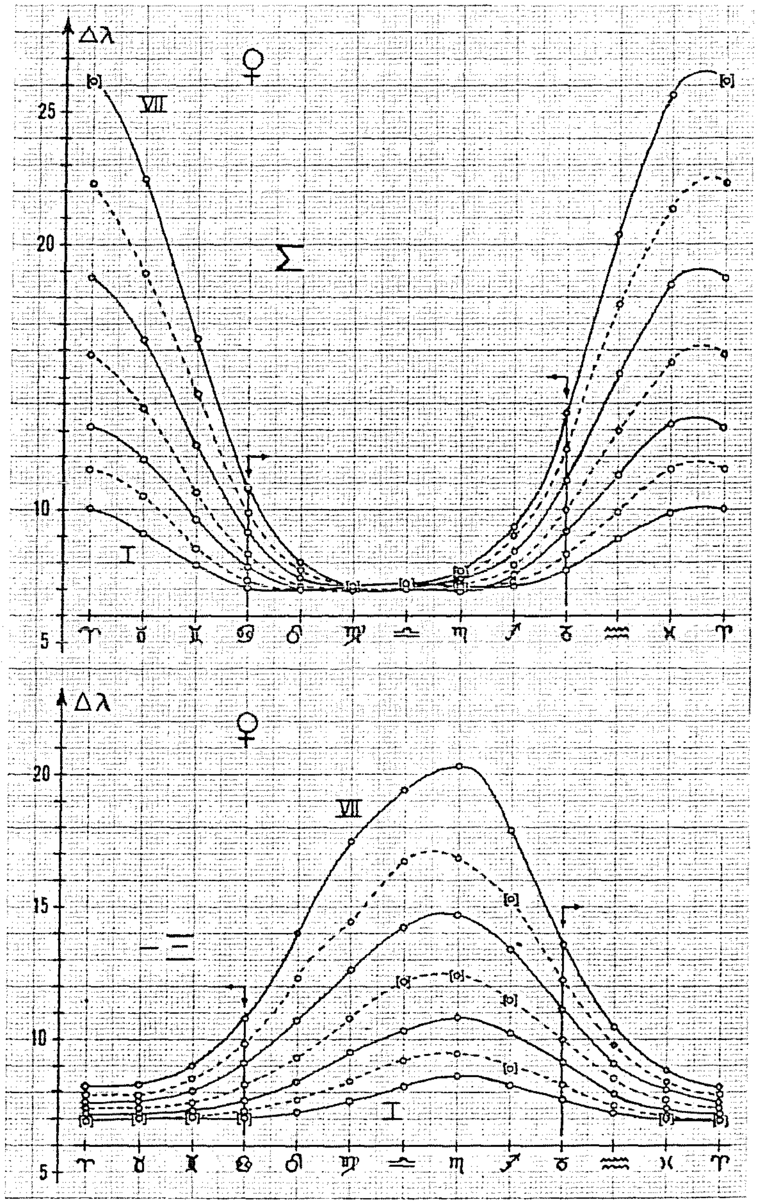

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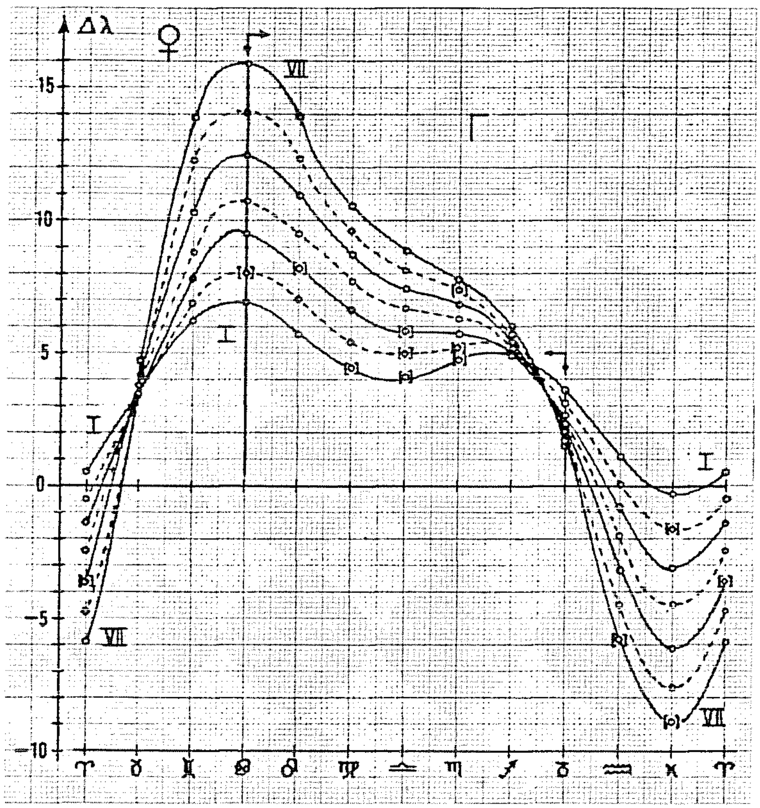

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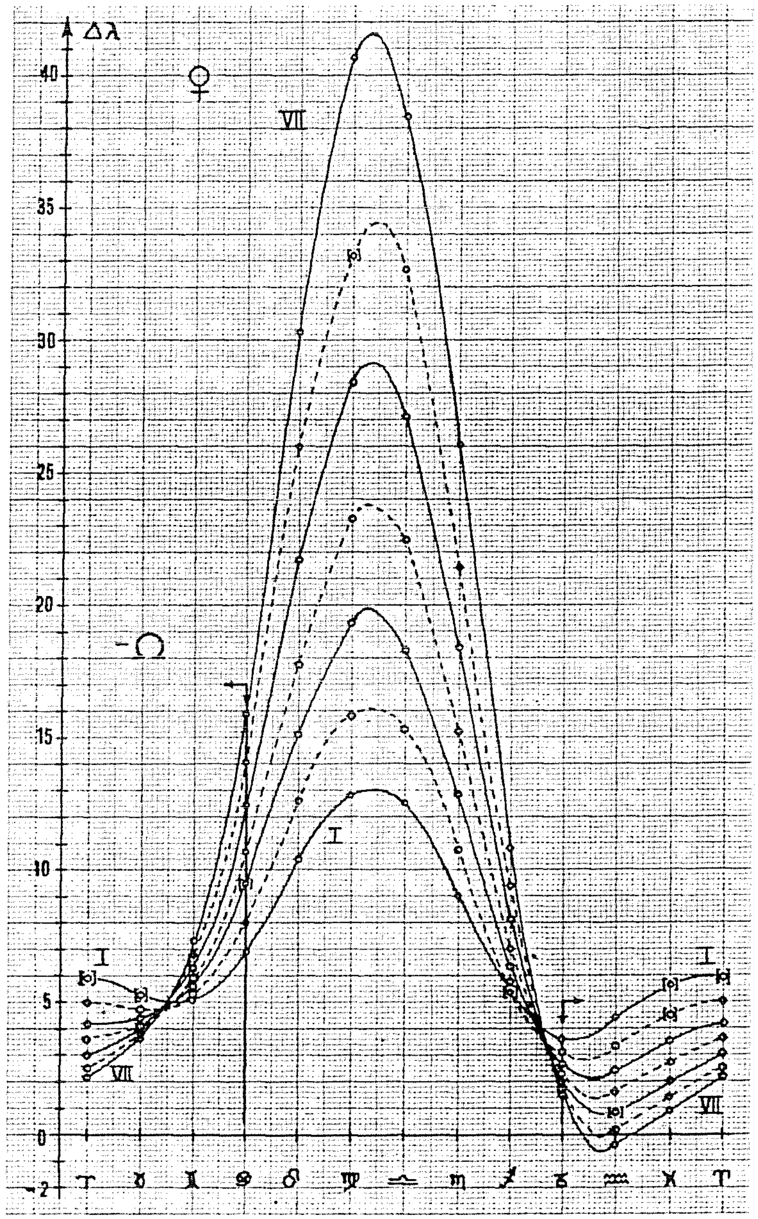


Fig. 127

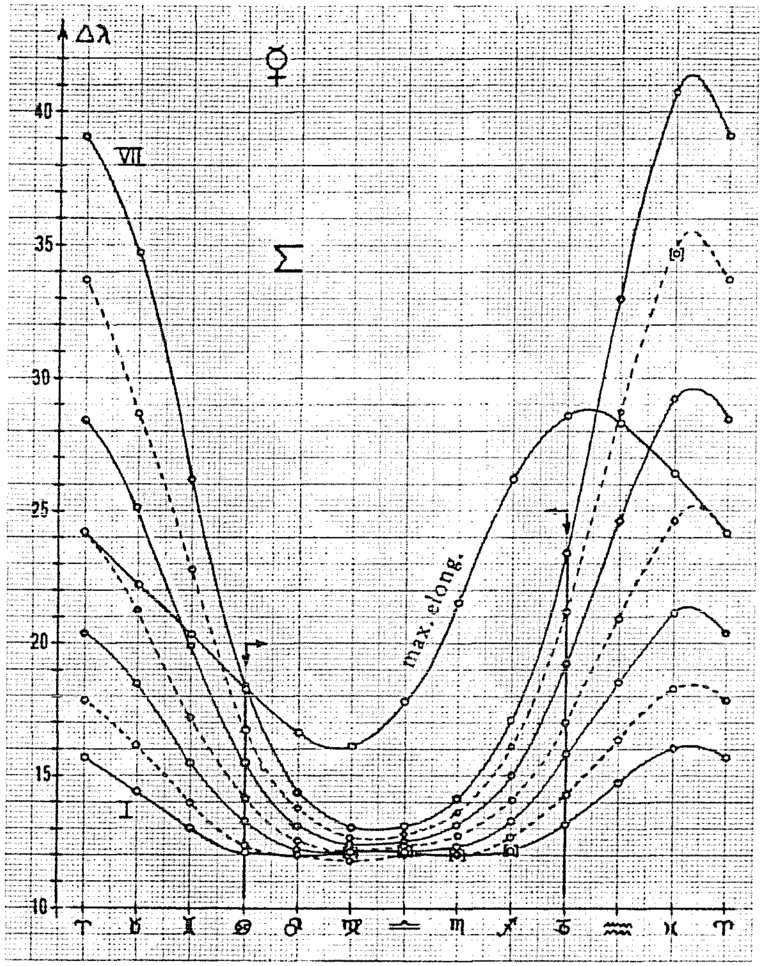
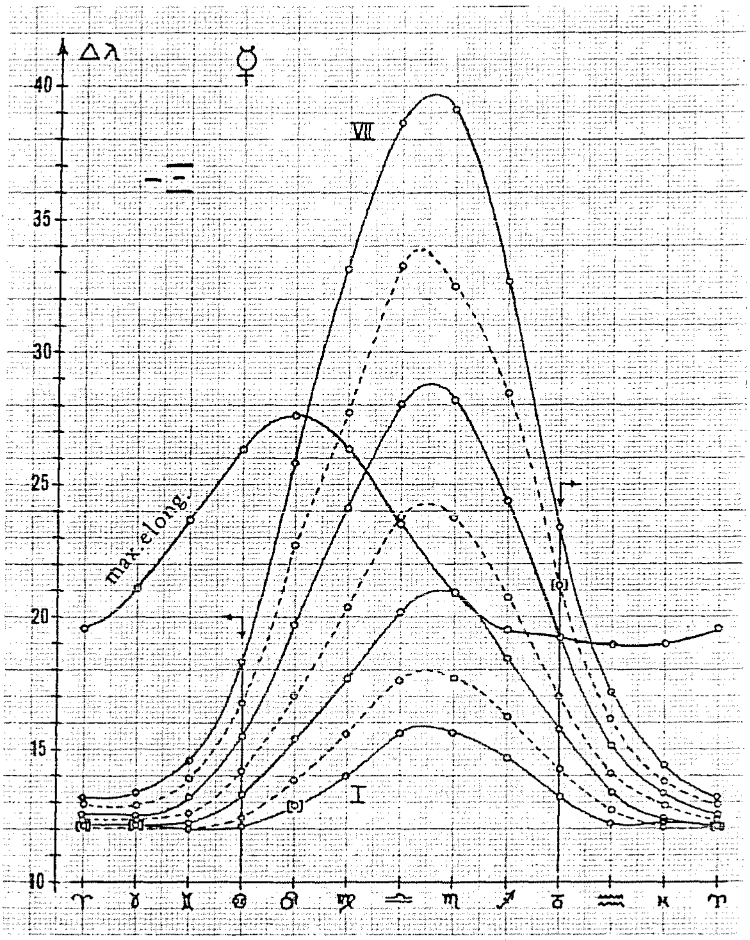


Fig. 128



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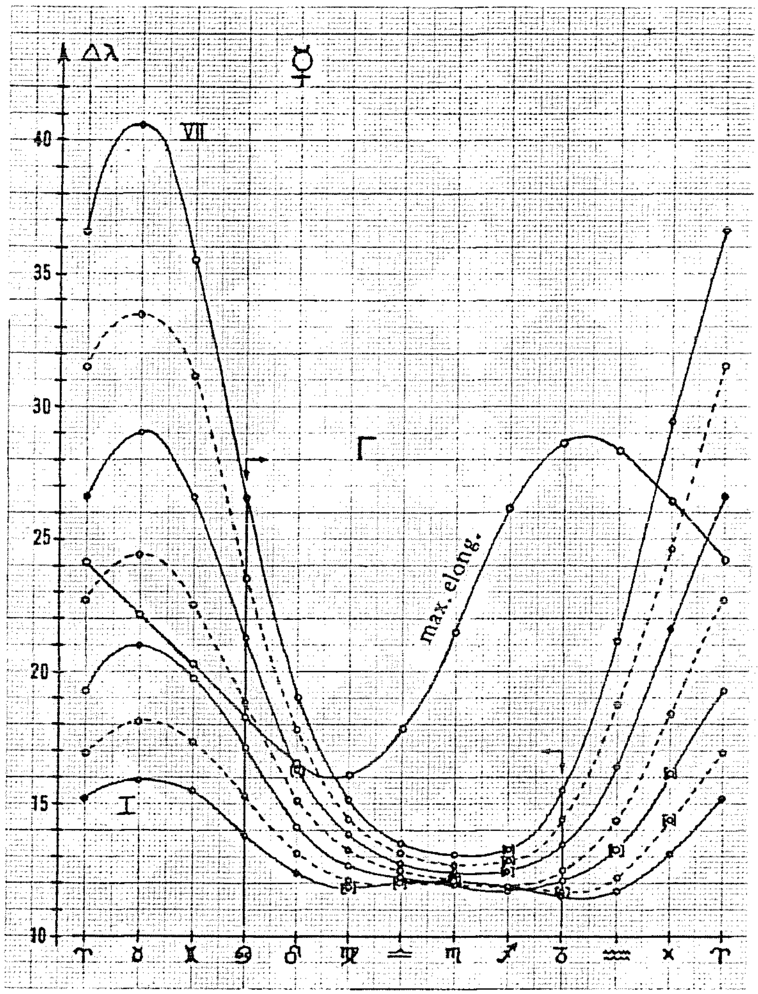

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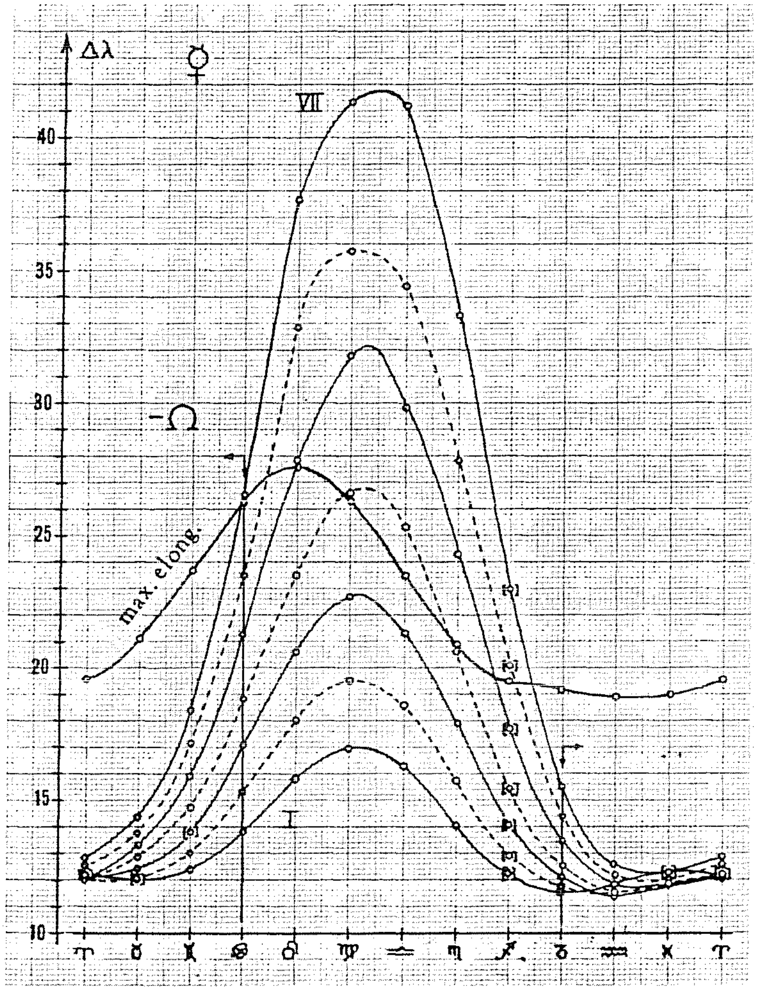

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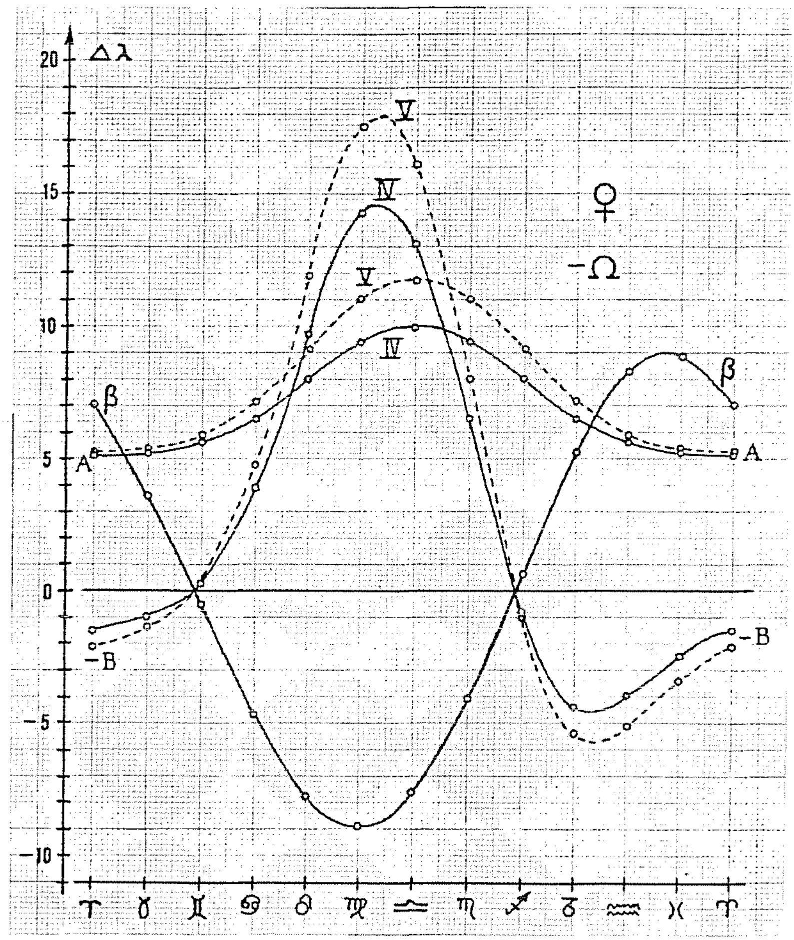

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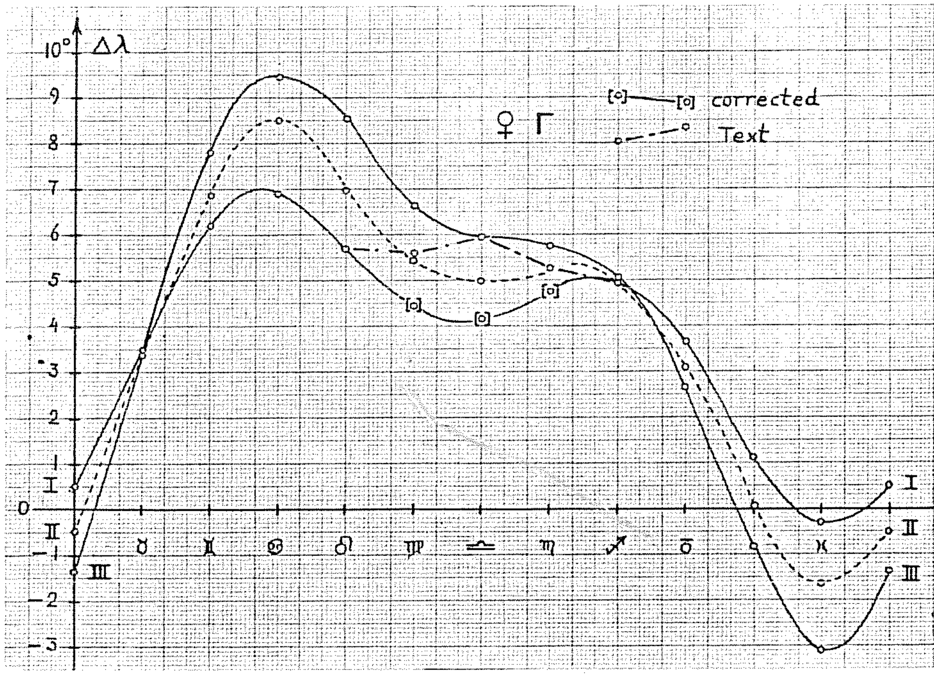

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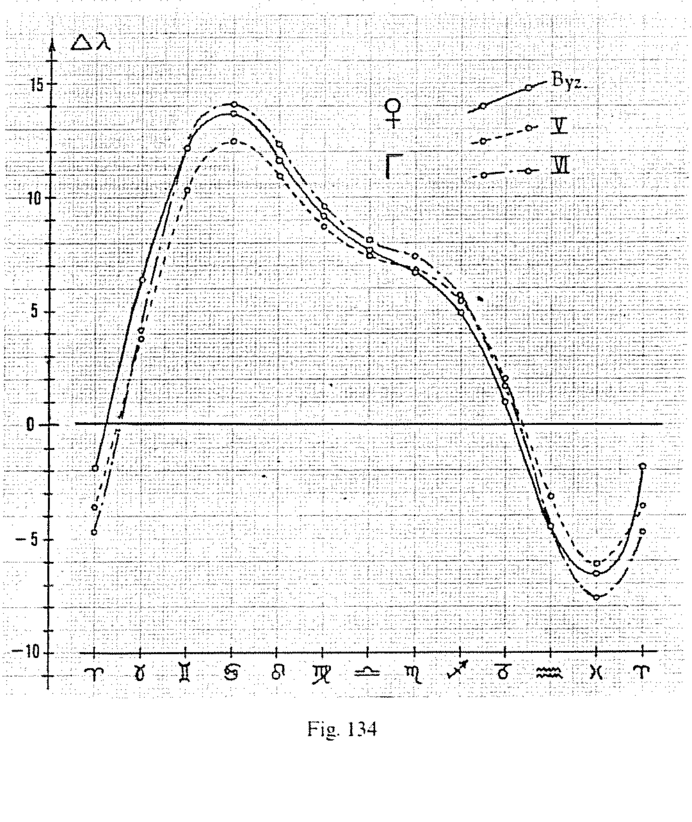

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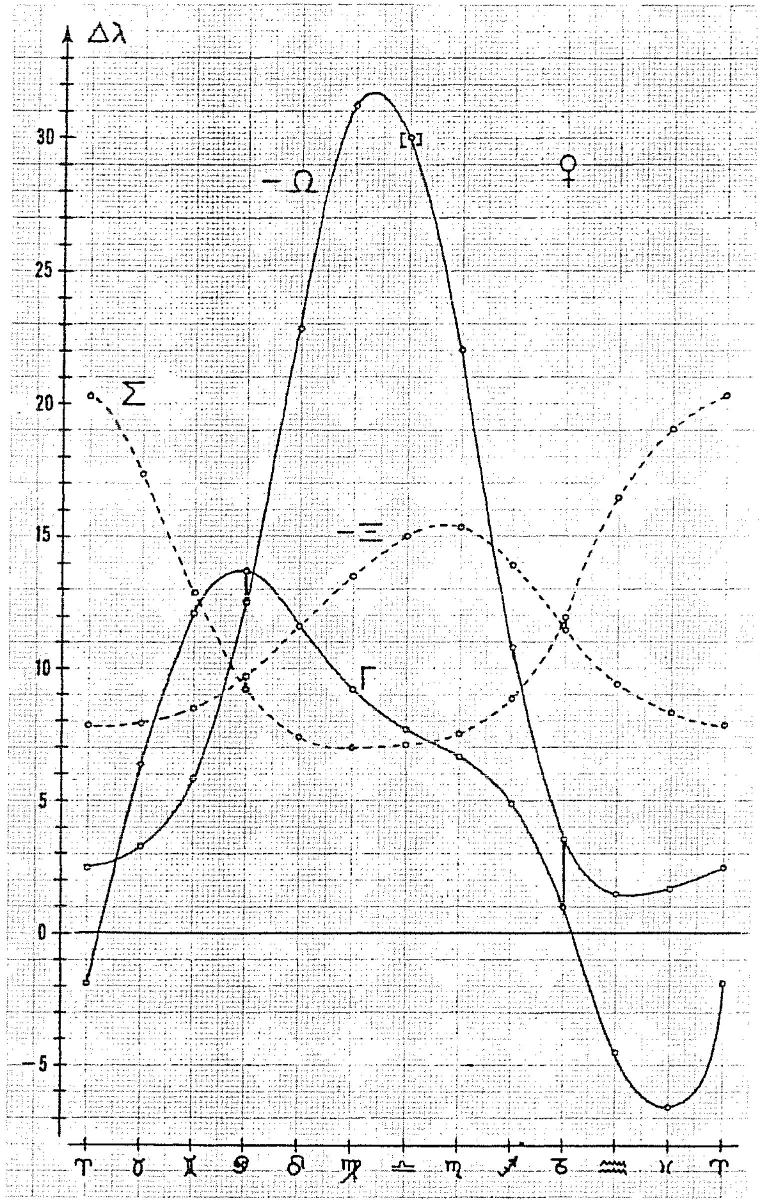

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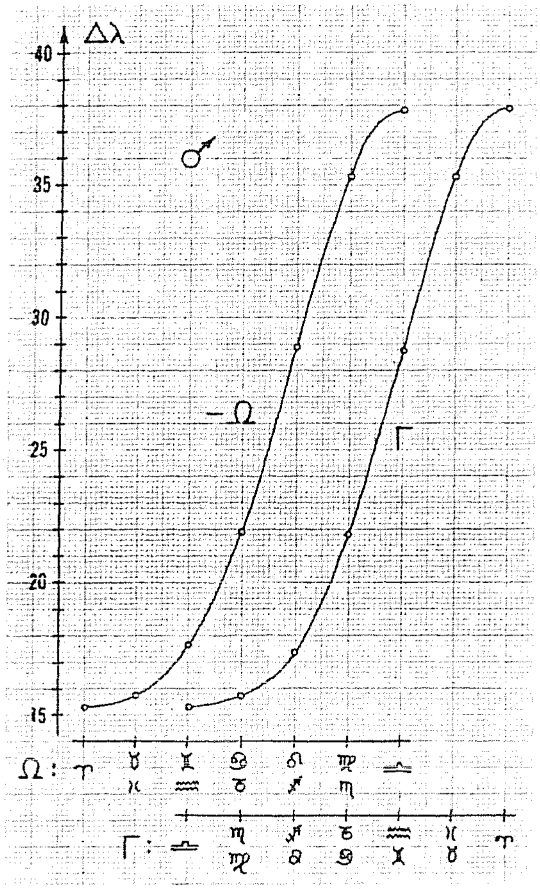

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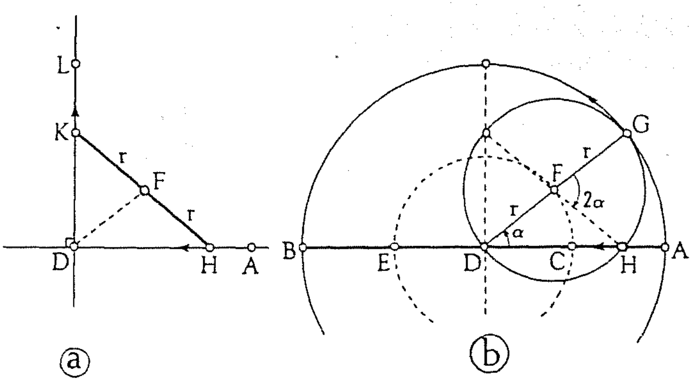

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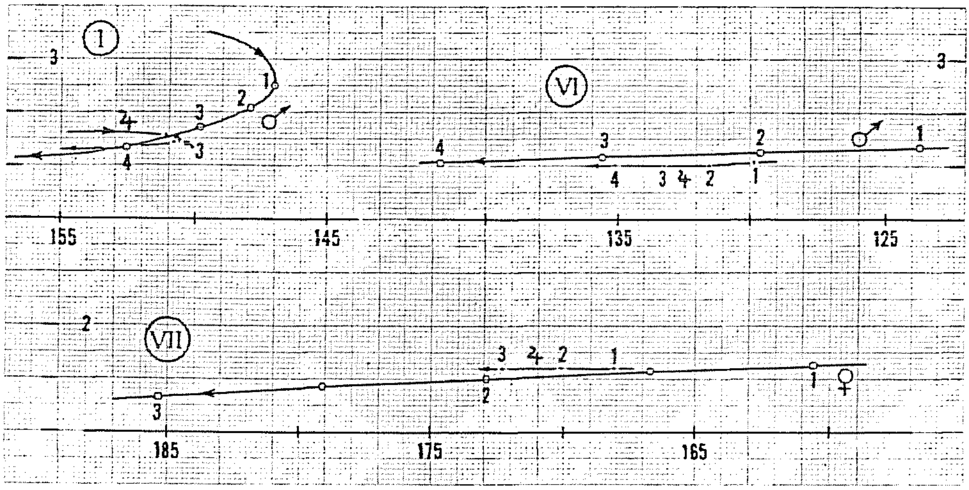
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Fig. 138

Figures to Book VI

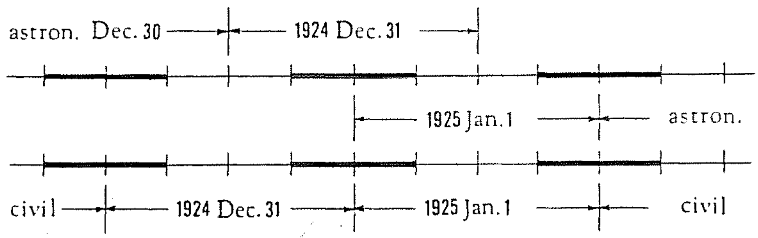


Fig. 1

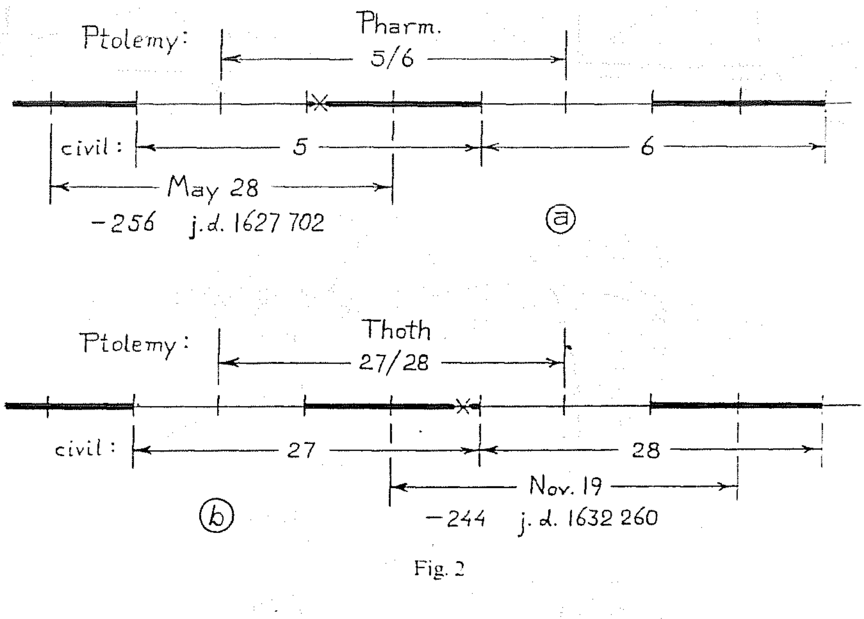


Fig. 2

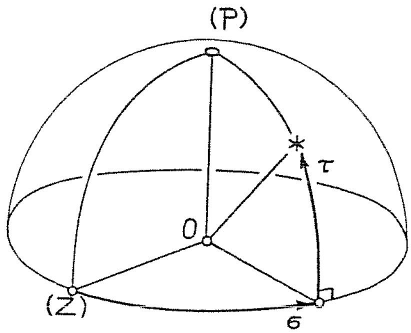


Fig. 3

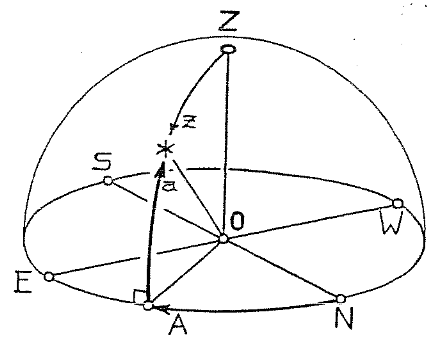


Fig. 4

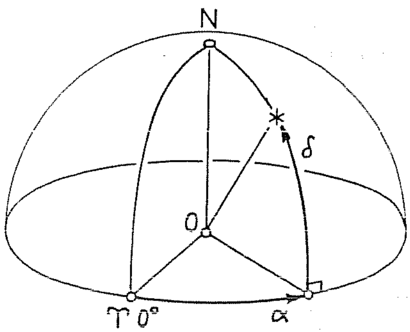


Fig. 5

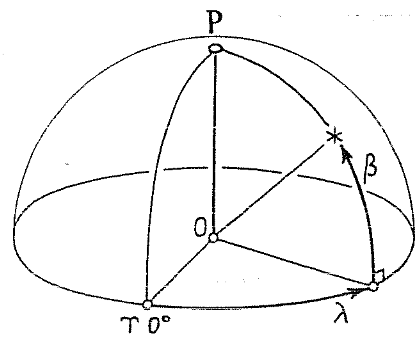


Fig. 6

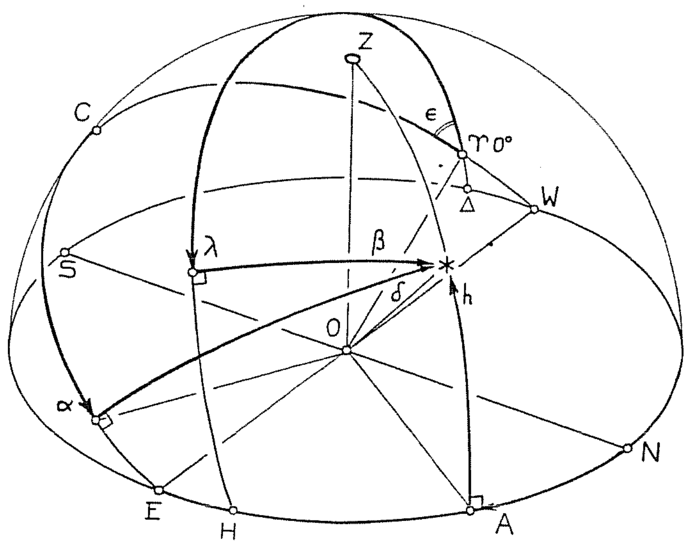


Fig. 7

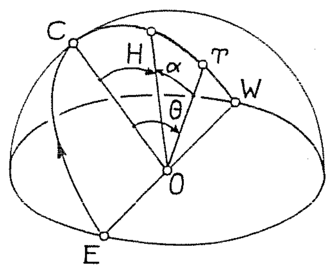


Fig. 8

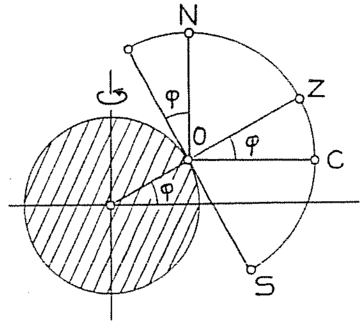


Fig. 9

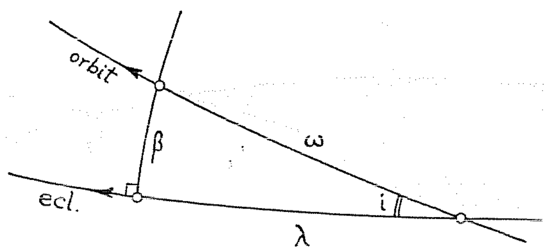


Fig. 10

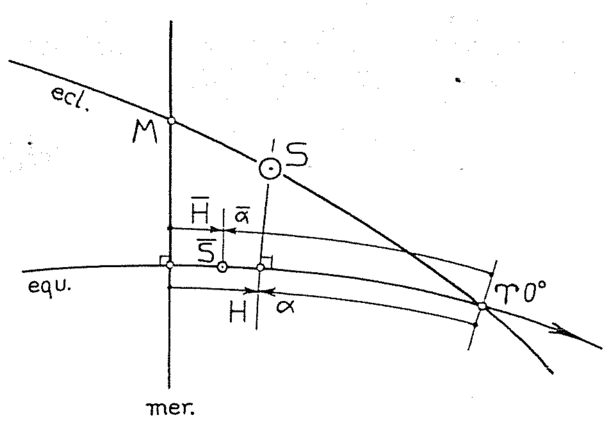
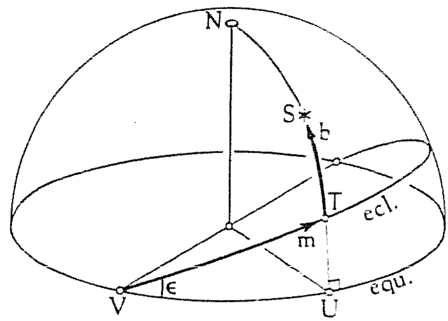


Fig. 11



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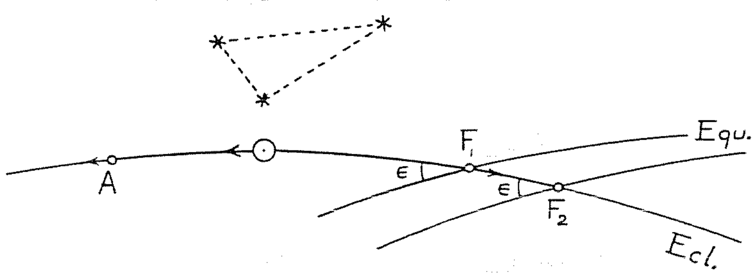

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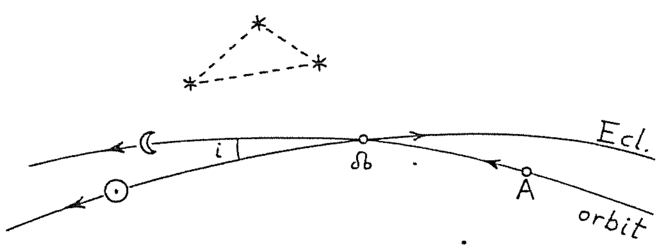

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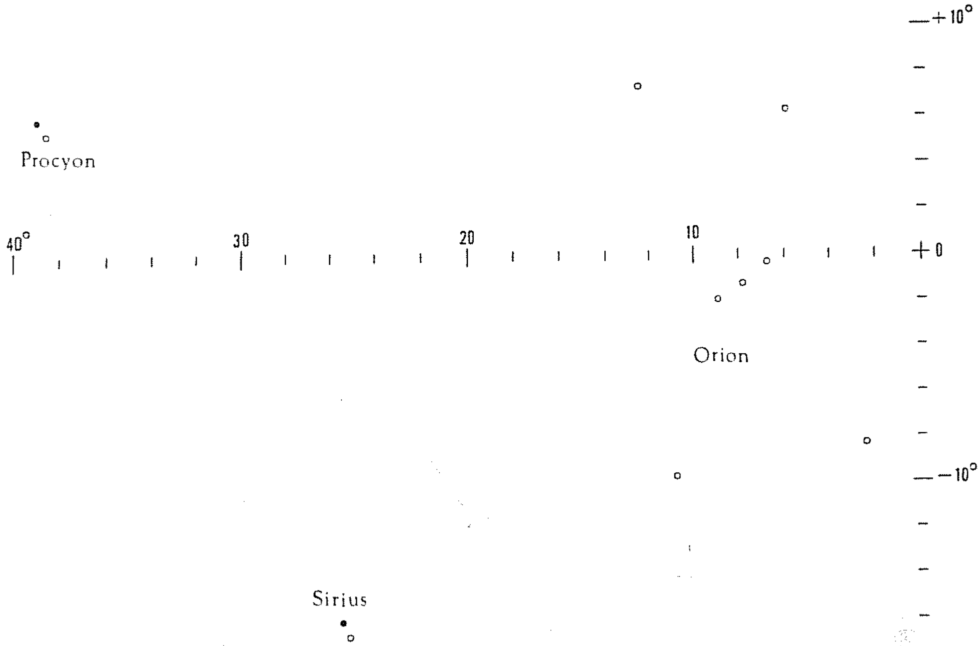


Fig. 15

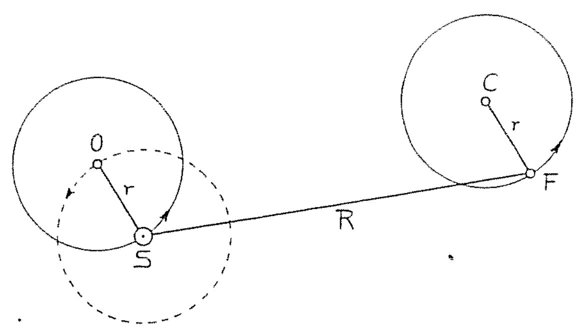


Fig. 16

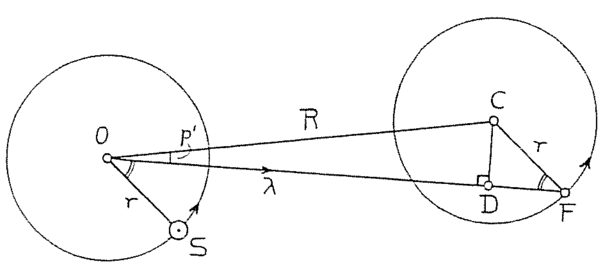
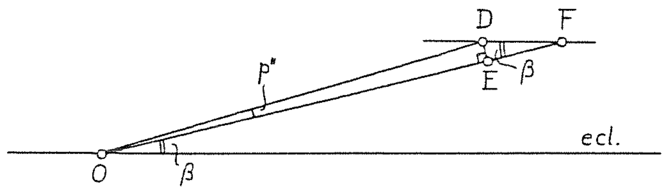


Fig. 17



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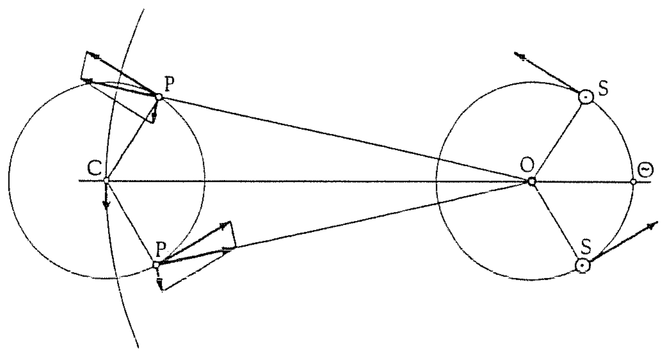

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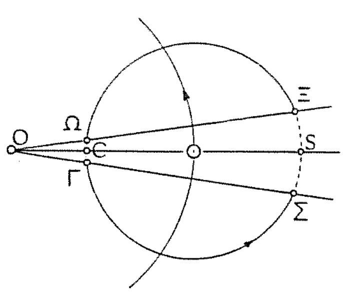


Fig. 20

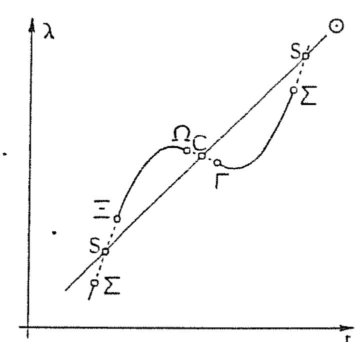


Fig. 21

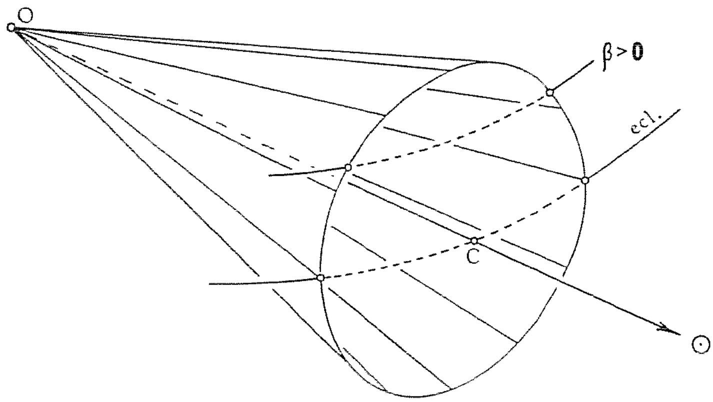


Fig. 22

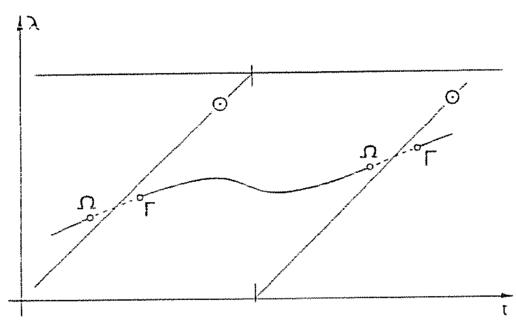
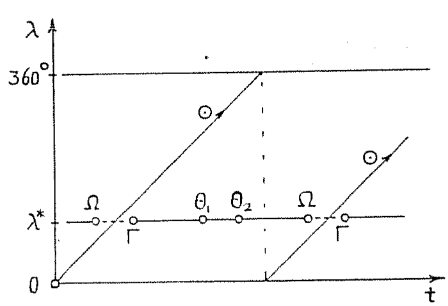


Fig. 23



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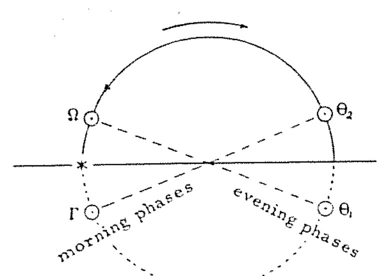


Fig. 25

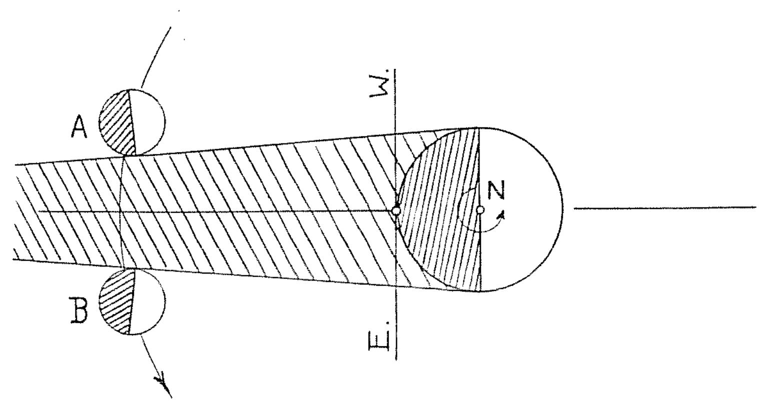
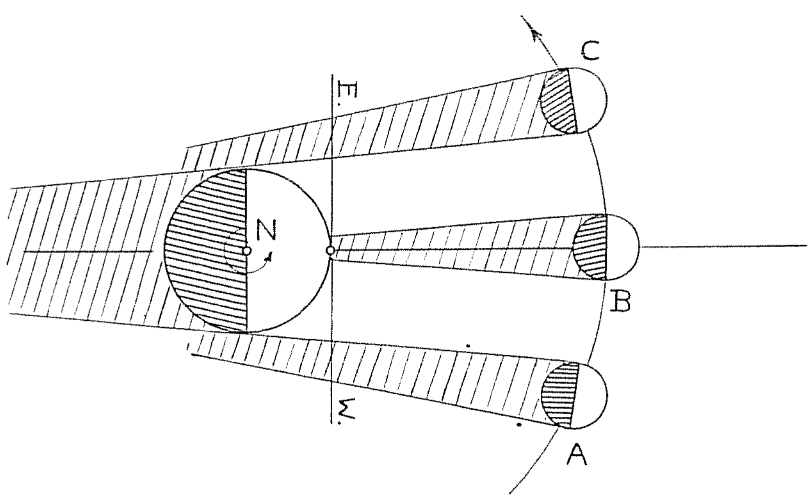


Fig. 26



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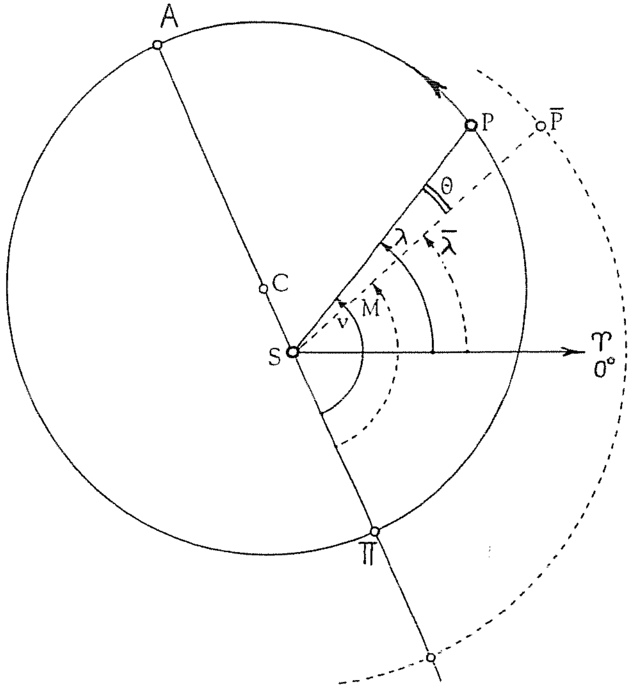

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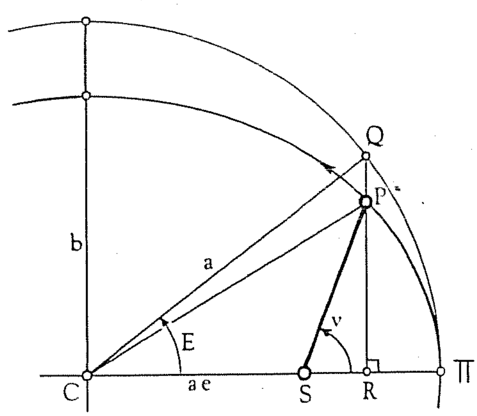

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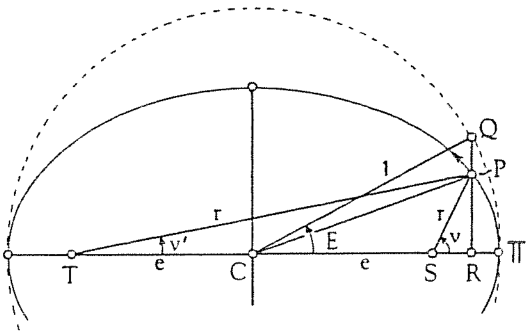

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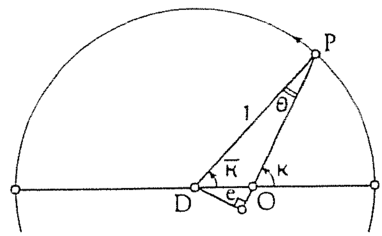

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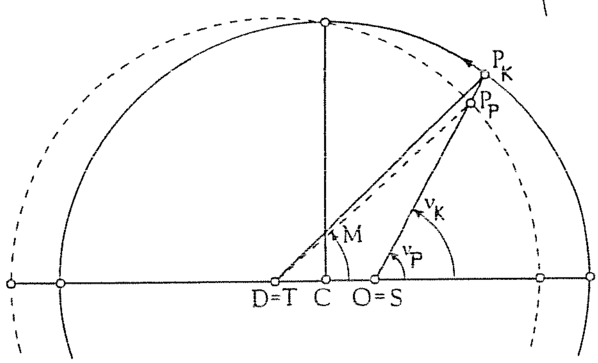

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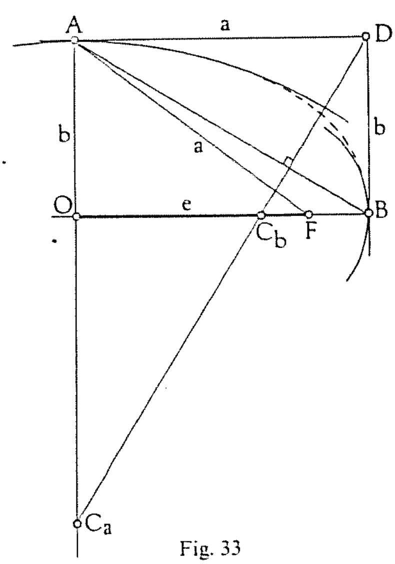

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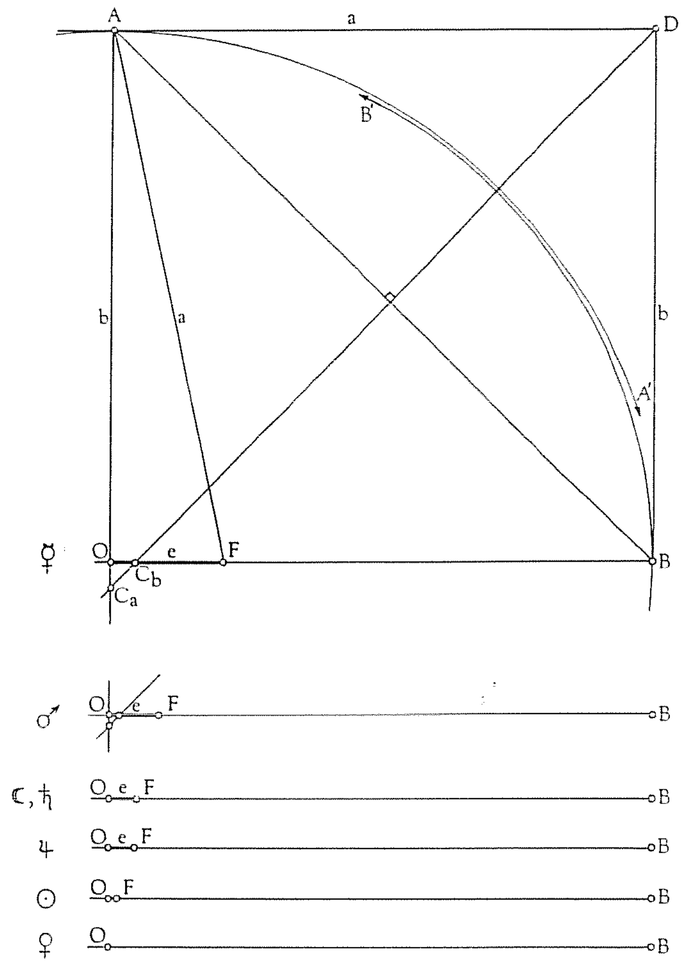

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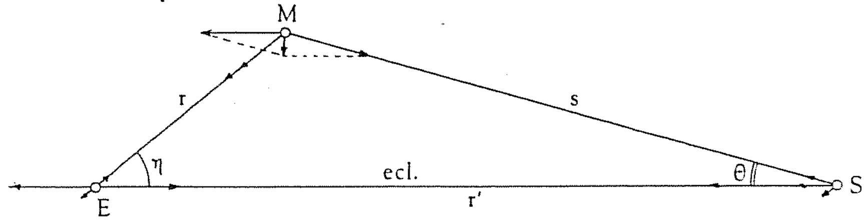

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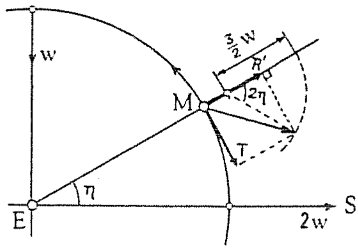


Fig. 36

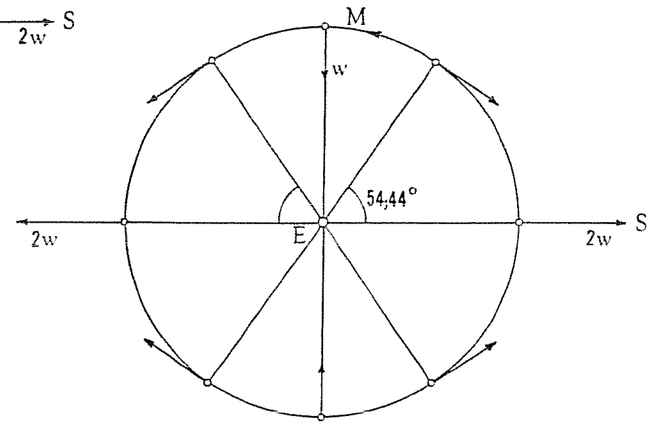
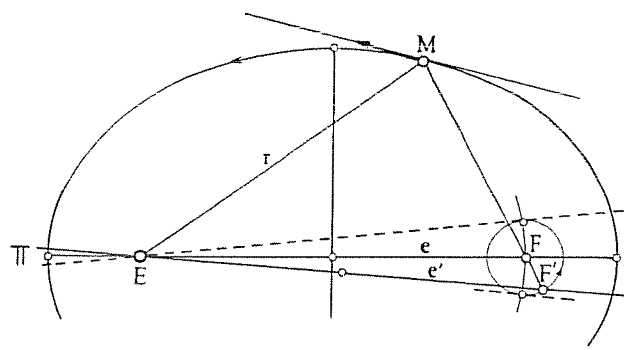


Fig. 37



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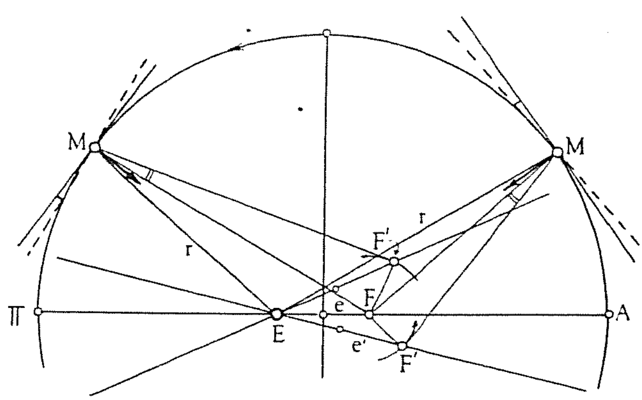

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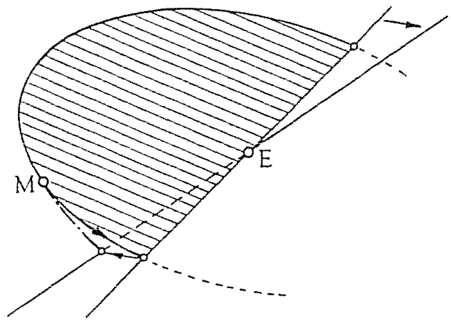


Fig. 40

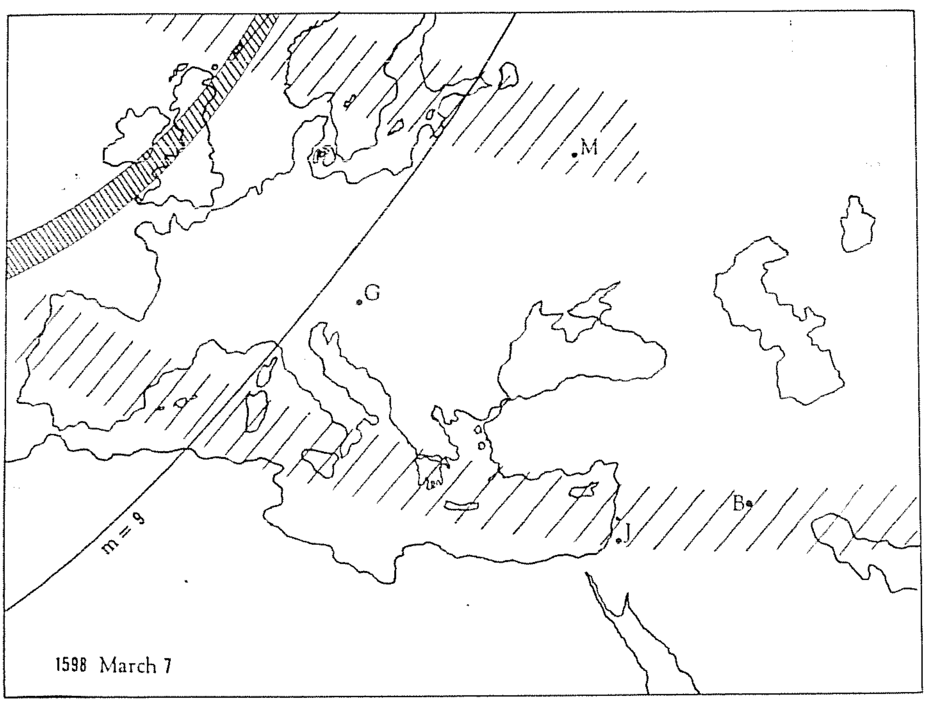
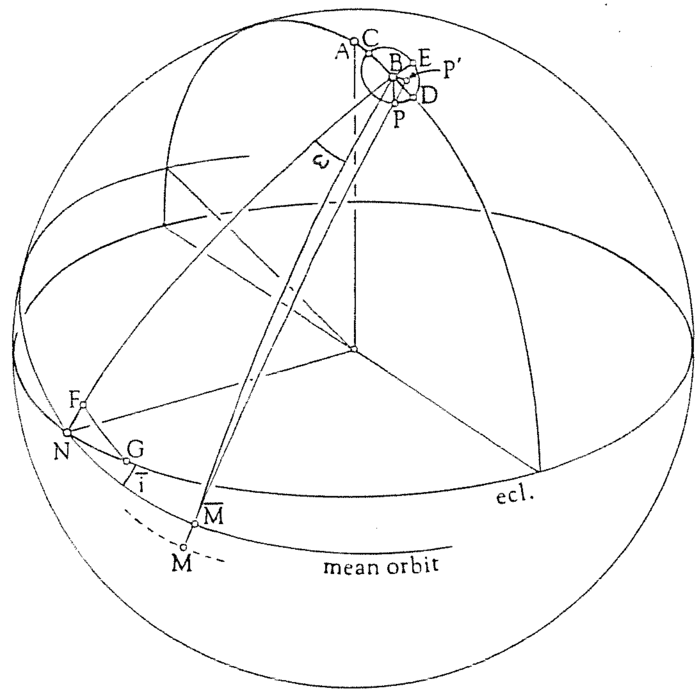


Fig. 41



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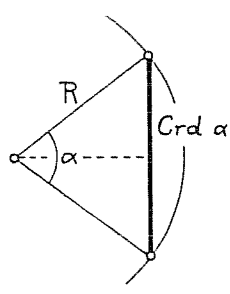
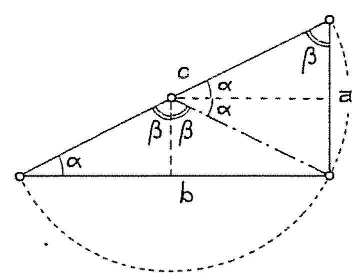


Fig. 43



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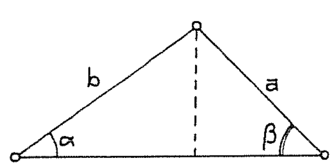


Fig. 45

Plates

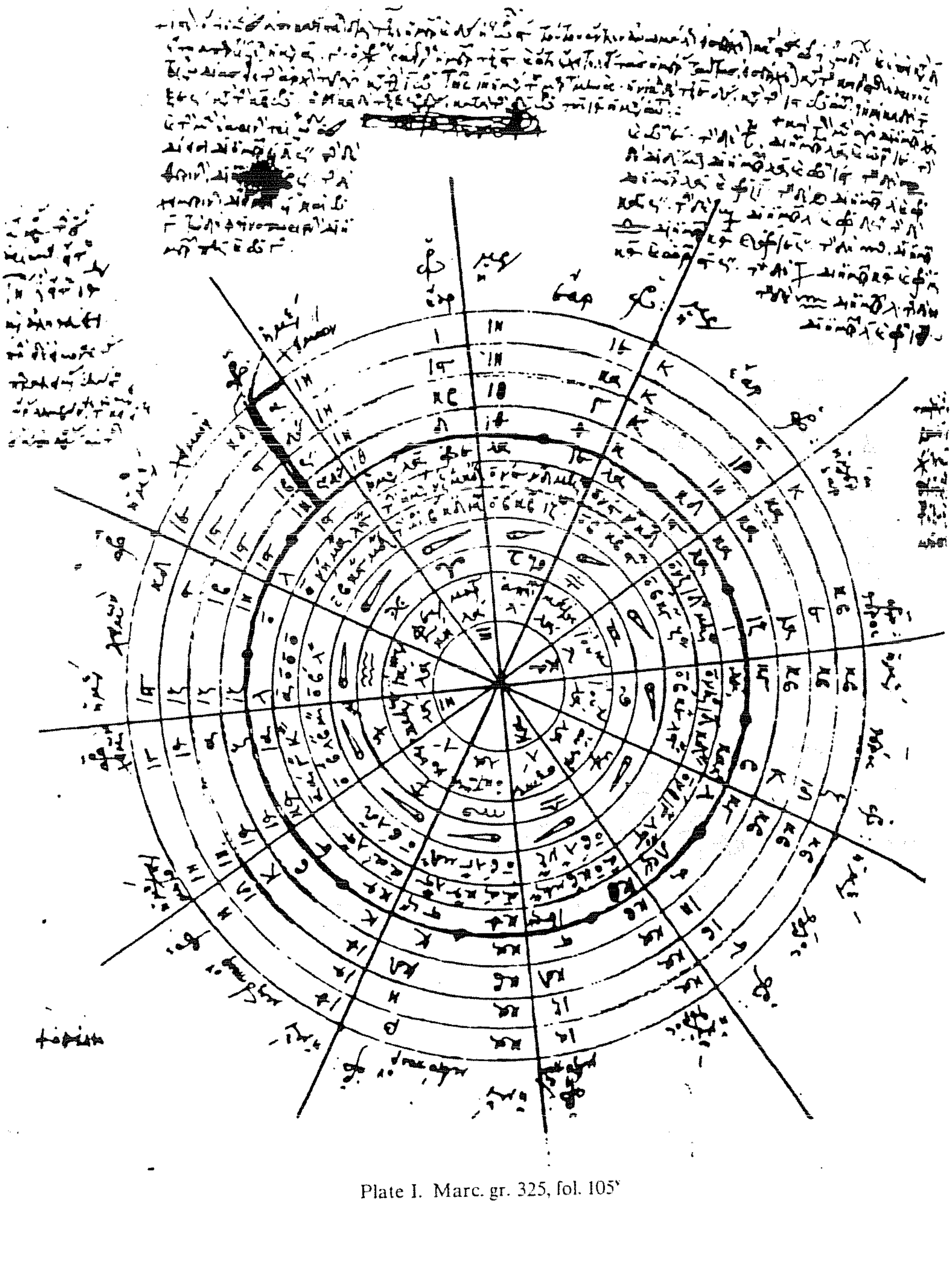


Plate I. Marc. gr. 325, fol. 105$^{\mathrm{v}}$

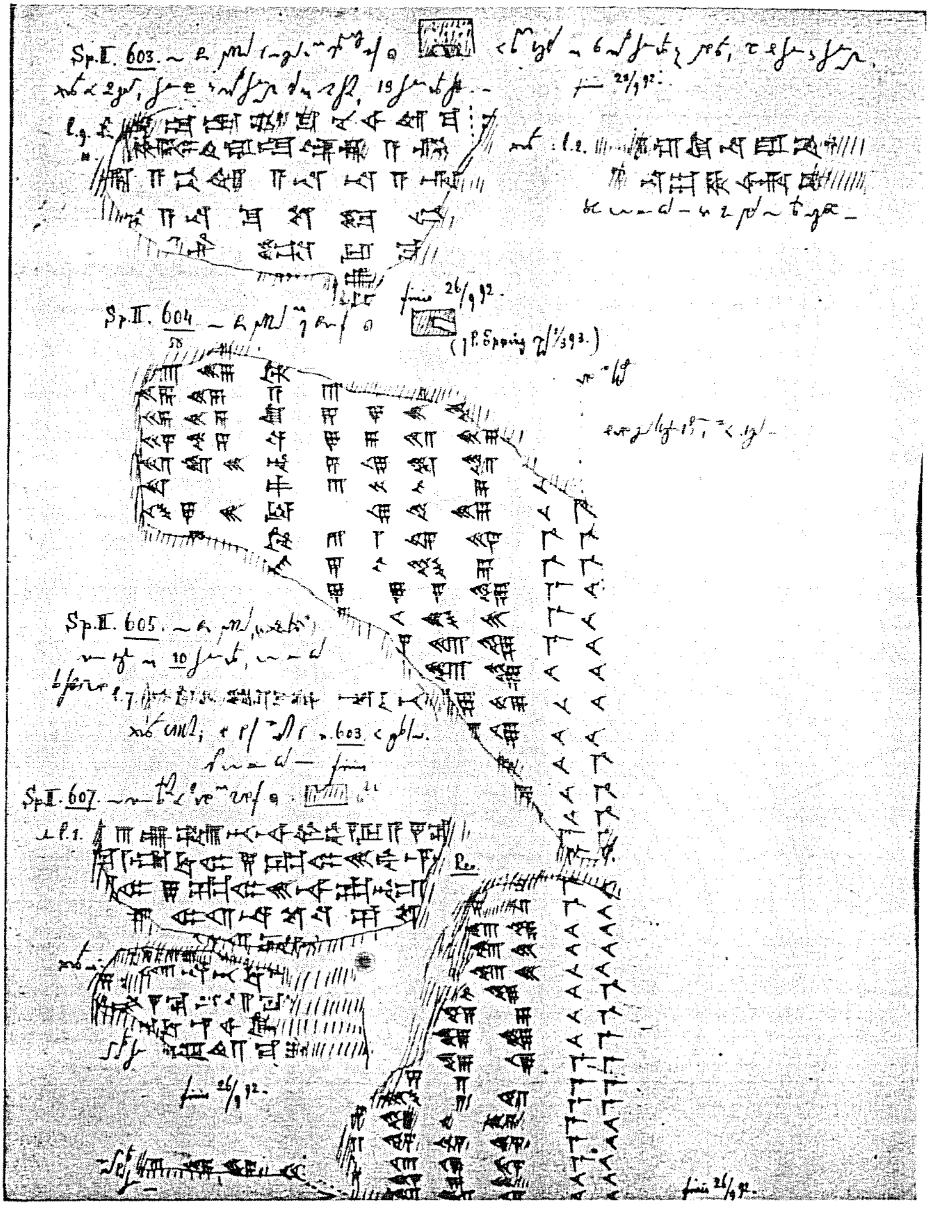


Plate II

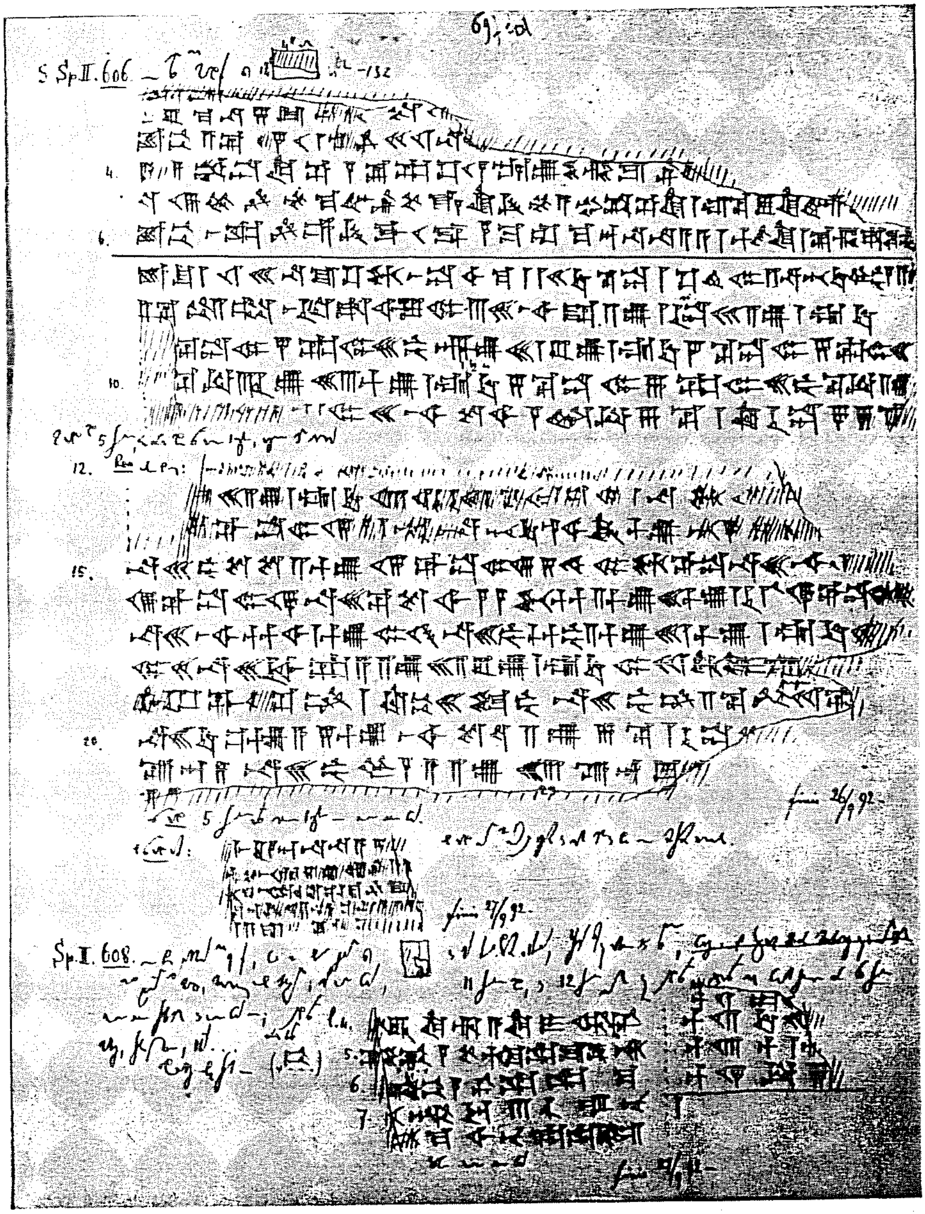
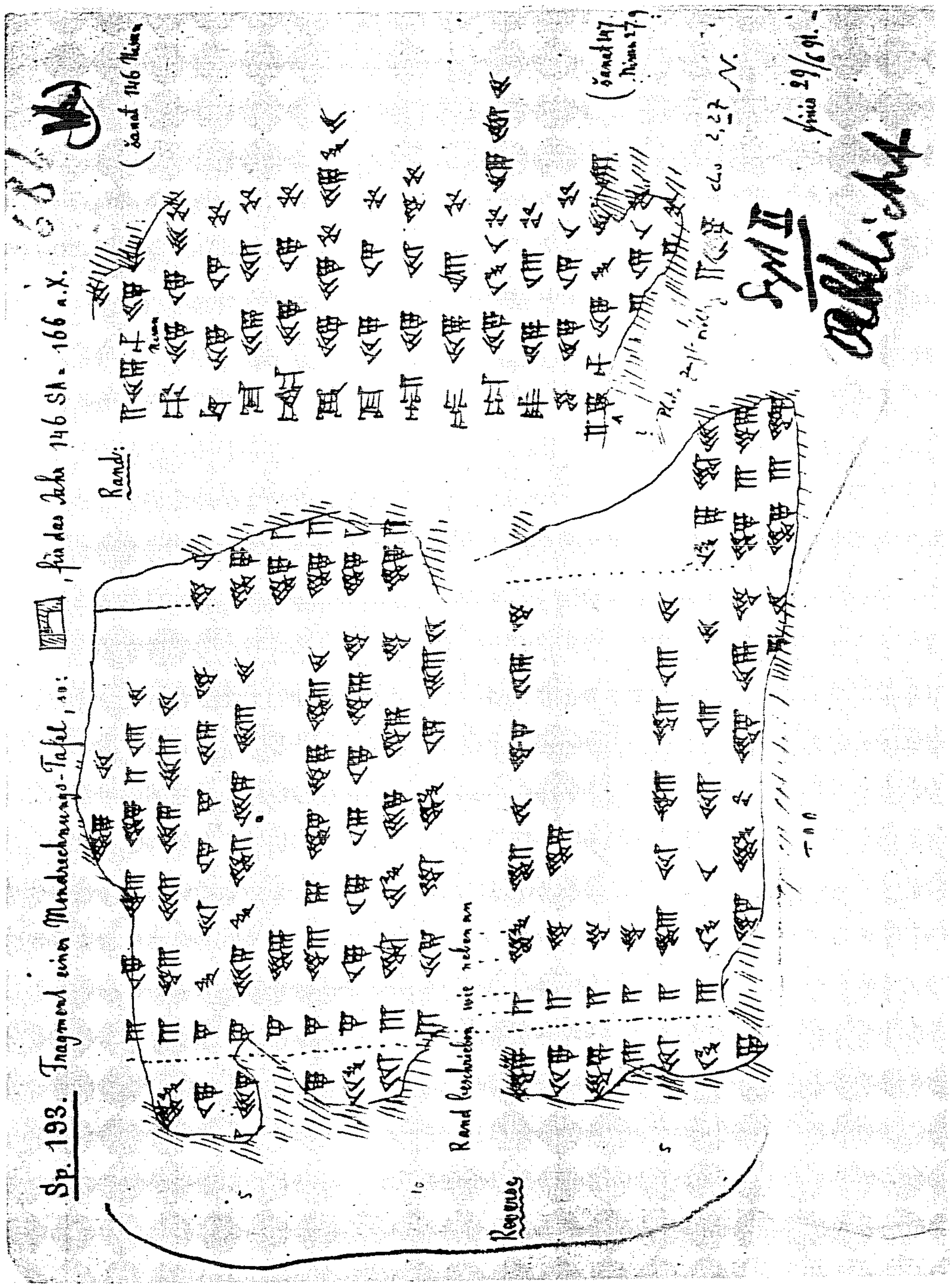


Plate II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Rand beschrieben wie neben an
Revers

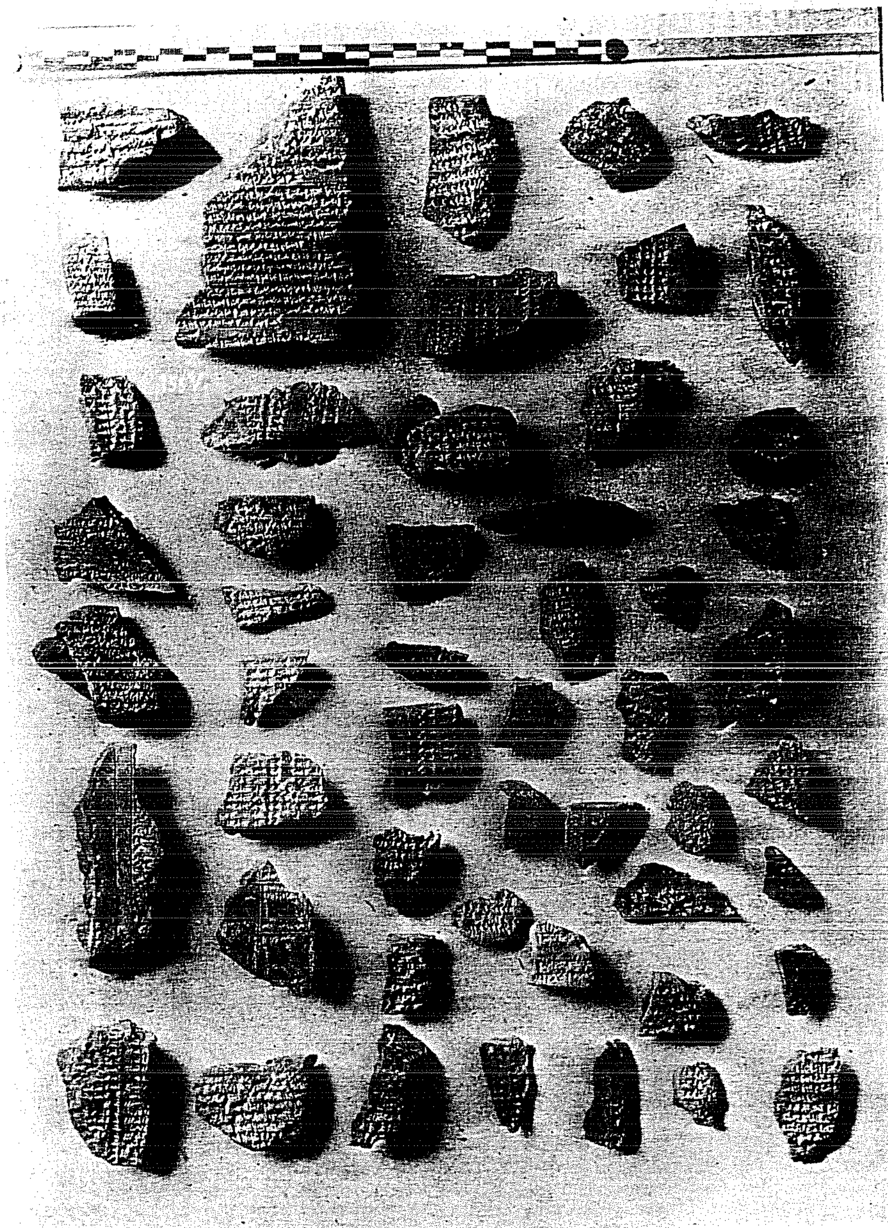

Plate V

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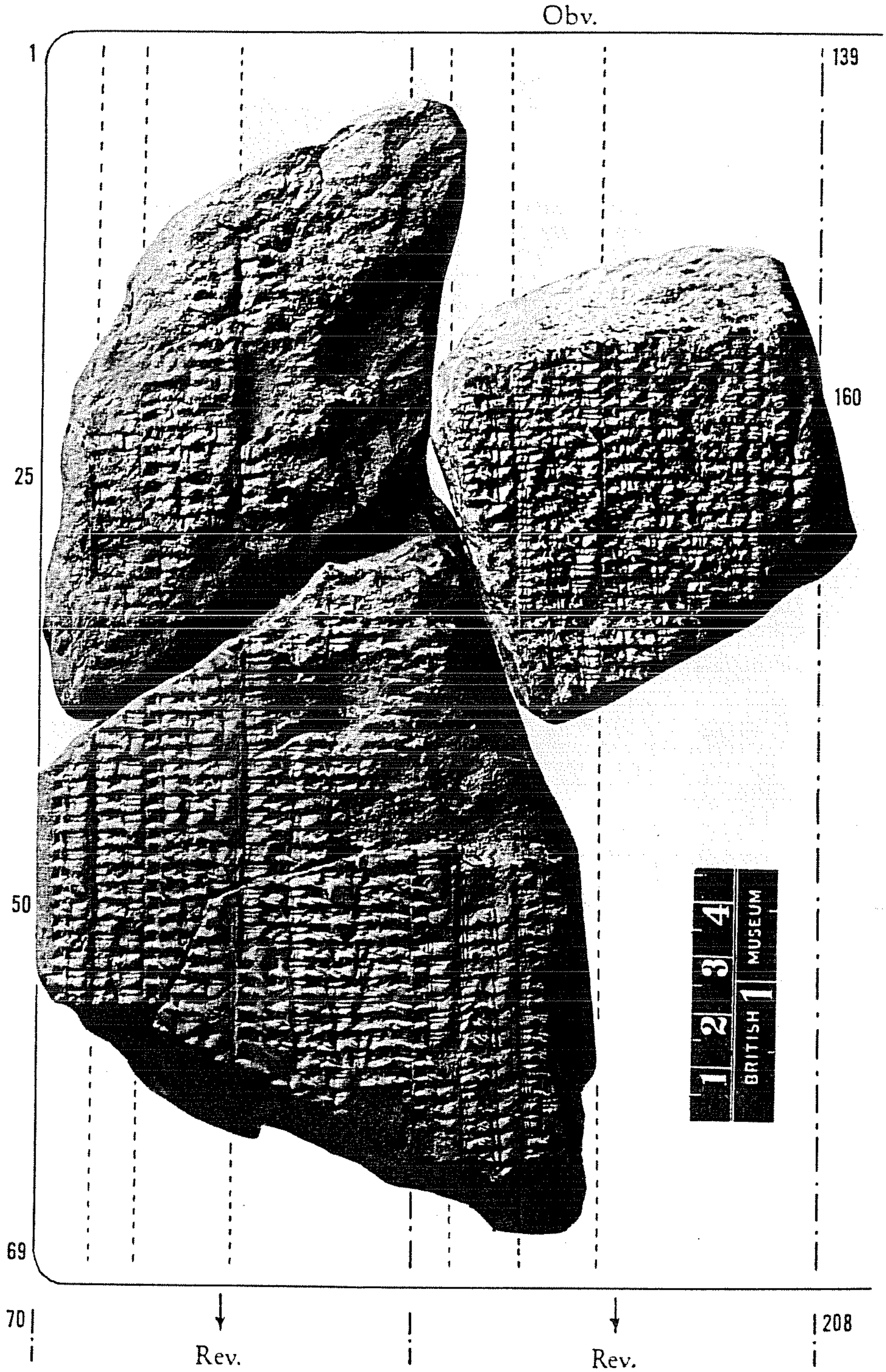

Plate VI

Col. X XI XII XIII XIV

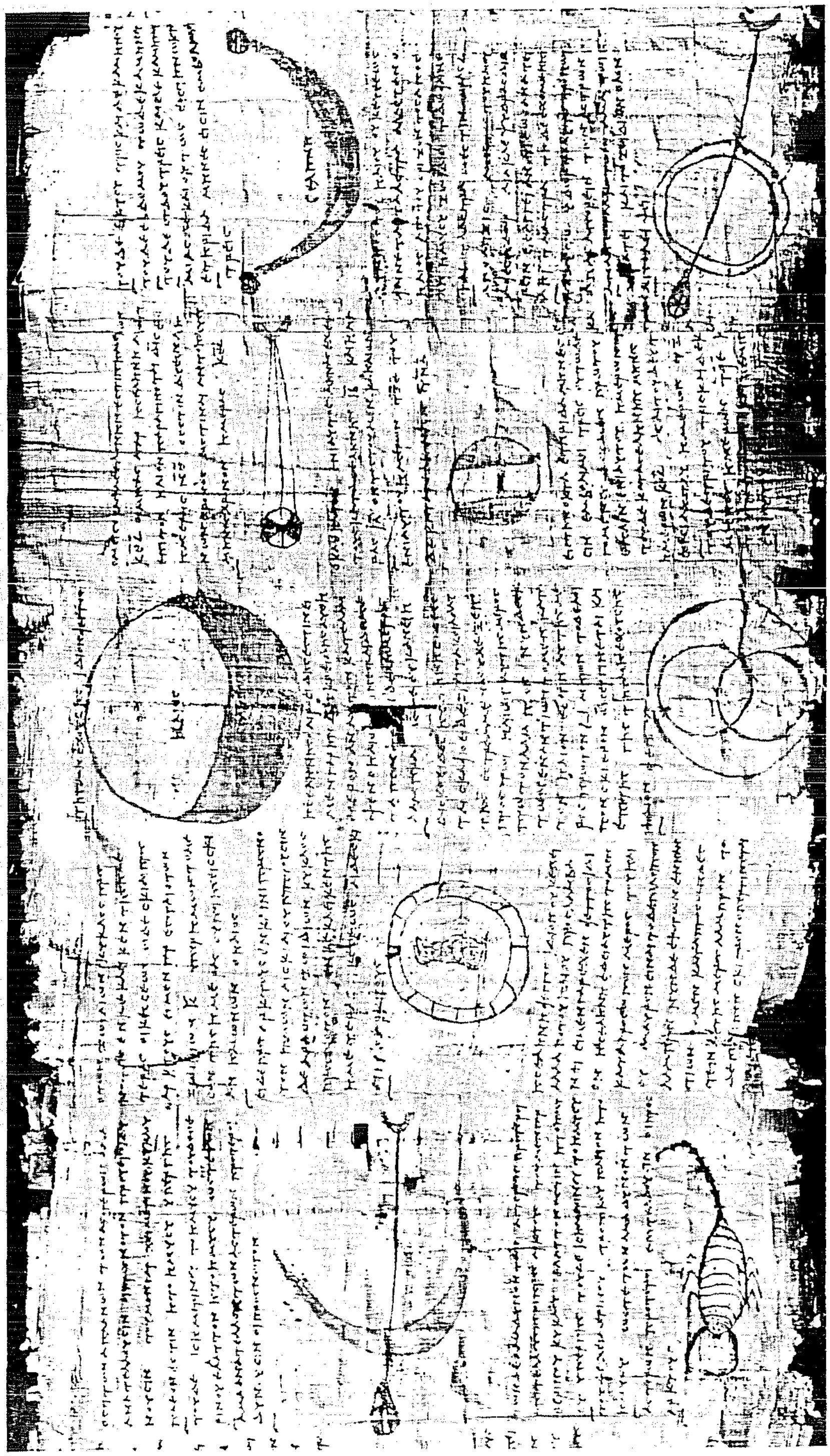

Fig. 11 12 13 14 15 16 17 18 19

Plate VII

Plate VIII. Vat. gr. 204, fol. 61ᵛ

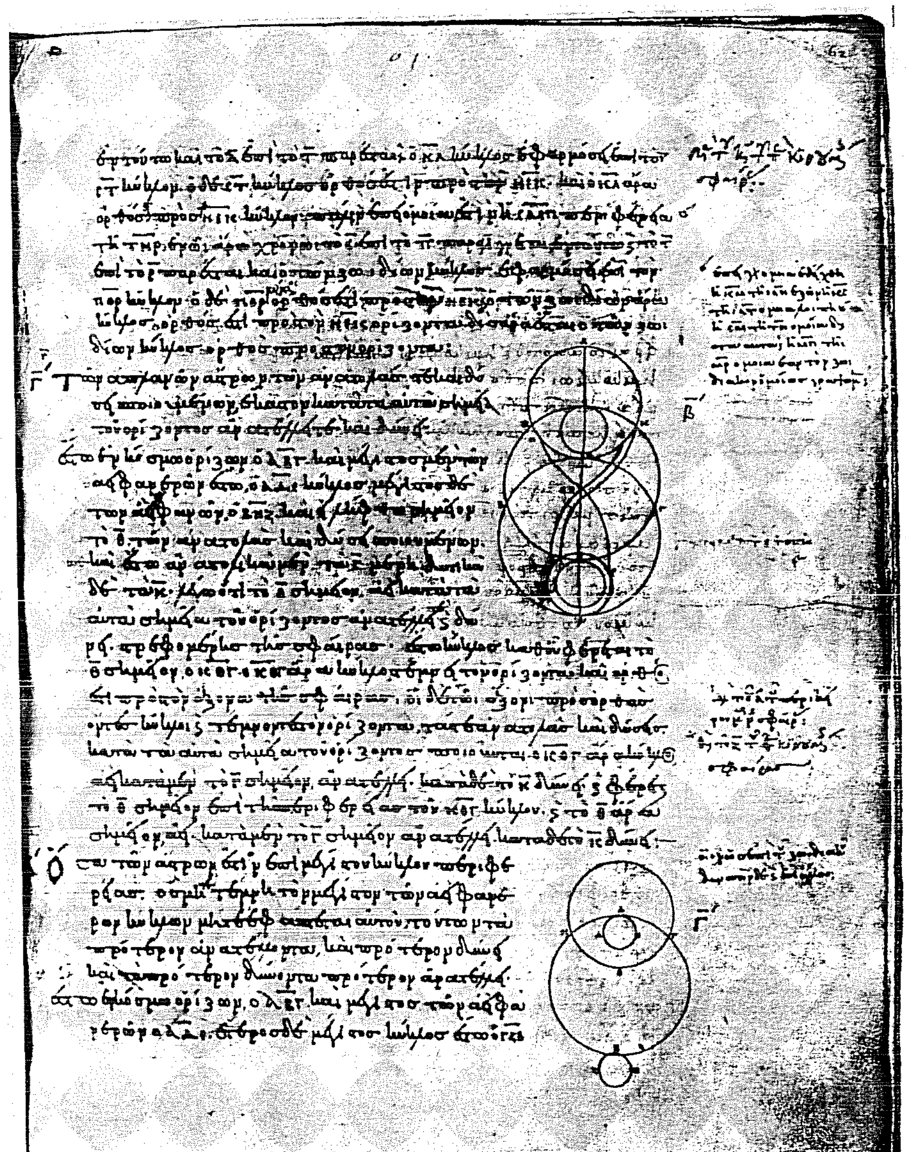

Plate VIII. Vat. gr. 204, fol. 62r

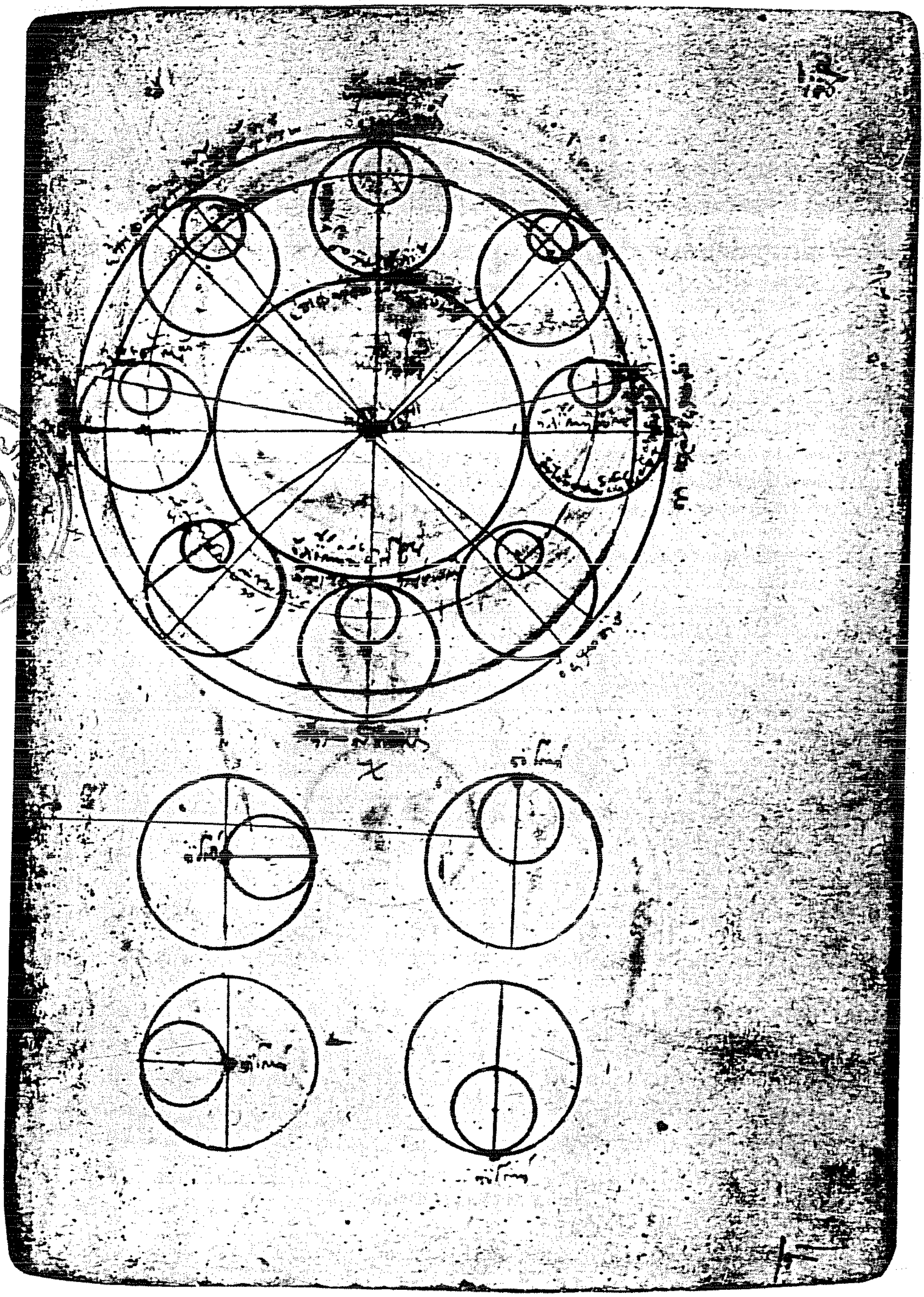

Plate IX. Vat. gr. 211, fol. 116r